JEE and NEET CRASH COURSE

Work, Energy and Power

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WORK

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

$$W = \int \vec{F}.\overrightarrow{dS} = \int FdScos\theta$$

Dimension : $M^1L^2T^{-2}$

SI UNIT : joule

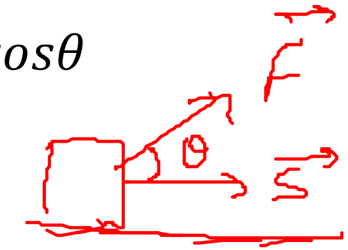
C.G.S. UNIT : erg

1 joule = 10^7 erg

Work is scalar quantity and can have +ve, -ve or zero value.

Work done by Constant Force

$$W = \vec{F}.\vec{S} = FS\cos\theta$$



If $\quad \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \quad$ and $\quad \vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$

then $\quad W = \vec{F}.\vec{s} = F_x x + F_y y + F_z z$

$$\vec{S} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$W = \int \vec{F} \cdot d\vec{S}$$

$$= \vec{F} \cdot \int d\vec{S}$$

$$= \vec{F} \cdot \vec{S}$$

B (x_2, y_2, z_2)

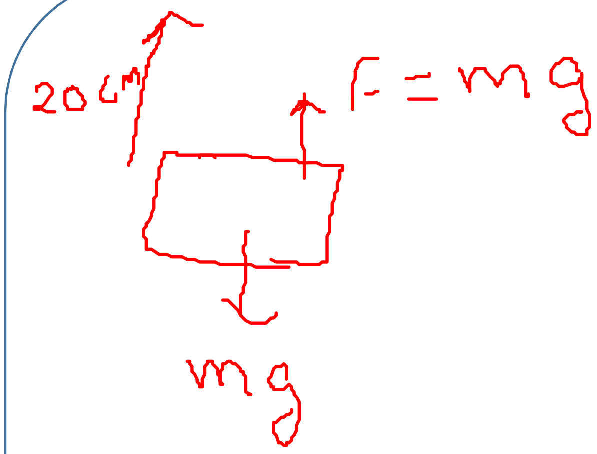
A (x_1, y_1, z_1)

Example

Calculate the amount of work done in raising a glass of water weighing 0.5 kg through a height of 20 cm. ($g = 10\ m/s^2$)

Sol. $W = \text{Force} \times \text{distance}$

$= mgh = 0.5 \times 10 \times 0.2 = 1\ J$

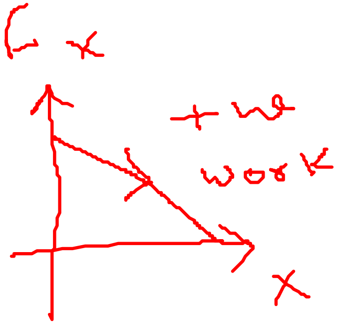


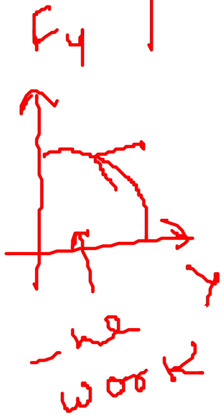
$W = F\,S$

$= 0.5 \times 1\cancel{0} \times \frac{2\cancel{0}}{1\cancel{0}\cancel{0}}$

$= 1\ J$

Work done by Variable Force





$$W = \int \vec{F}.\overrightarrow{dS}$$

$$\vec{F} = F_x\tilde{i} + F_y\tilde{j} + F_z\tilde{k}\ ,\ \ d\vec{s} = dx\tilde{i} + dy\tilde{j} + dz\tilde{k}$$

$$W_{AB} = \int_A^B (F_x\tilde{i} + F_y\tilde{j} + F_z\tilde{k}).(dx\tilde{i} + dy\tilde{j} + dz\tilde{k})$$

$$= \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

NOTE:- Area under force-displacement graph gives magnitude of work done by the force.

$F(t)$

$$a(t) = \frac{F(t)}{m}$$

$$v = \int a\,dt$$

$$= \int \frac{F}{m}\,dt$$

$$W = K_f - K_i$$

Example

A force F = (10 + 0.5x) N acts on a particle in x direction, where x is in metre. Find the work done by this force during a displacement from x = 0 to x = 2.

Sol.

$$W = \int_{x=0}^{x=2} (10 + 0.5x)dx = \left[10x + \frac{0.5x^2}{2}\right]_0^2$$

$$= 10(2-0) + \frac{0.5}{2}(2^2 - 0) = 21$$

$$W = \int_0^2 \left(10 + \frac{x}{2}\right)dx$$

$$= \left[10x + \frac{x^2}{4}\right]_0^2$$

$$= \left(20 + \frac{4}{4}\right) - 0$$

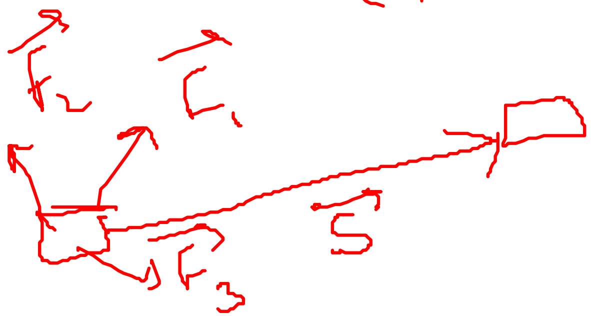
$$= 21 J$$

Work done by Several Forces

When several forces acts on a body then the net work done on the body is the algebraic sum of work done by individual force.

$$W_{net} = \vec{F}_1.\vec{s}_1 + \vec{F}_2.\vec{s}_2 + \ldots..\vec{F}_n.\vec{s}_n$$

$$= (\vec{F}_1 + \vec{F}_2 + .. + \vec{F}_n)\vec{S}$$



Net work done on system of bodies

$$W_{net} = W_1 + W_2 + \ldots$$

$$= \vec{F}_1.\vec{S}_1 + \vec{F}_2.\vec{S}_2 + \ldots$$

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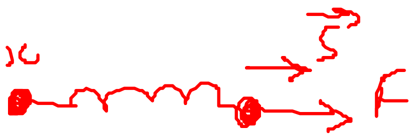


Power

It is defined as the rate at which the body can do the work.

$$\text{Power} = \frac{\text{work}}{\text{time}} \quad \text{or} \quad P = \frac{W}{t}$$

This power P is also called average power.

Instantaneous power (P)

$$P = \frac{dW}{dt} \quad \text{or} \quad P = \frac{\vec{F}.d\vec{s}}{dt} \quad (\because dW = \vec{F}.d\vec{s})$$

$$\text{or} \quad P = \vec{F}.\frac{d\vec{s}}{dt} \quad \text{or} \quad \boxed{P_t = \vec{F}.\vec{v}}$$

N – m/s

Dimensions : $[M^1L^2T^{-3}]$ SI UNIT : $\text{watt} = \frac{\text{joule}}{\text{second}}$

Bigger UNITS of power :

1 kilo watt (kW) = 1000 watt = 10^3W

1 maga watt = 1000,000 watt or 1MW = 10^3kW $= 10^6$ W

1 horse power (h.p.) = 746 W

Important Points

The slope of work–time graph gives the instantaneous power.

$$\text{slope} = \tan\theta = \frac{dW}{dt} = P$$

Area under power–time graph gives the work done.

$$\text{Area under power–time graph} = \int P\,dt = W$$

The efficiency η of a machine $\quad \eta = \dfrac{\text{work done}}{\text{energy input}}$

Example

A pump can take out 7200 kg of water per hour from a well 100 m deep. Calculate the power of the pump, assuming that its efficiency is 50%. (g = 10 m/s^2)

Sol.

$$\text{Output power} = \frac{mgh}{t} = \frac{7200 \times 10 \times 100}{3600} = 2000 \text{ W}$$

$$\text{Efficiency} \quad \eta = \frac{\text{output power}}{\text{input power}}$$

$$\text{Input power} = \frac{\text{output power}}{\eta} = \frac{2000 \times 100}{50} = 4 \text{ kW}$$

$$P_{o/p} = \frac{\text{Work}}{\text{time}}$$

$$= \frac{mgh}{t}$$

$$= \frac{72\cancel{00} \times 10 \times 100}{\cancel{36}\cancel{00}\ 2}$$

$$= 2000 \text{ W}$$

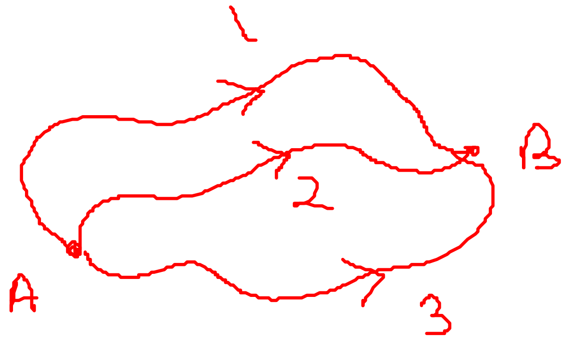
$$\frac{P_o}{P_I} = \frac{1}{2} \Rightarrow P_I = 4 \text{ kW}$$

Conservative Force

A force is said to be conservative if work done by the force on a particle moving between two points does not depends on the path taken by the particle.

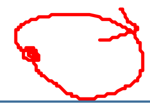
<u>Examples</u>

1. Gravitational force
2. Elastic force in a stretched or compressed spring
3. Electrostatic force



$W_1 = W_2 = W_3$

In closed path, W.D. is zero



Non-Conservative Force

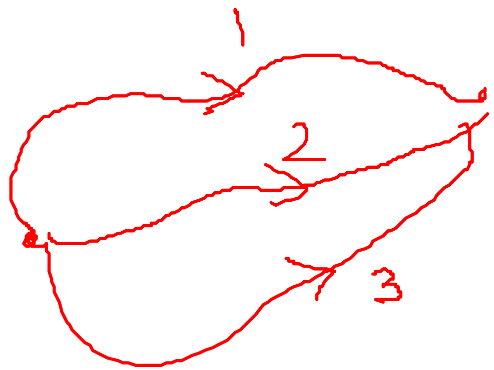
$W = \vec{F} \cdot \vec{S}$

A force is said to be non conservative, if work done or against the force in moving a body from one position to another, depends on the path.

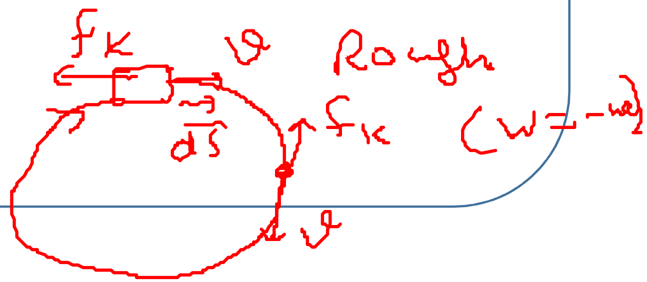
<u>Examples</u>

1. The velocity dependent forces such as : air resistance and viscous force

2. Retarding forces such as friction force and drag force.



$W_1 = W_2 \neq W_3$



Energy

Energy is defined as the internal capacity for doing work.

When we say that a body has energy it mean that it can do work.

Energy is a scalar quantity

Dimensions : $[M^1L^2T^{-2}]$

S I UNIT : joule

1 unit

Other units $1\ erg = 10^{-7}$ joule

$1\ eV = 1.6 \times 10^{-19}$ joule

$1 kWh = 36 \times 10^5$ joule $= 3.6\ MJ$

$1\ cal = 4.2$ joule $= 4.18\ J$

Kinetic Energy

If a particle of mass m is moving with velocity 'v' (much less than the velocity of the light) then the kinetic energy K.E. is given by

$$K.E. = \frac{1}{2}mv^2$$

1. It is always non negative
2. depends on the frame of reference

Relation between K.E. (K) and linear momentum (p) :

$$p = mv \quad \text{and} \quad K = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{p^2}{2m}$$

For particles

$$\boxed{K = \frac{p^2}{2m}} \quad \text{or} \quad \boxed{p = \sqrt{2mK}}$$

$$\vec{v}^2 = \vec{v}\cdot\vec{v}$$
$$= v^2$$

Work Energy Theorem

According to this theorem work done by net force on a body is equal to change in its kinetic energy.

$$W = \Delta KE \quad \text{or} \quad \boxed{W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2} = K_f - K_i$$

W.D. by all forces

Example

A bullet weighing 10 g is fired with a velocity 800 m/s. After passing through a mud wall 1m thick, its velocity decreases to 100 m/s. Find the average resistance offered by the mud wall.

Sol.

Work done by the average resistance offered by the wall

= change in K.E. of the bullet.

$$F\,s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad F = \frac{m(v^2 - u^2)}{2s} = \frac{0.01(100^2 - 800^2)}{2 \times 1} = -\,3150\ \text{N}$$

$\Rightarrow$ Resistance offered = 3150 N

$F_{avg} \longleftarrow \quad \vec{v} \quad \vec{dS}$

$W_{all} = \Delta K = K_f - K_i$

$-F_{avg} \cdot S = \frac{1}{2}m(100)^2 - \frac{1}{2}m(800)^2$

$-F_{avg} = \frac{1}{2}m(100)^2(1 - 64)$

$= \frac{1}{2} \times \frac{1\cancel{0}}{1\cancel{000}} \times 1\cancel{00} \times 100 \times (-63)$

$F_{avg} = \frac{6300}{2}$

$= 3150$ N

Potential Energy (P.E.)

Definition 1:- Potential energy is the internal capacity of doing work of a system by virtue of its configuration.

Definition 2:- The energy which a body has by virtue of its position or configuration in a conservative force field.

Mathematical Definition:- Change in potential energy is negative of work done by internal conservative forces.

$$\Delta U = Uf - Ui = -Win_{ternal\ conservative\ forces}$$

NOTE:- $W_{internal\ conservative\ forces} = Ui - Uf$

q_1 q_2

$\Delta U = U_f - U_i$

$= -W_{int.\ cons.\ forces}$

$W_{grav.} = U_i - U_f$

$W_{spring} = U_i - U_f$

$W_{electro.} = U_i - U_f$

Potential energy in different conservative fields

(a) Gravitational P.E.:-

Near the surface of earth assuming zero P.E. at earth's surface

$$U = mgh$$

(b) Spring P.E.:-

$$U = \frac{1}{2}kx^2$$

where, x is extension or compression in the spring

Work done by spring force = $U_i - U_f$

Example

Find out work done by applied force to slowly extend the spring from x to 2x.

Sol.

$$W_{\text{applied force}} = -W_{\text{spring force}}$$
$$= U_f - U_i$$
$$= \frac{1}{2}k(2x)^2 - \frac{1}{2}kx^2$$
$$= \frac{3}{2}kx^2$$

$W_{\text{app. force}} + W_{\text{s.f.}} = \Delta K$

$W_A + (U_i - U_f) = 0$

$W_{\text{A.F.}} = U_f - U_i$

$= \frac{1}{2}K(2x)^2 - \frac{1}{2}Kx^2$

$= \frac{3}{2}Kx^2$

Relation between force and P.E.

Force from P.E.

If U(r); $F = -\frac{dU}{dr}$

If U(x, y, z)

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

$\frac{\partial U}{\partial x}$ – Partial differentiation of U w.r.t. x (keeping y, z constant)

$\frac{\partial U}{\partial y}$ – Partial differentiation of U w.r.t. y (x, z constant)

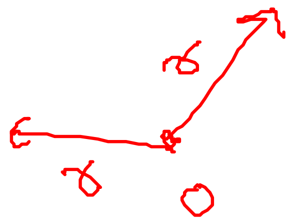
$\frac{\partial U}{\partial z}$ – Partial differentiation of U w.r.t. z (x, y constant)

P.E. from Force

$$U = -\int \vec{F}.\overrightarrow{dr} = -\int (Fxdx + Fydy + Fzdz)$$

$\Delta U = -W$ int. cons. force

$= -\int \vec{F}.\vec{dS}$



Example

The potential energy of a particle in a space is given by U = $x^2 + y^2$.

Find the force associated with this potential energy.

Sol.

$$F_x = \frac{-\partial u}{\partial x} = -[2x + 0] = -2x$$

$$F_y = \frac{-\partial u}{\partial y} = -(2y + 0) = -2y$$

$$\vec{F} = -2x\,\hat{i} - 2y\,\hat{j}$$

$$F_x = -\frac{\partial U}{\partial x}$$

$$= -(2x) = -2x$$

$$F_y = -\frac{\partial U}{\partial y} = -2y = -2y$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$= -2x\hat{i} - 2y\hat{j}$$

$$= -2(x\hat{i} + y\hat{j})$$

Example

Find out the potential energy of given force $\vec{F} = -2x\,\hat{i} - 2y\,\hat{j}$.

Sol.

$$dU = -dW$$

$$\int dU = \int -(-2x\hat{i} - 2y\hat{j}) \bullet (dx\hat{i} + dy\hat{j})$$

$$\int dU = \int 2xdx + \int 2ydy$$

$$\therefore \quad U = x^2 + y^2 + C$$

$$U = -\int (F_x dx + F_y dy)$$

$$= -\int (-2x dx - 2y dy)$$

$$= \cancel{2}\frac{x^2}{\cancel{2}} + \cancel{2}\frac{y^2}{\cancel{2}} + C$$

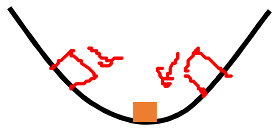
$$U = x^2 + y^2 + C$$

Equilibrium

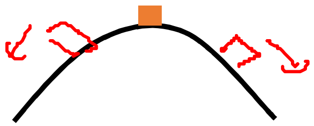
Equilibrium means; $F_{net} = 0$

Types of Equilibrium

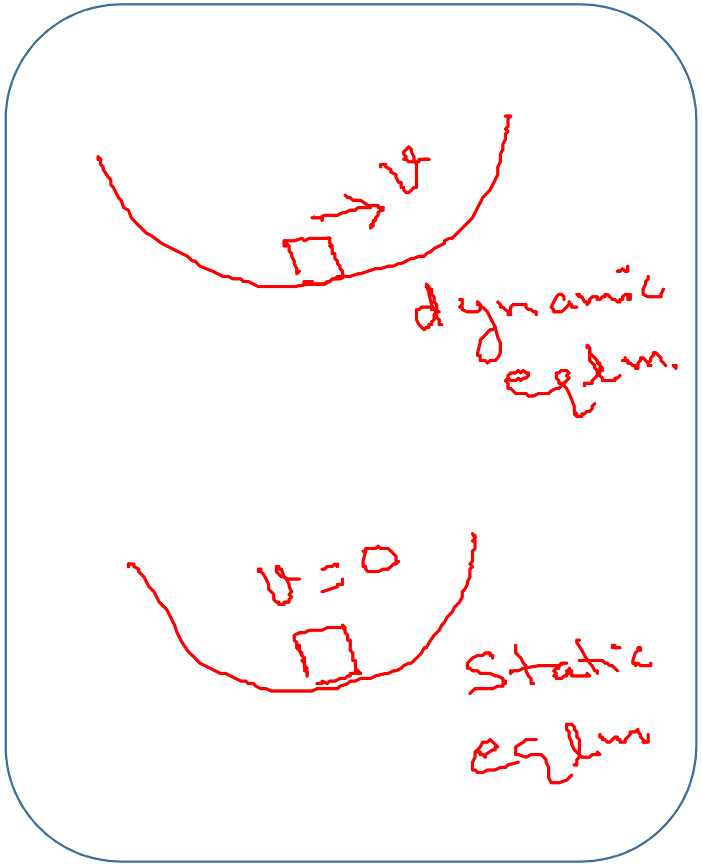
(a) Stable equilibrium
If on slight displacement from equilibrium position a body has tendency to regain its original position.



(b) Unstable equilibrium
If on slight displacement from equilibrium position body moves in the direction of displacement.



(c) Neutral equilibrium
If on slight displacement from equilibrium position a body has no tendency to come back to original position or to move in the direction of displacement.



Mechanical Energy

Sum of kinetic and potential energy

$$M.E. = K.E. + P.E.$$

- Scalar quantity
- S.I. Unit : Joule
- Depends on frame of reference
- $M.E. \geq P.E.$

$K.E. \geq 0$

$M.E \geq P.E.$

Can be +ve, -ve and zero.

Law of Conservation of Mechanical Energy

If work done by non-conservative forces and external forces is zero, then M.E. of system will remain conserved.

$$ME_{initial} = ME_{final}$$

$$KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$$