JEE and NEET CRASH COURSE

Law's of Motion & Friction

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Force

- A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object.

- It is a vector quantity.

Effect of resultant force :

(1) may change only speed

(2) may change only direction of motion.

(3) may change both the speed and direction of motion

(4) may change size and shape of a body

Units and dimension Force

SI Unit : newton (N) and kg-m/s^2
CGS Unit : dyne and g-cm/s^2

Kgf
(50 kg × g) N

$$1 \text{ Newton} = 10^5 \text{ dyne}$$

kilogram force (kgf)

The force with which earth attracts a 1kg body towards its centre is called kilogram force, thus

$$\text{kgf} = \frac{\text{Force in newton}}{g}$$

So, to find force in Newton from kgf, we have to multiply by g

Dimensional Formula of force : $[M L T^{-2}]$

Classification of forces on the basis of contact

$1\,kgf \times g = 9.8\,N$

$1\,N = \frac{1}{9.8}\,kgf$

(A) Field Force:

Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force.

Examples: (a) Gravitational force (b) Electromagnetic force

(B) Contact Force:

Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

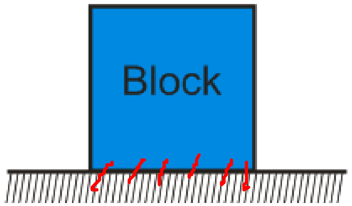
Examples: (a) Normal force (b) Tension (c) Friction

Normal force (N)

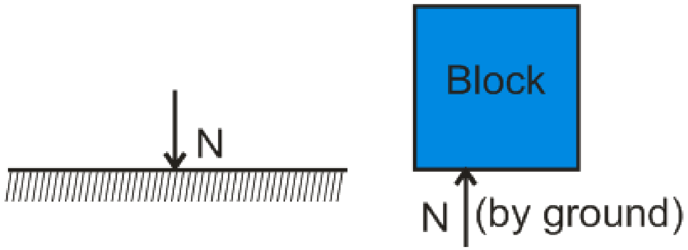
Kg wt. × g = N
60kg × g = 600N

It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is electromagnetic in nature.

Consider a block kept on Earth

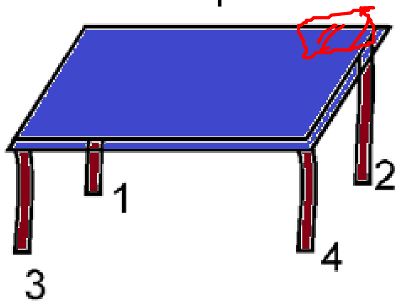


Here block presses the earth so normal force exerted by block on earth and by earth on block is as shown

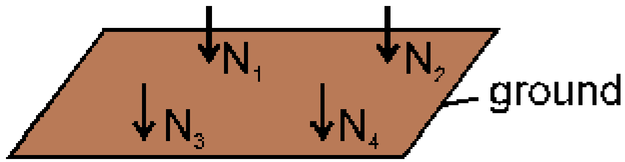


Normal force (N)

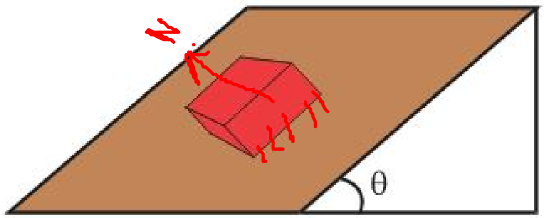
Consider a table placed on Earth

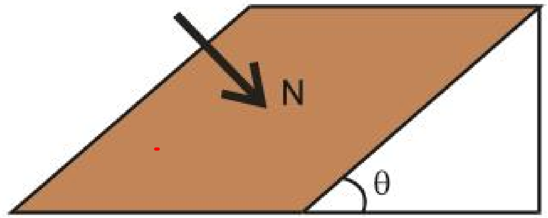


Here table presses the earth so normal force exerted by four legs of table on earth is as shown



Consider a block kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.

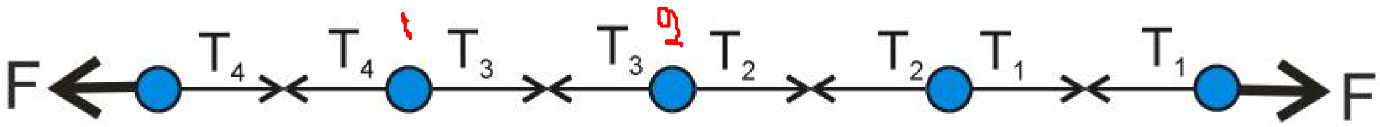




Tension (T)

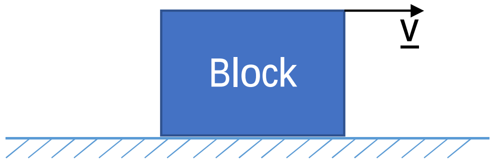
Tension in a string is an electromagnetic force. It arises when a string is pulled to increase its length.

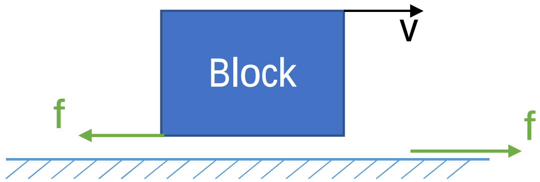
String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown



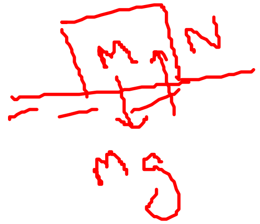
friction (f)

It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.





Third Law of Motion



To every action, there is always an equal and opposite reaction.

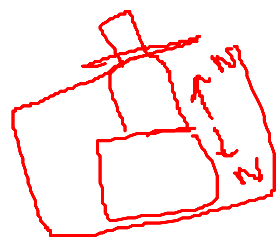
Important points about the Third Law

(a) The terms 'action' and 'reaction' in the Third Law mean nothing else but 'force' i.e. Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

(b) The terms 'action' and 'reaction' in the Third Law may give a wrong impression that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause-effect relation implied in the Third Law. The force on A by B and the force on B by A act at the same instant. Any one of them may be called action and the other reaction.

(c) Action and reaction forces act on different bodies, not on the same body. Thus if we are considering the motion of any one body (A or B), only one of the two forces is relevant. It is an error to add up the two forces and claim that the net force is zero.

System



Two or more than two objects which interact with each other form a system.

Classification of forces on the basis of boundary of system

(A)Internal Forces

Forces acting with in a system among its constituents.

(B) External Forces

Forces exerted on the constituents of a system by the outside surroundings are called as external forces.

Note: If you are considering the system of two bodies as a whole, F_{AB} (force on A due to B) and F_{BA} (force on B due to A) are internal forces of the system (A + B). They add up to give a null force. Internal forces in a body or a system of particles thus cancel away in pairs.

Free Body Diagram

A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Steps for F.B.D.

Step 1: Identify the object or system and isolate it from other objects clearly specify its boundary.

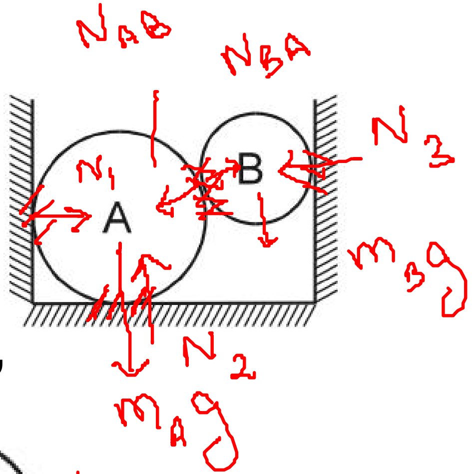
Step 2: First draw non-contact external force in the diagram. Generally it is weight.

Step 3: Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.

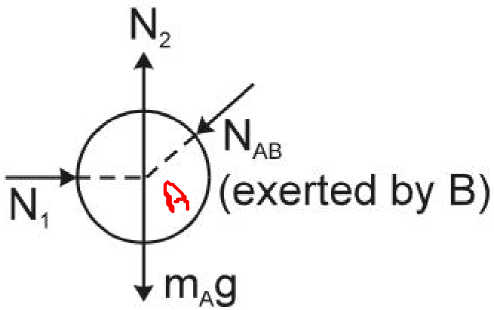
<u>In F.B.D, internal forces are not drawn, only external are drawn.</u>

Example

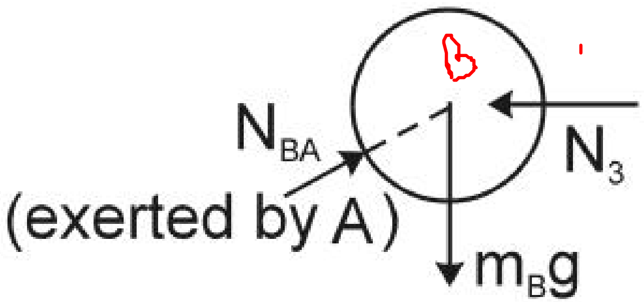
Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



Sol. F.B.D. of sphere 'A'

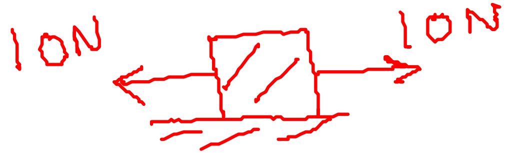


F.B.D. of sphere 'B'

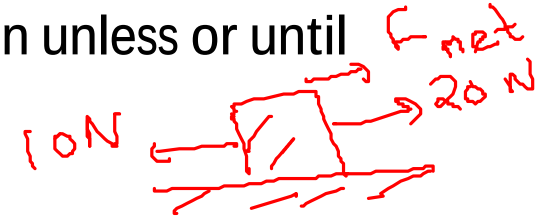


Note : Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

First Law of Motion (Law of Inertia)



Each body continues to be in its state of rest or of uniform motion unless or until an unbalanced external force acts on it.



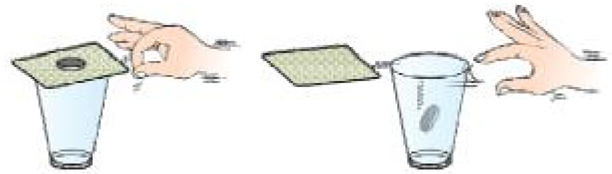
Examples of First Law

(a) A passenger sitting in a bus gets a jerk when the bus starts or stops suddenly.

(b) A passenger feels outward movement when vehicle takes a sharp turn.

(c) A bullet fired on a glass window makes a clean hole through it while a stone breaks the whole of it. The speed of bullet is very high. Due to its large inertia of motion, it cuts a clean hole through the glass. When a stone is thrown, it inertia is much lower so it cannot cut through the glass.

(d) Put a postcard on glass and keep a five rupee coin on it. Give the card a sharp horizontal flick with a finger. If we do it fast then the card shoots away, allowing the coin to fall vertically into the glass tumbler due to its inertia.



Second Law of Motion

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Mathematically $\vec{F} = \frac{d\vec{p}}{dt}$ where $\vec{p} = m\vec{v}$, $\vec{p}$ = Linear momentum.

Or,

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d(\vec{v})}{dt} + \frac{d(m)}{dt}\vec{v}$$

If, m = constant

$$\vec{F} = m\frac{d(\vec{v})}{dt} = m\vec{a}$$

Important points about second law

(a)The Second Law is obviously consistent with the First Law as F = 0 implies a = 0.

(b) The Second Law of motion is a vector law. It is actually a combination of three equations, one for each component of the vectors :

$$F_x = \frac{dp_x}{dt} = ma_x \qquad F_y = \frac{dp_y}{dt} = ma_y \qquad F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

Equilibrium

If net force acting on an object is zero, then it is said to be in equilibrium. $\sum \vec{F}_{ext} = 0$

To solve problems involving objects in equilibrium:

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the free-body diagram for the object. Label all external forces acting on it.

Step 3: Choose a convenient coordinate system and resolve all forces into rectangular components along x and y direction.

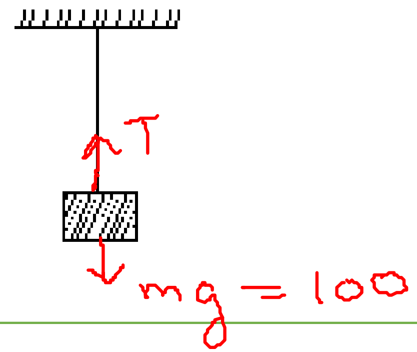
Step 4: Apply the equations $\sum F_x = 0$ and $\sum F_y = 0$

Step 5: Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Example

A ‘block’ of mass 10 kg is suspended with string as shown in figure. Find tension in the string. ($g = 10\ m/s^2$)



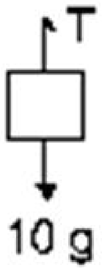
Sol.

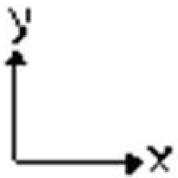
F.B.D. of block

$\Sigma F_y = 0$

$T - 10\ g = 0$

$\therefore \quad T = 100\ N$





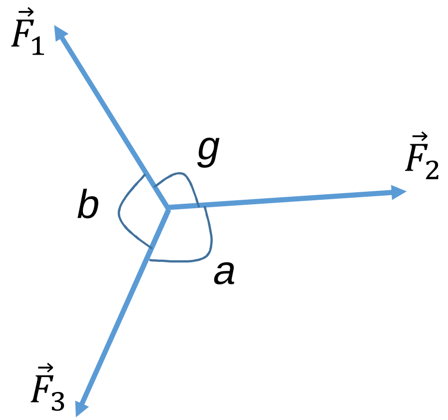
$T - 100 = 0$

$T = 100\ N$

LAMI'S THEOREM

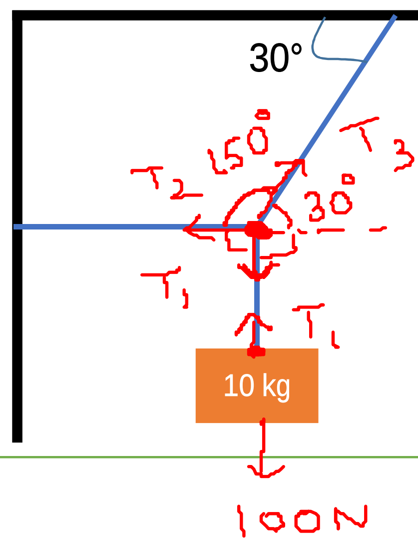
For equilibrium of 3 forces

$$\frac{F_1}{\sin a} = \frac{F_2}{\sin b} = \frac{F_3}{\sin g}$$



Example

For tension in all strings if system is in equilibrium



$T_1 = 100\,N$

Sol.

$$\frac{T_1}{\sin(180-30°)} = \frac{T_2}{\sin(90+30°)} = \frac{T_3}{\sin 90°}$$

$$\frac{100}{\sin 30°} = \frac{T_2}{\cos 30°} = \frac{T_3}{1}$$

$$\frac{100}{1} \times 2 \times \frac{\sqrt{3}}{2} = T_2$$

$$T_2 = 100\sqrt{3}$$

$$T_3 = 200\,N$$

ACCELERATED OBJECTS

$$\vec{F}_{net} = m\vec{a}$$

It is vector equation and can be applied independently in mutually perpendicular direction .

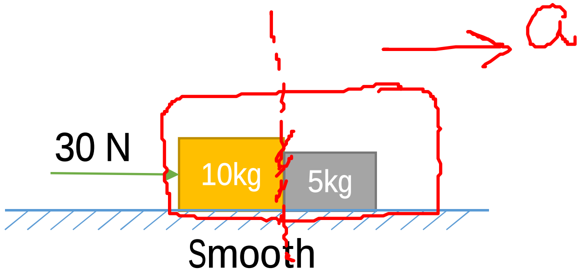
$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

Example

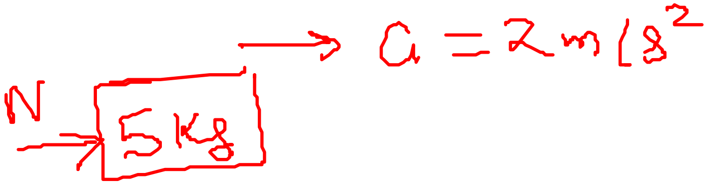
Two blocks of mass 5 kg and 10 kg are kept as shown. A constant force is applied as shown. Find acceleration of blocks and normal force acting between the blocks.



Sol.

$30 = 15a$

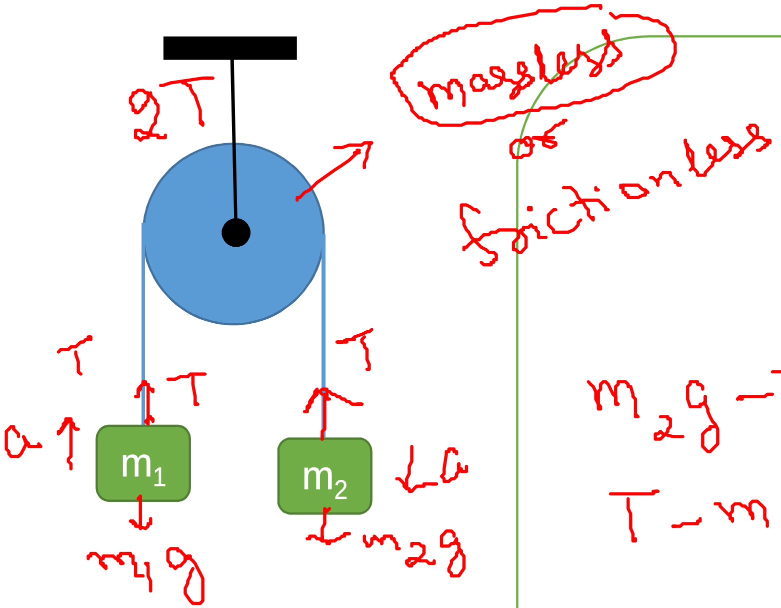
$a = 2\ m/s^2$



$N = 5 \times 2$

$= 10\ N$

ATWOOD MACHINE

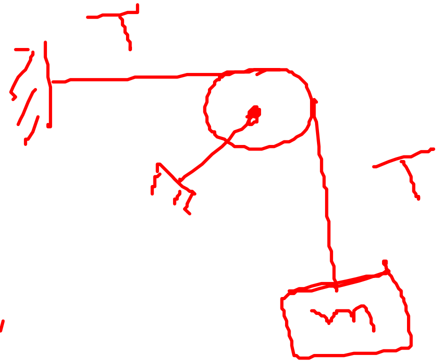


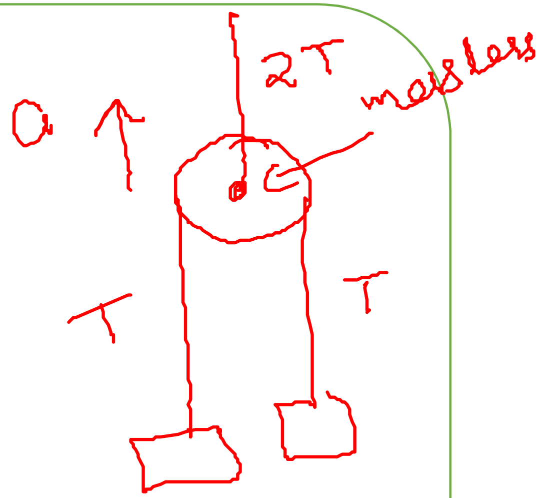
$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

$$T = \left(\frac{2m_1 m_2}{m_2 + m_1}\right) g$$

$m_2 g - T = m_2 a$

$T - m_1 g = m_1 a$





SPRING FORCE

The extension or compression produced in a spring is directly proportional to the force applied on it.

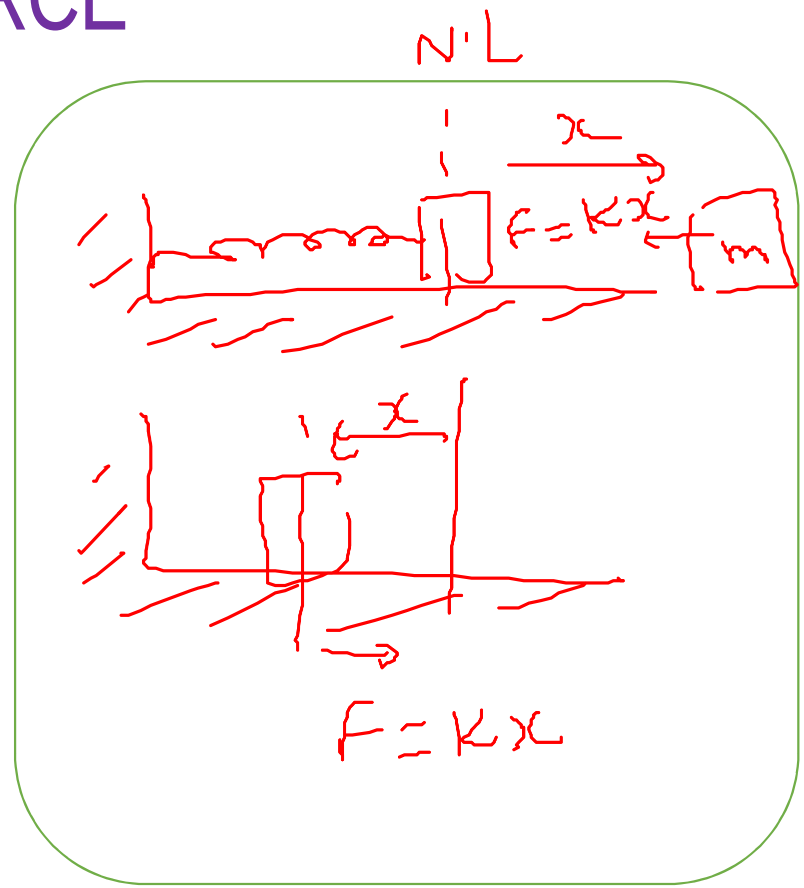
If x is the deformation in the spring then

$F \mu x$

or $F = -kx$

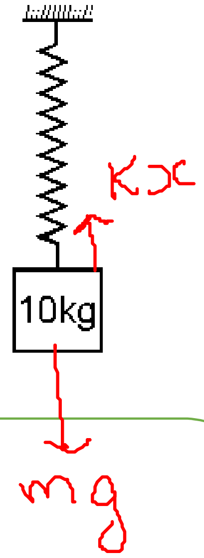
where k is constant and is called the spring constant or force constant.

The negative sign shows that the force and the extension or compression are opposite in their directions.



Example

Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10\ m/s^2$)



$$Kx = mg$$

$$x = \frac{mg}{K} = \frac{100}{100} = 1\ m$$

PSEUDO FORCE

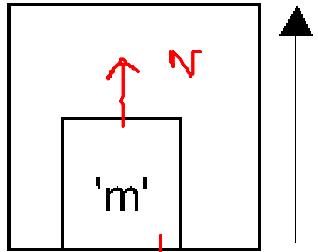
Hypothetical force used to apply law's of motion in accelerated (non-inertial) frames.

$$\vec{F}_{Pseudo} = -m\ \vec{a}_{Frame}$$

$$\vec{F}_p = -m_{object}\ \vec{a}_{frame}$$

Example

A box is moving upward with retardation 'a' <g, find the direction and magnitude of " pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



w.r.t. ground

$N - mg = ma$

$N = m(g+a)$

w.r.t. lift

rest

$N - mg - ma = 0$

$N = m(g+a)$

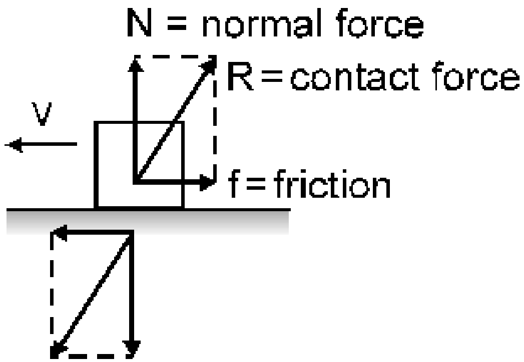
$F_{pseudo} = ma$

$g_{eff} = g + a$ (a ↑)

$g_{eff} = g - a$ (a ↓)

FRICTION

The component of contact force parallel to the surface is called friction (generally written as f).



Net contact force is

$$R = \sqrt{f^2 + N^2}$$

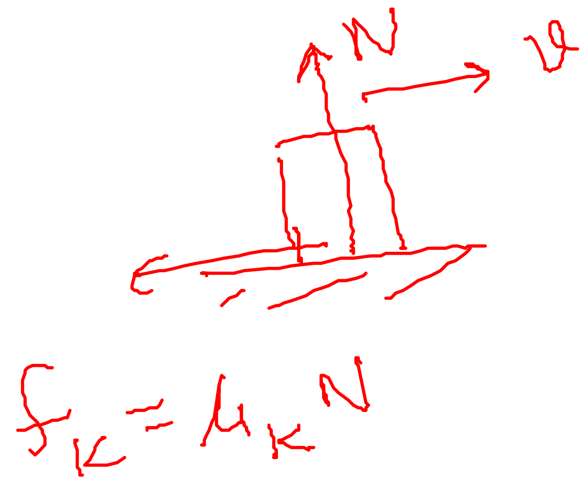
Kinetic Friction

It exists due to relative motion between surfaces in contact and opposes the relative motion.

Magnitude of Kinetic Friction

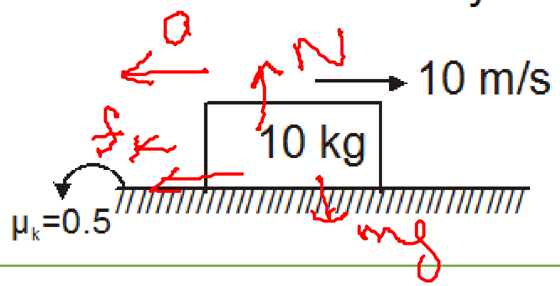
$$f_k = \mu_k N$$

m_k is called the coefficient of kinetic friction



Example

Find out the distance travelled by the blocks shown in the figure before it stops.



$N = mg = mg$

$f_k = \mu_k N = \mu_k mg$

$a = \frac{f_k}{m} = \frac{\mu_k m g}{m} = 0.5 \times 10 = 5$

$u = 10$

$a = -5$

$v^2 = u^2 + 2as$

$0 = 100 - 10s$

$s = 10\ m$

Static Friction

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

Magnitude of Static Friction

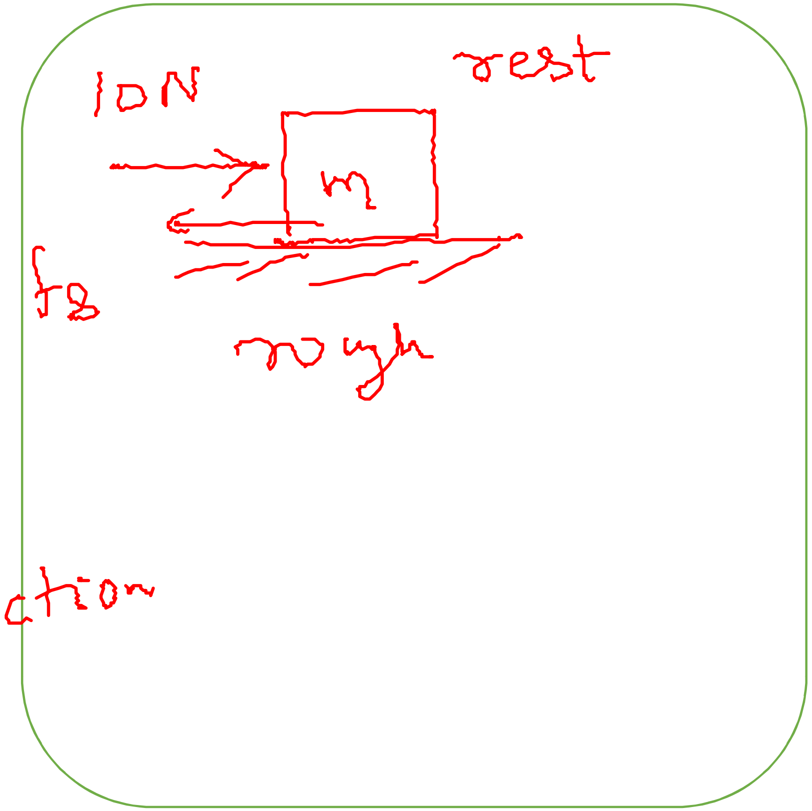
$$f_{max} = \mu_s N$$

m_s is called the coefficient of static friction

$$0 \leq f_s \leq f_{smax}$$

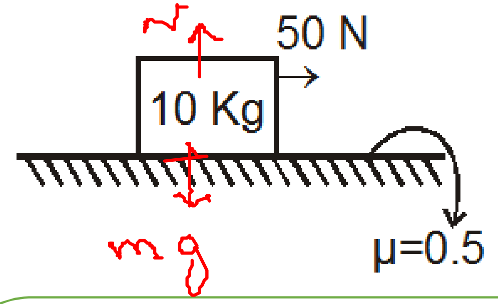
← Limiting friction

In general, $m_s > m_k$



Example

Find acceleration of block and friction force acting on the block. Initially the block is at rest.



$W = mg = 100$

$N = 100$

$f_{max} = \mu N$

$= 0.5 \times 100$

$= 50N$

Body will not move

$f_s = 50N$

$a = 0$

$g = 9.8$

$N = 98N$

$f = 49$

$f_s = 49$

$a = 0$