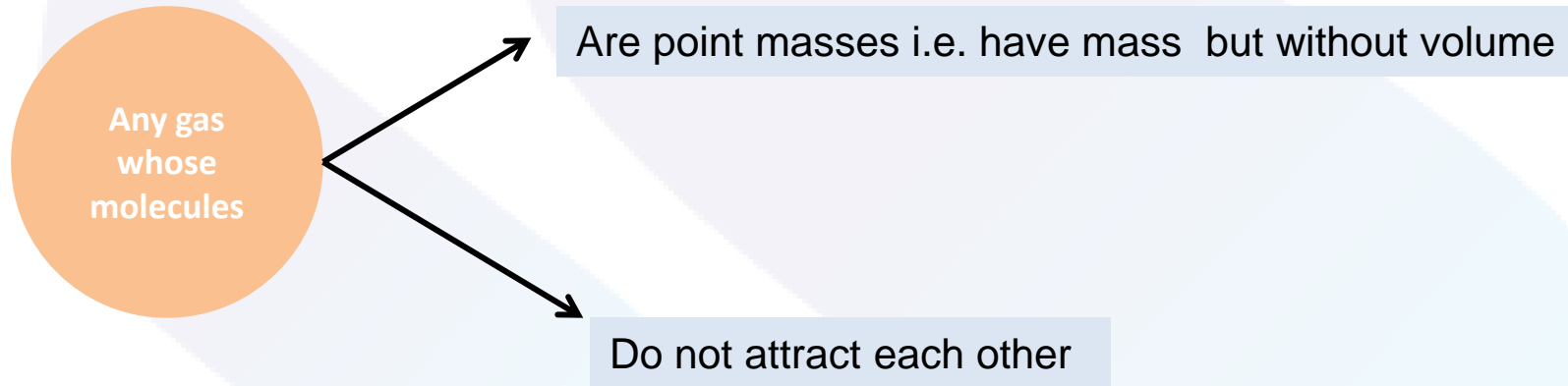


# Gaseous State



By  
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# Concept of an Ideal Gas



**NOTE:** It is a hypothetical concept, and an **actual gas** behaves as ideal gas, most closely **at low pressure & high temperature**

Permanent gases are gases which cannot be liquefied easily. Eg: Hydrogen

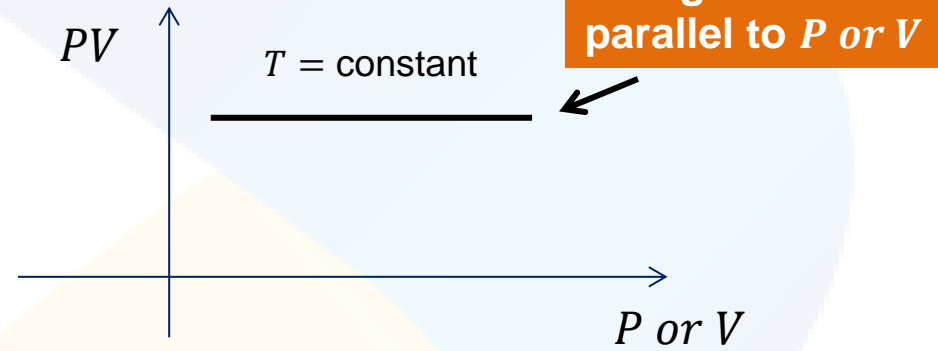
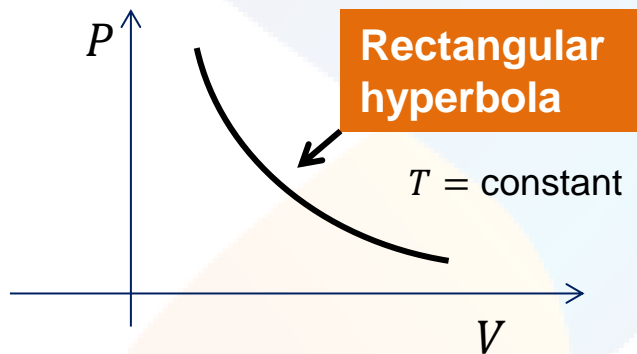
# Boyle's Law

$P$ ,  $V$ ,  $T$  are pressure, volume and temperature of gas

$$V \propto \frac{1}{P}$$

For  $T = \text{constant}$

Isothermal process



Gases, irrespective of their nature obey this law

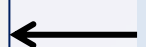
# Charle's Law

$P$ ,  $V$ ,  $T$  are pressure, volume and temperature of gas

$$V \propto T$$

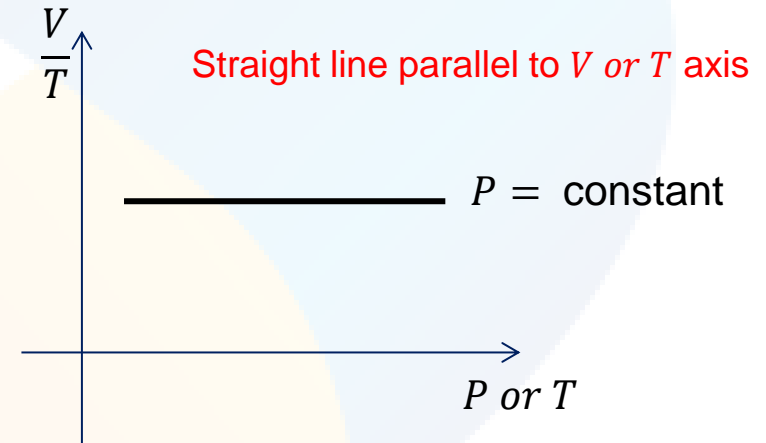
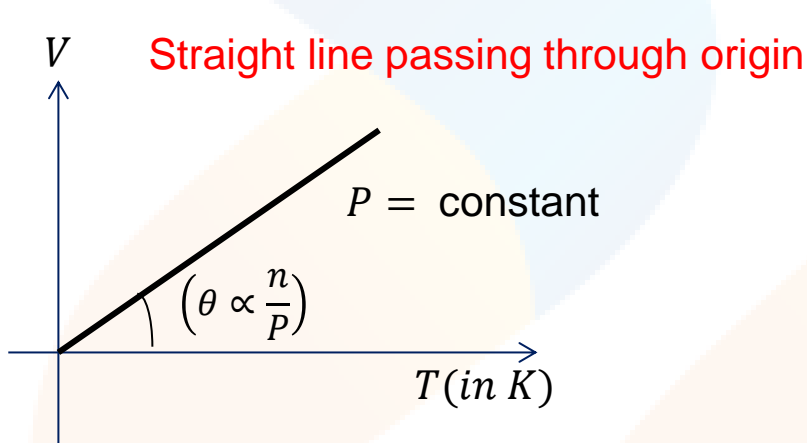


For  $P = \text{constant}$



Isobaric process

$$\frac{V}{T} = \text{constant}$$



All gases irrespective of their nature, obey this law



# Gay Lussac's Law

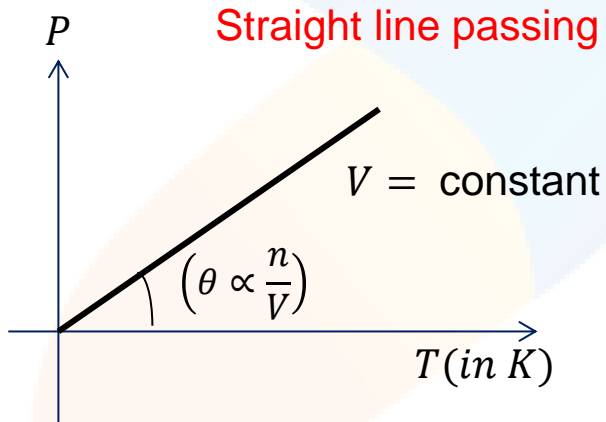
$P$ ,  $V$ ,  $T$  are pressure, volume and temperature of gas

$$P \propto T$$

For  $V = \text{constant}$

Isochoric process

$$\frac{P}{T} = \text{constant}$$

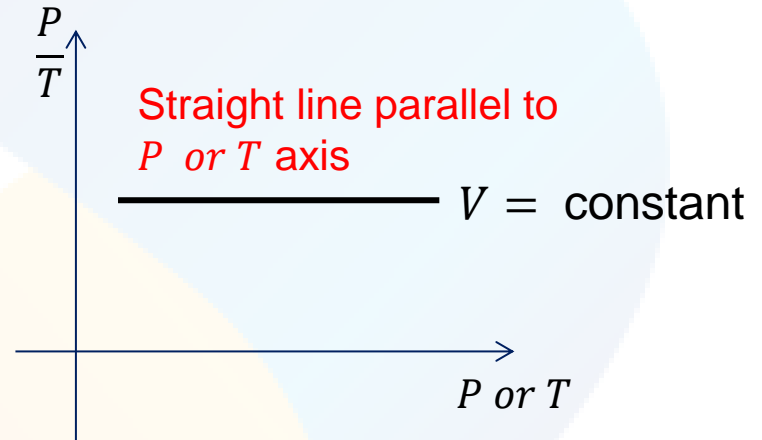


Straight line passing through origin

$V = \text{constant}$

$$(\theta \propto \frac{n}{V})$$

$T$  (in  $K$ )



Straight line parallel to  $P$  or  $T$  axis

$V = \text{constant}$

$P$  or  $T$

All gases irrespective of their nature, obey this law

# Avogadro's Law

At same temperature and pressure



Equal volumes of all gases contain equal number of molecules



Application of the concept is in getting volume relationships of reactants and products in a chemical reaction

$$\frac{V}{n} = \text{constant}$$

$$\frac{p_1 \cdot V_1}{T_1 \cdot n_1} = \frac{p_2 \cdot V_2}{T_2 \cdot n_2} = \text{constant}$$





# Ideal Gas Equation

$$P.V = n.R.T = \frac{m}{M}.R.T$$

Where ,

$n$  = number of moles of the gas

$$= \frac{m}{M} = \frac{\text{total mass of gas}}{\text{molecular mass of gas}}$$

$R$  = universal gas constant

$$= 8.31 \frac{J}{mol} - K$$

$$= 2.0 \frac{cal}{mol} - K$$

**Note that**, all 4 gas laws, i.e. Charle's law, Boyle's law, Gay-Lussac's law and Avogadro's law can be derived from Ideal gas equation

# Charles's Law in Modified Form

On heating a given mass of gas at constant  $P$ .

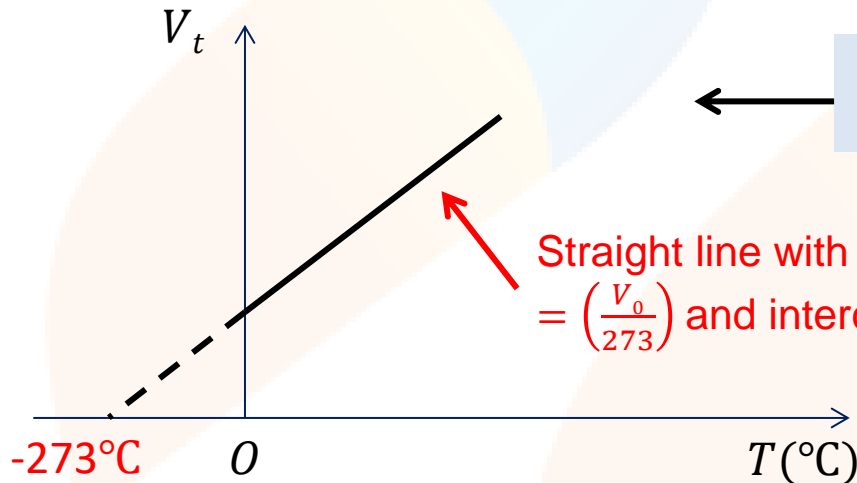
$V$  of gas increases by a fraction  $\alpha$ , for each  $1^\circ\text{C}$  rise in temperature.

$$V_T = V_0 + V_0 \cdot \alpha \cdot T$$

$\alpha$  is volume coefficient of gas

**NOTE:** for all gases, experiment value of  $\alpha = \frac{1}{273}$  per  $^\circ\text{C}$

$$V_t = V_0 \left( 1 + \frac{T}{273} \right)$$



$V_T = 0$ , at  
 $T = -273^\circ\text{C}$

# Gay Lussac's Law of pressure in Modified Form

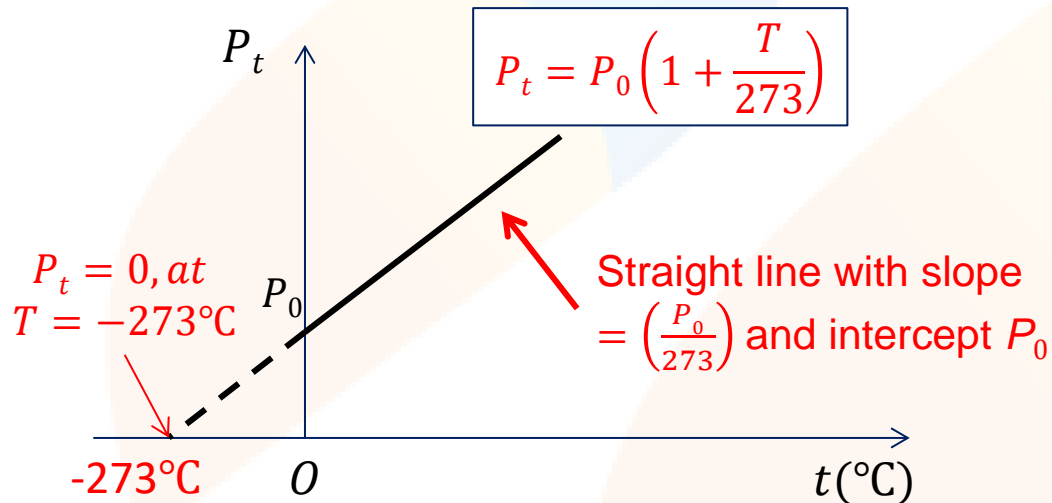
On heating a given mass of a gas at constant volume

Pressure of gas increases by a fraction  $\beta$ , for each  $1^\circ\text{C}$  rise in temperature

$$P_t = P_0(1 + \beta T)$$

$\beta$  is pressure coefficient of gas

**NOTE:** for all gases, experimental value of  $\beta = \frac{1}{273}$  per  $^\circ\text{C}$





# Density of a Gas ( $\rho$ )

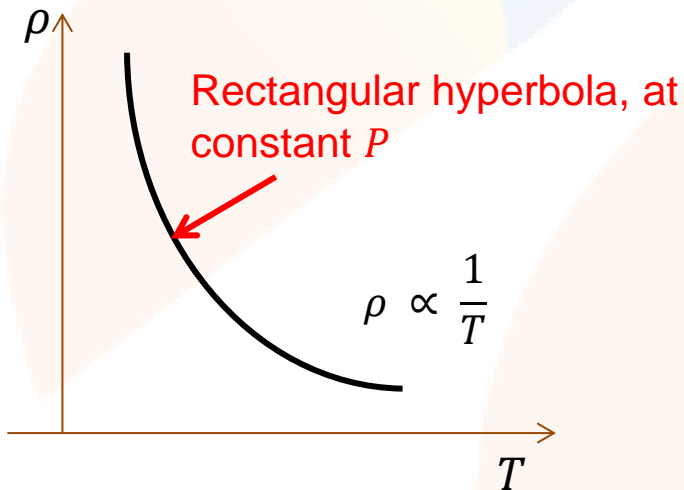
$$P.V = n.R.T = \frac{m}{M}.R.T$$

→ Ideal gas equation

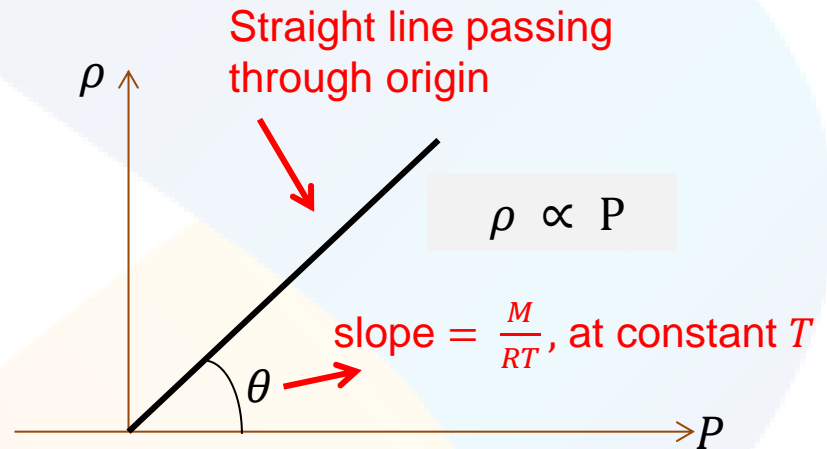
$$\frac{m}{V} = \rho = \frac{P.M}{R.T}$$

$$\rho = \frac{P.M}{R.T}$$

$\rho$  vs.  $T$



$\rho$  vs.  $P$



# Assumptions of Kinetic Theory of Gases

1) All gases are made of molecules moving randomly in all directions

2) Size of molecules  $\ll$  average separation between the molecules

3) Molecules exert no force on each other, except on wall of container during collision

4) Molecules obey Newton's law of motion

5) When a gas is left for sufficient time, it comes to a steady state

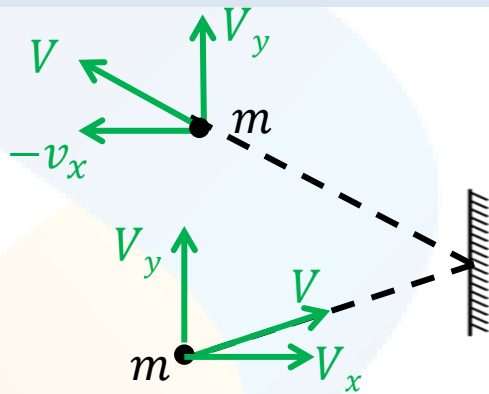
It means density and distribution of molecules with different velocities are independent of position, direction & time

It is valid only if the number of molecules is very large

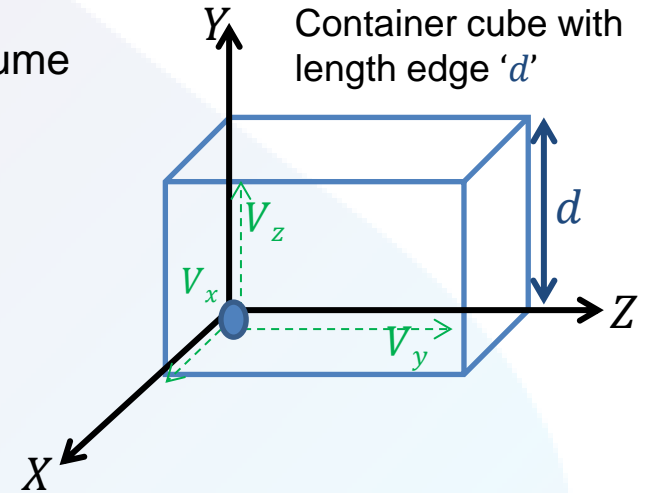
# Impulse of an Ideal Gas

Let an ideal gas consist of  $N$  molecules in a container of volume  $V$ , with mass  $m$  of each molecule

$$\begin{aligned} \text{Change in momentum } (\Delta p_x) &= -m \cdot v_x - (m \cdot v_x) \\ &= -2 \cdot m \cdot v_x \end{aligned}$$



**Note that  $x$  component of momentum is reversed while  $y$  component remains unchanged**



$$\Delta p_x = F \cdot \Delta t = -2 \cdot m \cdot v_x$$

$$\Delta t \text{ is time interval between 2 collisions} = \frac{2 \cdot d}{v_x}$$

$F$  is the magnitude of average force exerted by wall on the molecules in time  $\Delta t$





# Translational K.E. of Ideal Gas

$$P.V = \frac{2}{3} N \left( \frac{1}{2} m.V^2 \right)$$

← From pressure of an Ideal gas

Avogadro number

Comparing with  $P.V = n.R.T$  →

$$T = \frac{2}{3} \left( \frac{N_A}{R} \right) \left( \frac{1}{2} m.V^2 \right), \text{ where } N_A = \frac{N}{n} \quad \dots 1$$

Boltzmann constant.  $k = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \quad \dots 2$

From 1 and 2

Average translational K.E. per molecule ←

$$\frac{1}{2} mV^2 = \frac{3}{2} k.T$$

For 1 mole of gas, total translational K.E =  $\frac{3}{2} (k.N_A)T$  →

$$\frac{3}{2} R.T$$

Since,  $v_x^2 = \frac{1}{3} v^2 = v_y^2 = v_z^2$  →

$$\frac{1}{2} m.V_x^2 = \frac{1}{2} k.T$$

**NOTE:** It means in each translational degree of freedom one gas molecule has an energy  $\frac{1}{2} kT$

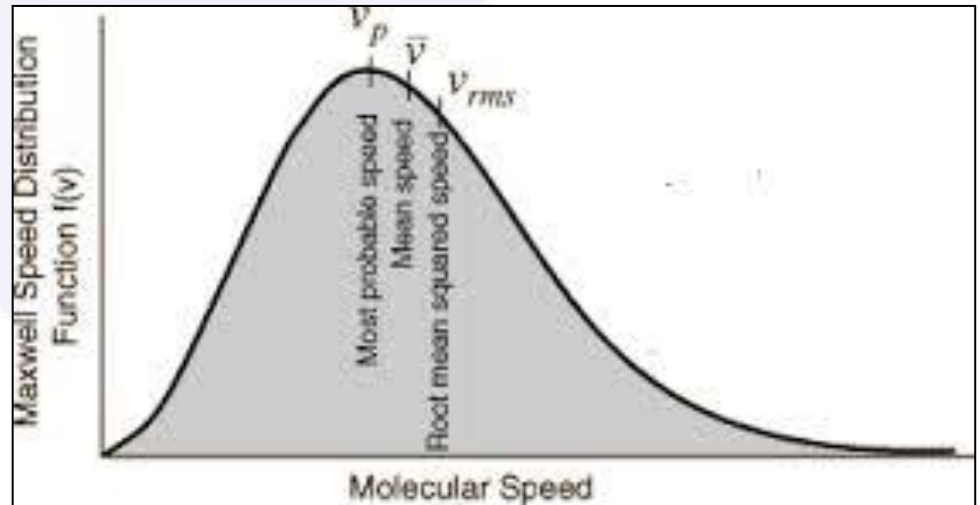
# Relation Between Different Speeds of Gas Molecules

$$V_{rms} = \sqrt{\frac{3.R.T}{M}}$$

$$V_{av} = \sqrt{\frac{8.R.T}{\pi M}}$$

$$V_{mp} = \sqrt{\frac{2.R.T}{M}}$$

$$V_{rms} > V_{av} > V_{mp}$$

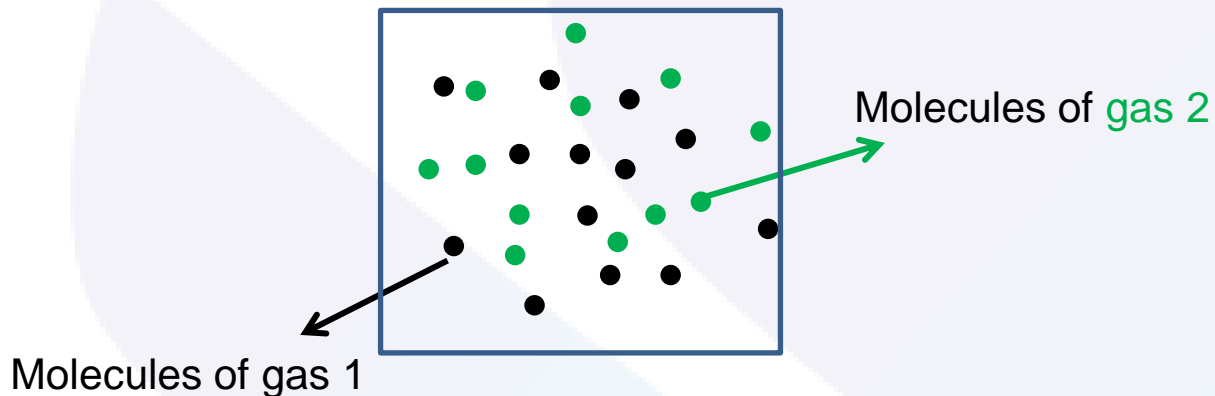


**NOTE:**  $V_{mp}$  is least can be derived logically as most of the particles will be lowest speed

$$V_{rms} : V_{av} : V_{amp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$$



# Dalton's Law of Partial Pressure



Assume that **gas 1** and **gas 2** do **not interact chemically** with one another

Total pressure  $P = P_1 + P_2$

$P_1$  and  $P_2$  are **partial pressure** of gas 1 and **gas 2** respectively

**NOTE:** This can be written for **n gases** also as  $P_{total} = P_1 + P_2 + P_3 + \dots + P_n$   
If the gases are **not interacting chemically**

# Van der Waal's Equation

Real gases deviate from Ideal gas behavior

Finite size of molecules

Intermolecular attraction

## Introducing correction terms

$$\left(P + \frac{a}{V^2}\right)(V - b) = R.T$$

Where 'a' and 'b' are constants

Correction for finite size: ***b***

Which depends on effective size and number of molecules of the gas

Correction for intermolecular attraction: ***a***

Which is to compensate for decrease in momentum of particles of a gas

The more the value of *a*, the more is the ease of the liquification of the gas

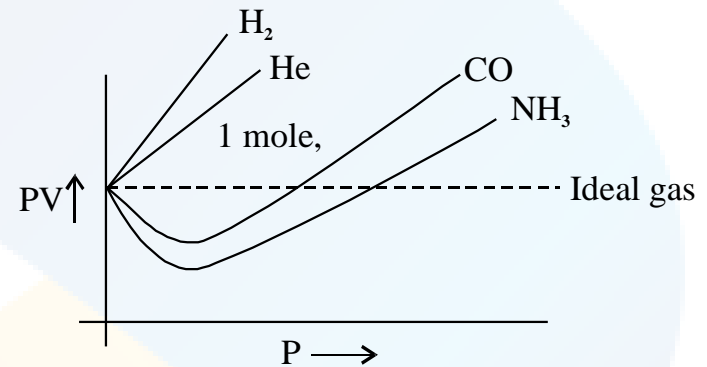


# Compressibility Factor (Z)

- None of the **gas in nature is ideal**. Hence, **all gases are called real gases**.
- The **deviation of any gas** from ideal behavior is given by **compressibility factor**

$$Z = \frac{(PV)_{\text{real}}}{(PV)_{\text{ideal}}} = \frac{PV}{nRT}$$

- For **ideal gases**,  $Z = 1$
- For **real gases**,  $Z < 1$  at **low pressure**
- $Z > 1$  at **high pressure**



# Critical Temperature, Pressure and Volume

Gases can't be liquefied above a temperature called critical temperature ( $T_c$ ) however large the pressure may be

$V$  of gas at  $P_c$  and  $T_c$

$$\Rightarrow V_c = 3 \cdot b$$

$P$  required to liquefy the gas at  $T_c$

$$\Rightarrow P_c = \frac{a}{27 \cdot b^2}$$

And

$$\Rightarrow T_c = \frac{8 \cdot a}{27 \cdot R \cdot b}$$

**NOTE:**  $\frac{R \cdot T_c}{P_c V_c} = \frac{8}{3}$  is called critical coefficient and same for all gases





## Quick References Table for $C_p, C_v$ Values

Nature of gas	$f$	$U = \frac{f}{2} R.T$	$C_v$	$C_p$	$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$
Monoatomic	3	$\frac{3}{2} R.T$	$\frac{3}{2} R$	$\frac{5}{2} R$	1.67
Dia and Linear polyatomic	5	$\frac{5}{2} R.T$	$\frac{5}{2} R$	$\frac{7}{2} R$	1.4
Non-linear Polyatomic	6	$3R.T$	$3R$	$4.R$	1.33

# Molar Heat Capacity for Ideal Gas

$C_p$  = molar heat capacity at constant pressure

$C_v$  = molar heat capacity at constant volume

$$\gamma = \frac{C_p}{C_v}$$

For an ideal gas

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p = C_v + R$$

And

$$\gamma = 1 + \frac{2}{f}$$

Where  $f$  = degrees of freedom

