Find the following limits

(i) $\lim\limits_{x\to 0} \dfrac{3^x - 1}{\tan x}$ $\left(\frac{0}{0}\right)$ (ii) $\lim\limits_{x\to 3} \dfrac{\sin(e^{x-3}-1)}{\log(x-2)}$

(i) $\lim\limits_{x\to 0} \dfrac{3^x - 1}{\tan x} = \lim\limits_{x\to 0} \dfrac{3^x - 1}{x} \times \dfrac{x}{\tan x} = \log 3 \times 1 = \log 3$

$\lim\limits_{x\to 0} \dfrac{3^x-1}{x} \cdot \dfrac{x}{\tan x}$

$\log 3 \cdot 1$

$\log 3$

(ii) $\lim\limits_{x\to 3} \dfrac{\sin(e^{x-3}-1)}{\log(x-2)} = \lim\limits_{x\to 3} \dfrac{\sin(e^{x-3}-1)}{e^{x-3}-1} \times \dfrac{e^{x-3}-1}{\log(x-2)}$

$= \log e$

$= (1)$

$\dfrac{\sin(e^{x-3}-1)}{\log(1+(x-3))} \times \dfrac{(x-3)}{(x-3)} \times \dfrac{(e^{x-3}-1)}{(e^{x-3}-1)}$

$= \lim\limits_{x\to 3} \dfrac{\sin(e^{x-3}-1)}{e^{x-3}-1} \times \dfrac{e^{x-3}-1}{x-3} \times \dfrac{x-3}{\log[1+(x-3)]}$

$= 1 \times 1 \times 1 = 1$

Evaluation of 1^∞ :

1. $$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

2. $$\lim_{x\to 0}(1+x)^{1/x} = e$$

$$\boxed{\lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a}[f(x)-1]\cdot g(x)}}$$

9) $$\lim_{x\to 0}(1+2x)^{1/x} = e^2$$

$$e^{\lim_{x\to 0} 2x\cdot\frac{1}{x}}$$

$$= e^2$$

General Method to solve :

(i) ~~$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$, then $\lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x\to a}\frac{f(x)}{g(x)}}$~~

(ii) If $\lim_{x\to a} f(x) = 1$, $\lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a}[f(x)-1]\times g(x)}$

1^∞

Evaluation of the limit form 0^0, 0^∞, or ∞^0:

If $\lim\limits_{x\to a}[f(x)]^{g(x)}$ gives any one form $(0^0, 0^\infty \text{ or } \infty^0)$ then $\lim\limits_{x\to a}[f(x)]^{g(x)} = e^{\lim\limits_{x\to a} g(x)\log f(x)}$

$e^{\log(f \cdot)} = (\ )$

Find the value of following limits

(i) $\lim\limits_{x\to 0}(1+3x)^{\frac{1}{5x}}$ (ii) $\lim\limits_{x\to 0}\left(\frac{1+3x^2}{1+5x^2}\right)^{\frac{1}{x^2}}$ (iii) $\lim\limits_{x\to 0}\left(\frac{1^x+2^x+3^x+\dots n^x}{n}\right)^{\frac{1}{x}}$

$e^{\lim\limits_{x\to 0}(3x)\cdot\frac{1}{5x}}$

$e^{3/5}$

$e^{\lim\limits_{x\to 0}\left(\frac{1+3x^2}{1+5x^2}-1\right)\cdot\frac{1}{x^2}}$

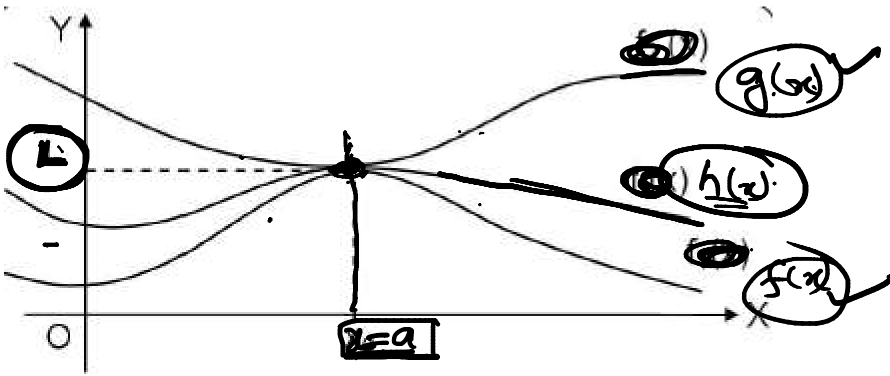
$e^{\lim\limits_{x\to 0}\left(\frac{-2\cancel{x^2}}{1+5x^2}\right)\cdot\frac{1}{\cancel{x^2}}}$

e^{-2}

Sandwich theorem or squezze play theorem :

If there is a function $h(x)$ such that $f(x) \le h(x) \le g(x)$ $\forall$ x and $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x)$ Then

$$\boxed{\lim_{x\to a} h(x) = \lim_{x\to a} g(x) \text{ or } \lim_{x\to a} f(x) = L}$$



Evaluate $\lim\limits_{n\to\infty}\left[\frac{1}{1+n^2}+\frac{2}{2+n^2}+\ldots\ldots+\frac{n}{n+n^2}\right]$

$\frac{1}{n+n^2} + \frac{2}{n+n^2} + \frac{3}{n+n^2} --- < P_n < \frac{1}{1+n^2} + \frac{2}{1+n^2} + \frac{3}{1+n^2} --- \frac{n}{1+n^2}$

$\frac{1}{n+n^2}(1+2+3 - - n) < P_n < \frac{1}{1+n^2}(1+2+3 - - n)$

Sandwich theorem or squezze play theorem :

If there is a function h(x) such that $f(x) \le h(x) \le g(x)\ \forall\ x$ and $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$ Then

$$\lim_{x\to a} h(x) = \lim_{x\to a} g(x) \text{ or } \lim_{x\to a} f(x)$$

$$\frac{n(n+1)}{2(n+n^2)} < P_n < \frac{n(n+1)}{2(1+n^2)}$$

$$\lim_{n\to\infty} \frac{n(n+1)}{2(n+n^2)} < \lim_{n\to\infty} P_n < \lim_{n\to\infty} \frac{n(n+1)}{2(1+n^2)}$$

$$\lim_{n\to\infty} \frac{n(n+1)}{2(n+n^2)} < \lim_{n\to\infty} P_n < \lim_{n\to\infty} \frac{n(n+1)}{2(1+n^2)}$$

$$\lim_{n\to\infty} \frac{1\left(1+\frac{1}{n}\right)}{2\left(\frac{1}{n}+1\right)} < \lim_{n\to\infty} P_n < \lim_{n\to\infty} \frac{1\left(1+\frac{1}{n}\right)}{2\left(\frac{1}{n^2}+1\right)}$$

$$\frac{1}{2} < \lim_{n\to\infty} P_n < \frac{1}{2} \Rightarrow \boxed{\lim_{n\to\infty} P_n = \frac{1}{2}}$$

L′ Hospital's Rule :

(☆ ☆ ☆ ☆ ☆)

If f(x) and g(x) are function of x such that f(a) = 0 and g(a) = 0, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$

Here f '(x) and g'(x) both are differentiation of f(x) and g(x) w.r.t. x.

$\left(\frac{0}{0}\right), \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)\left(\frac{0}{0}\right) = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

$\lim_{x\to 0}(2x)^{3x}$

$= \frac{\quad}{3x \log 2x}$

L'Hospital $= \frac{\log 2x}{\frac{1}{3x}}$

Forms $0\times\infty$ and $\infty-\infty$, reduced either to form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ for using L. Hospital's rule =

0^0, 1^∞, ∞^0 such types of form can be reduced to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ by taking log of the given limit.

9) The value of $\lim\limits_{x\to 64}\left(\dfrac{\sqrt{x}-8}{\sqrt[3]{x}-4}\right)$ is- $\left(\frac{0}{0}\right)$

(A) 1 (B) 2 (C) 3 (D) Does not exist

$$\lim_{x\to 64}\frac{\frac{1}{2\sqrt{x}}}{\frac{1}{3}x^{-2/3}} = \frac{\frac{1}{2\cdot 8}}{\frac{1}{3}\cdot(64)^{-2/3}} = \frac{3}{16}(4)^{3\times -2/3}$$

$$= \frac{3}{\cancel{16}}\times\cancel{16}$$

$$= 3$$

$\lim\limits_{x\to 64}\left(\dfrac{\sqrt{x}-8}{\sqrt[3]{x}-4}\right)$ (0/0 form)

$$= \lim_{x\to 64}\frac{\frac{1}{2\sqrt{x}}}{\frac{1}{3}x^{-2/3}} = \lim_{x\to 64}\frac{3}{2}\times\frac{x^{2/3}}{x^{1/2}} = \frac{3}{2}\times\frac{16}{8} = 3$$

End of Lecture

8)

If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to (2019 Main, 10 April II)

(a) -4 (b) 1 (c) -7 (d) 5

$1 - a + b = 0$

$\boxed{a - b = 1}$

$b = a - 1$

$b = -3 - 1$

$\boxed{b = -4}$

$\lim_{x \to 1} \frac{2x - a}{1} = 5$

$2 - a = 5$

$\boxed{a = -3}$

Q)

If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is

(2019 Main, 10 April I)

(a) $\frac{4}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{3}$

$$\lim_{x \to 1} \frac{4x^3}{1} = \lim_{x \to K} \frac{3x^{2}}{2x}$$

$$4 = \frac{3K}{2}$$

$$\boxed{K = 8/3}$$

$f(x) = ax^2 + bx + c$ $\boxed{c = 0}$

$\boxed{a + b + c = 3}$

$b = 5/2$

$f(x+y) = f(x) + f(y) + xy$

$\sum_{1}^{10} f(n)$

$\sum \left(\frac{1}{2}x^2 + \frac{5}{2}x\right)$

$f(2) = 2f(1) + 1$

$4a + \cancel{2b} = 2a + \cancel{2b} + 1$

$2a = 1$

$\boxed{a = 1/2}$