COMPOSITE FUNCTIONS

Let $f : A \to B$ & $g : B \to C$ be two functions. Then the function $gof : A \to C$ defined by $(gof)(x) = g(f(x))\ \forall\ x \in A$ is called the composite of the two functions f & g.

Diagramatically $\xrightarrow{x} \boxed{f} \xrightarrow{f(x)} \boxed{g} \longrightarrow g(f(x))$.

8) If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, the composition $f o g(x)$ is

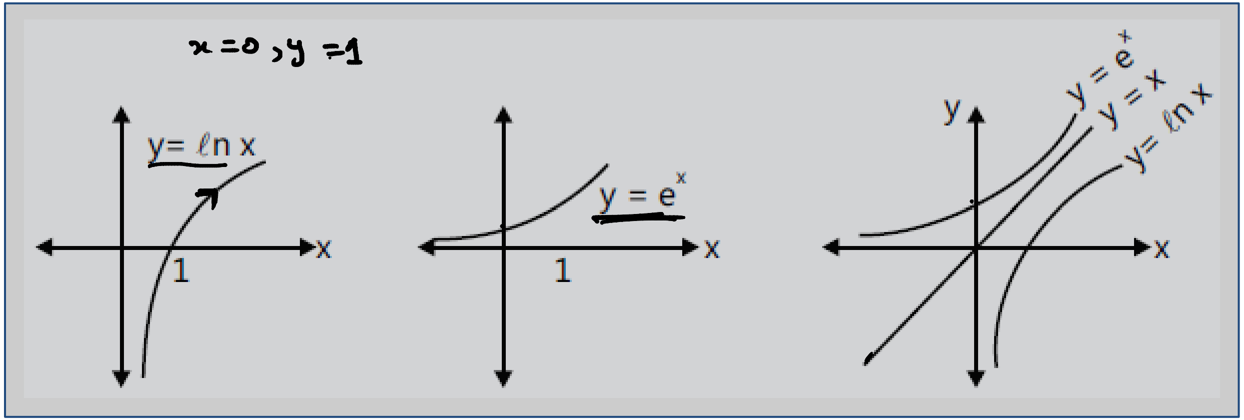
(a) $\sqrt{(2-2x)}$ (b) $(2-x)^{\frac{1}{4}}$ (c) $x^{\frac{1}{4}}$ (d) $\sqrt{2-\sqrt{x}}$

$$f(g(x)) = f(\sqrt{2-x})$$
$$= \sqrt{\sqrt{2-x}}$$
$$= (2-x)^{1/4}$$

⇒ INVERSE OF A FUNCTION :

bijective

Let $f: A \to B$ be a $\boxed{\text{one–one \& onto function}}$ then their exists a unique function $g: B \to A$ such that $f(x) = y \Leftrightarrow g(y) = x,\ \forall\ x \in A\ \&\ y \in B$. Then g is said to be inverse of f.



$f(g(x)) = x$

$\boxed{f(f^{-1}(x)) = x}$

$\boxed{f^{-1}(f(x)) = x}$

→ Note that the graphs of f & g are the mirror images of each other in the line $y = x$.

→ If $f: A \to B$ is a bijection & $g: B \to A$ is the inverse of f, then $\boxed{fog = I_B}$ and $gof = I_A$, where I_A & I_B are identity functions on the sets A & B respectively.

$g = f^{-1}(x)$

INVERSE OF A FUNCTION :

Let $f: A \to B$ be a one–one & onto function, then their exists a unique function $g: B \to A$ such that $f(x) = y \Leftrightarrow g(y) = x$, $\forall\ x \in A\ \&\ y \in B$. Then g is said to be inverse of f.

Properties

$f(x) \longrightarrow \boxed{f^{-1}(x)} =$

→ The inverse of a bijection is also a bijection.

→ If f & g are two bijections $f: A \to B$, $g: B \to C$ then the inverse of gof exists and $(gof)^{-1} = f^{-1} o\, g^{-1}$.

$$(gof)^{-1} = f^{-1}(g^{-1}(x))$$

Q) If $f(x) = \dfrac{6x-3}{2x+4}$, then $f^{-1}(x)$ is

Range( )

(a) $\dfrac{2x+4}{6x-3}$ (b) $\dfrac{6x-4}{2x+3}$ (c) $\dfrac{4x+3}{6-2x}$ (d) Does not exist

①

$$y = \frac{6x-3}{2x+4}$$

$$2xy + 4y = 6x - 3$$

$$x(2y-6) = -3 - 4y$$

$$\boxed{x = \frac{4y+3}{6-2y}}$$

$$y = \frac{4x+3}{6-2x}$$

$$\boxed{f^{-1}(x) = \frac{4x+3}{6-2x}}$$

ODD & EVEN FUNCTIONS :

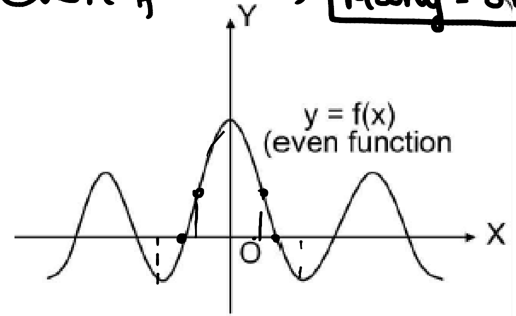
If $f(-x) = f(x)$ for all x in the domain of 'f' then f is said to be an even function.

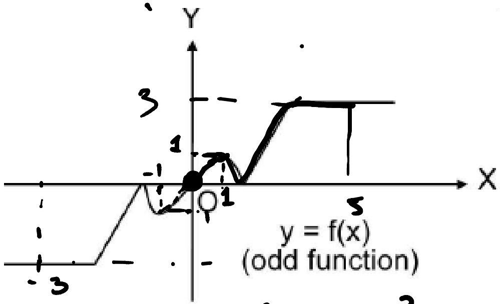
If $f(-x) = -f(x)$ for all x in the domain of 'f' then f is said to be an odd function.

$\boxed{f(-x) = f(x)}$ even

$\boxed{f(-x) = -f(x)}$ odd

even f^n → $\boxed{\text{Many - one}}$





$$y = \begin{cases} x^2, & x > 0 \\ -x^2, & x \le 0 \end{cases}$$

→ even f^n are symmetric about y- axis.

→ odd f^n are symmetric about origin.

$f(x) = x^3$

$f(-x) = (-x)^3$

$= -x^3$

$\boxed{f(-x) = -f(x)}$

odd

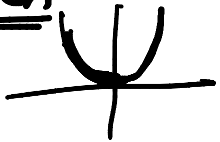
$f(x) = x^2$

$f(-x) = (-x)^2$

$f(-x) = x^2$

$\boxed{f(-x) = f(x)}$

even



If $f(x) = \dfrac{x+2}{x-3}$; then f (x) is $\quad f(-x) = \dfrac{-x+2}{-x-3}$

(a) even function
(b) odd function
(c) neither even function nor odd function ✓
(d) periodic function

Problem

If $f(x)$ is a function that is odd and even simultaneously, then $f(3)-f(2)$ is

a) 1 b) -1 c) 0 d) 2

$$\boxed{f(x) = 0}$$

$f(x) = 2$

$f(-x) = 2$

$$\boxed{f(-x) = f(x)}$$

even

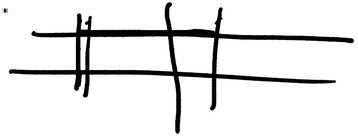
PERIODIC FUNCTION : (If a f^n repeats its nature after a fixed interval).

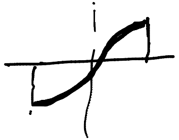
→ A function f(x) is called periodic if there exists a positive number T (T > 0) called the period of the function such that f(x+T) = f(x), for all values of x within the domain of x.

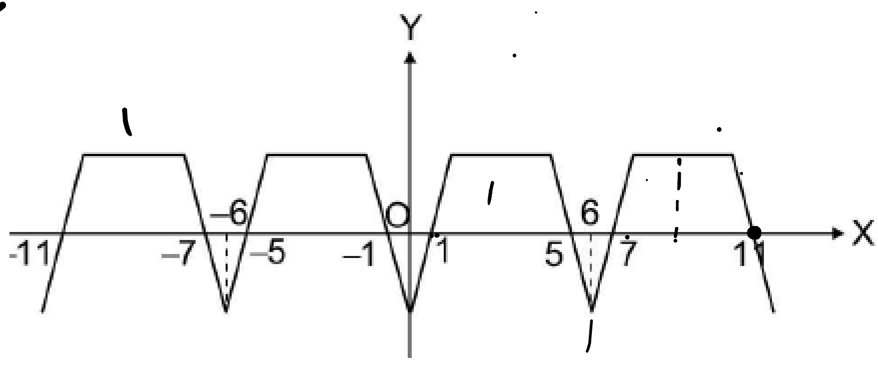
e.g. The function sin x & cos x both are periodic over 2π & tan x is periodic over π.

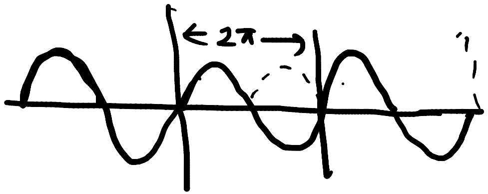
$$\boxed{f(x+T) = f(x)}$$ ✓

$f(x+2) = f(x)$









(a) Inverse of a periodic function does not exist (Many - one)

(b) Every constant function is always periodic, with no fundamental period.

(c) If f(x) has a period T & g (x) also has a period T then it does not mean that f(x)+g(x) must have a period T. e.g. f(x) = |sinx| + |cosx|.

(d) if f(x) has a period T then f(ax + b) has a period T/|a| (a > 0).

$$\boxed{\begin{array}{l} \sin x, \cos x \to 2\pi \\ \tan x, \cot x \to \pi \end{array}}$$

$|\sin x| \to \pi$, $|\cos x| \to \pi$

$f(x+\pi/2) = |\cos x| + |\sin x|$

$= f(x)$

Problem

T_1 T_2

9) If $f(x) = \cos x + \{x\}$ *where* $\{.\}$ **is fractional part function then the period of** $f(x)$ **is**

a) 2π b) 1 c) $\frac{\pi}{2}$ ~~d)~~ Does not exist

$\cos x \rightarrow 2\pi$

$\{x\} \rightarrow (1)$

L.C.M of (T_1, T_2)

$\pi \rightarrow \left(\frac{22}{7}\right)$, 3.14

$e \sim 2.7$

LCM $(2\pi, 1)$

Irrational Rational

$\sin x \rightarrow 2\pi$

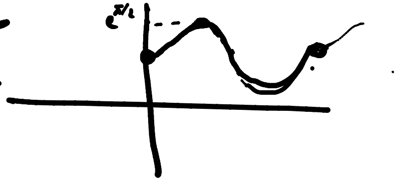
$\sin(2x+3) \rightarrow \frac{2\pi}{2}$ π

$2\sin x + 3 \rightarrow 2\pi$

$[af(x)+b] \rightarrow$ Period of $f(x)$

Problem

$e^{\sin x}$



The non-periodic function from among the following is

$f_1(x) = x - [x]$; $\quad$ $f_2(x) = \log(2 + \cos 2x)$,

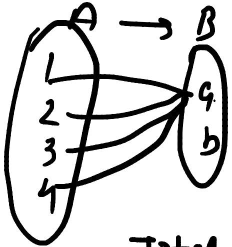
$f_3(x) = \tan(3x + 2)$; $\quad$ $f_4(x) = x^2 - x + \tan x$

(a) $f_1(x)$ $\quad$ (b) $f_2(x)$ $\quad$ (c) $f_3(x)$ $\quad$ (d) $f_4(x)$

$\boxed{x^3 \le x}$

$x^3 - x < 0$

x

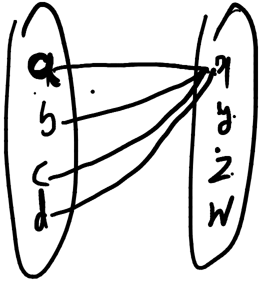


Total $= 2^4$ (16)

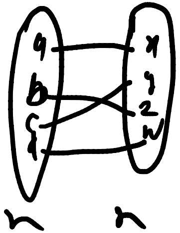
Into $= 2$

Onto $= 16 - 2$

$= 14$



$\frac{4^4}{4!}$



$4 \times 3 \times 2 \times 1 = 4!$

$n!$