Functions

By
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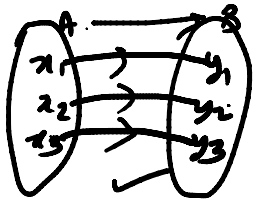
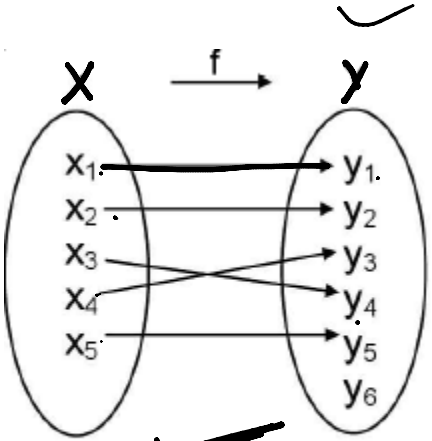


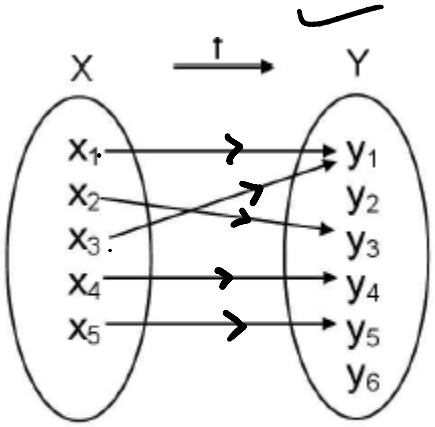
Functions

only & only one-image

A relation R from a set A to a set B is called a function if each element of A has unique image in B.
It is denoted by the symbol $f : A \to B$ which reads 'f' is a function from A to B or' f maps A to B

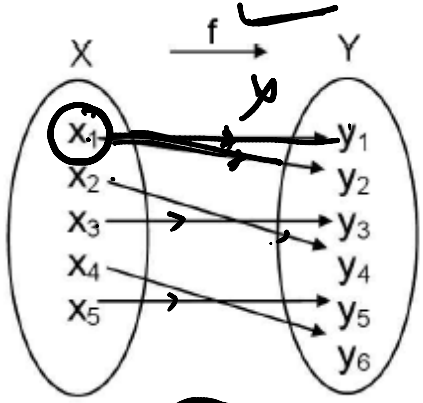


(i)

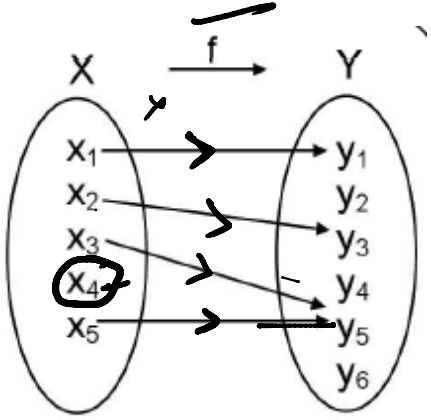


(ii)

Functions

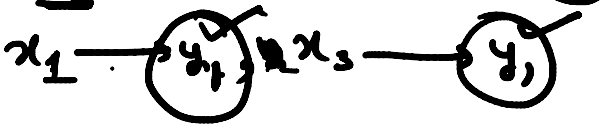


(iii)



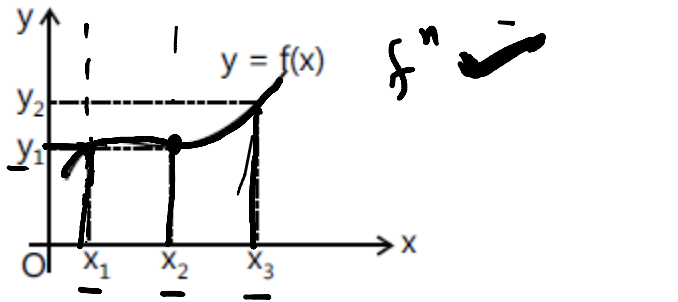
(iv)

Not Functions



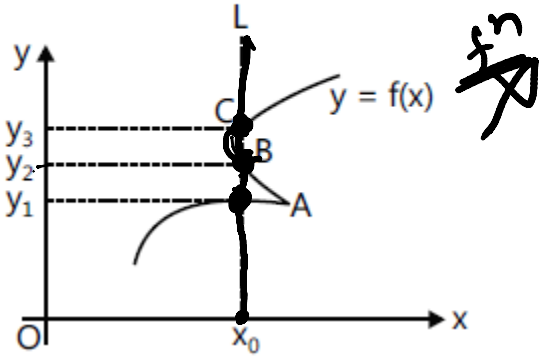
Functions

RELATION VS FUNCTION (Vertical Line Test)



f^n ✓

$x_1 \to y_1$, $x_2 \to y_1$, $x_3 \to y_2$



f^n ✗

$x_0 \to y_1, y_2, y_3$

Note :

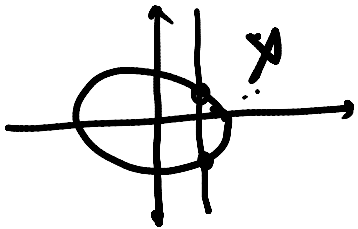
(i) If a vertical line cuts a given graph at more than one point then it can not be the graph of a function.

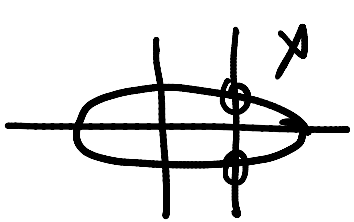
(ii) Every function is a relation but every relation is not necessarily a function.

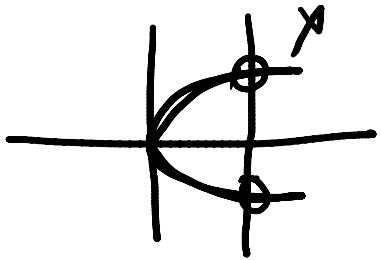
Problems

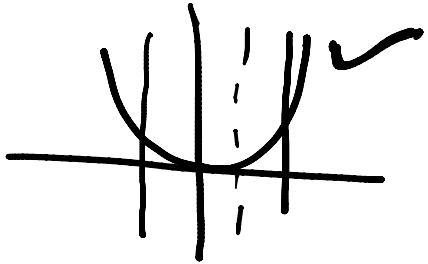
9 Which among the following relations is a function?

(a) $x^2 + y^2 = r^2$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$ (c) $y^2 = 4ax$ (d) $x^2 = 4ay$









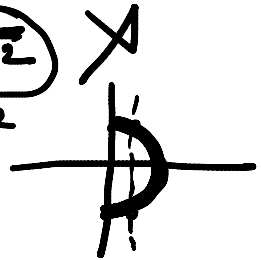
(A) $x = \sqrt{9 - y^2}$

$x^2 = 9 - y^2$

$x^2 + y^2 = 9$

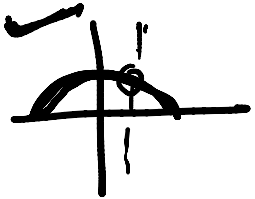
$x^2 = 9 - y^2$

$x = \pm\sqrt{9 - y^2}$



(B) $y = \sqrt{9 - x^2}$

$x^2 + y^2 = 9$



Note :

(i) If a vertical line cuts a given graph at more than one point then it can not be the graph of a function.

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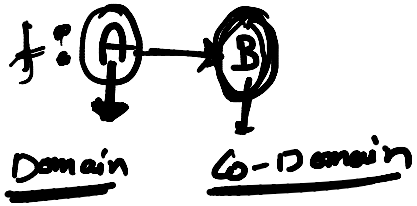
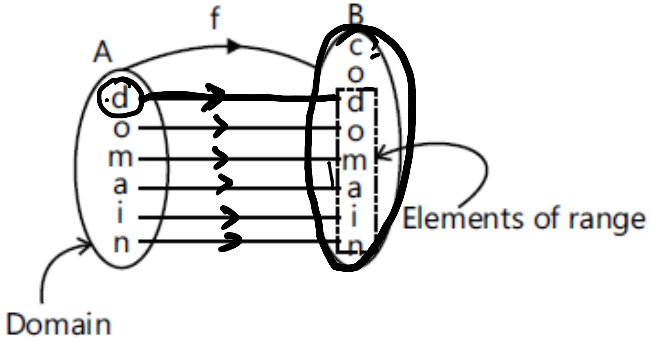
Functions

DOMAIN, CO–DOMAIN & RANGE OF A FUNCTION :

Let $f: A \to B$, then the set A is known as the domain of f & the set B is known as co-domain of f. The set of all f images of elements of A is known as the range of f .





Range

{d, o, m, a, i, n} = Range

- It should be noted that range is a subset of co–domain
- the domain of the function is the set of those real numbers, where function is defined.

$f(x) = \frac{1}{x-2}$

Domain $x \in R - \{2\}$

For a continuous function, the interval from minimum to maximum value of a function gives the range.

1. If $y = \frac{f(x)}{g(x)}$ Domain → $g(x) \neq 0$

$\frac{x+2}{(x-3)(x-4)}$ $x \in R - \{3,4\}$

$x \neq 3,4$

2. If $y = \sqrt[n]{f(x)}$ Domain → $f(x) \geq 0$ if n is even

3. If $y = \log_a x$ Domain → $x > 0, a > 0, a \neq 1$

$f(x)^{1/2}$, $f(x)^{1/4}$, $f(x)^{1/6}$

$f(x) = \sqrt{(x+2)}$ $x + 2 \geq 0$

Domain $x \geq -2$

$[-2, \infty)$

↳ If $y = \sin^{-1}[f(x)]$ Domain → $-1 \leq f(x) \leq 1$

↳ If $y = \cos^{-1} f(x)$ Domain → $-1 \leq f(x) \leq 1$

↳ If $y = {}^{n}C_r$ or ${}^{n}P_r$ Domain → $n \geq 0, r \geq 0, n \geq r$

$f(x) = \sin^{-1}(2x)$

$-1 \leq 2x \leq 1$

$\left(-\frac{1}{2} \leq x \leq \frac{1}{2}\right)$

${}^{n}C_r$, ${}^{n}P_r$

${}^{2}C_1$, ${}^{3}C_2$ ${}^{1}C_2$ ✗

Exponential/Logarithmic Function

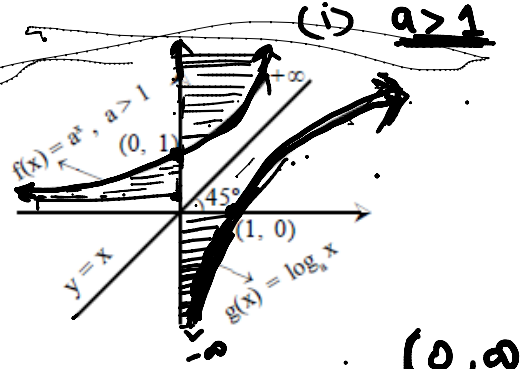
A function $f(x) = a^x = e^{x \ln a}$ $(a > 0,\ a \neq 1,\ x \in R)$ is called an exponential function. The inverse of the exponential function is called the logarithmic function . i.e. $g(x) = \log_a x$.

Note that f(x) & g(x) are inverse of each other & their graphs are as shown .

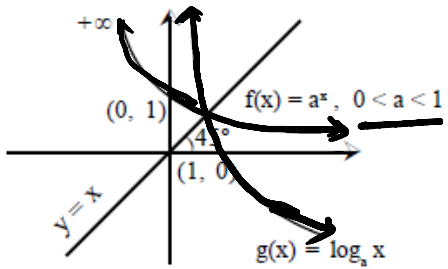
a^x, $\log_a(x)$

$x > 0$

(i) $a > 1$



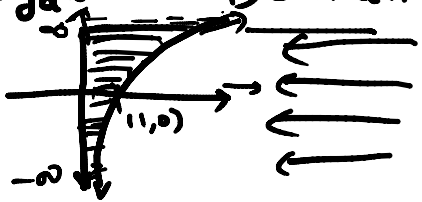
(ii) $0 < a < 1$



$(0, \infty)$

$f(x) = e^x$ domain is R and range is R^+.

$f(x) = \log_a x$



Domain $= x > 0$

R^+

$(0, \infty)$

Range $= (-\infty, \infty)$

R

Absolute Value Function : (Modulus)

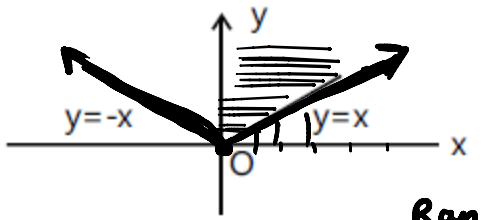
A function $y = f(x) = |x|$ is called the absolute value function or Modulus function. It is defined as

$$: y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$|-2| = -(-2) = 2$

$|-3| = 3$

$|3| = 3$



Range $[0, \infty)$

$f(x) = |x|$, domain is R and range is $R^+ \cup \{0\}$.

$(0, \infty)$

Problem

10 log (x² + x + 1)

9) The domain of the function $f(x) = \log(x^2 + x + 1) + \sin\sqrt{x - 1}$ is

(a) $(-2, 1)$ (b) $(-2, \infty)$ (c) $[1, \infty)$ (d) None of these

$x^2 + x + 1 > 0$

$D = 1 - 4(1)(1)$

$= -3 < 0$

$x - 1 \geq 0$

$x \geq 1$

$D_1 \cap D_2$

Domain : $x \geq 1$

$[1, \infty)$

9) Range of the function $y = \dfrac{x + 3}{x - 3}$ is ?

(a) [3, -3] (b) R (c) R – {1} (d) $[3, \infty]$

① Find x in terms of y

$xy - 3y = x + 3$

$x(y - 1) = 3y + 3$

$x = \dfrac{3y + 3}{y - 1}$

② Find Domain of x

$y - 1 \neq 0$

$y \neq 1$

$y \in R - \{1\}$

③ This is your Range