JEE and NEET CRASH COURSE

Rectilinear Motion

Part 02

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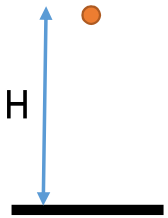
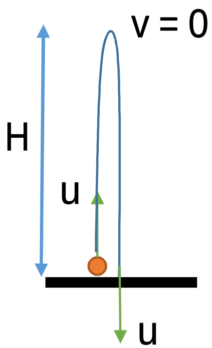
MOTION UNDER GRAVITY

Case I : Particle is dropped at rest from height H.



$$T = \sqrt{\frac{2H}{g}} \qquad v = \sqrt{2gH}$$

Case II : Particle is thrown with speed u from ground.



$$T = \frac{2u}{g} \qquad H = \frac{u^2}{2g}$$

Example

Two balls are dropped from heights h and 2h respectively from the earth surface. The ratio of time of these balls to reach the earth is : **[2003]**

(1) $1 : \sqrt{2}$ (2) $\sqrt{2} : 1$ (3) $2 : 1$ (4) $1 : 4$

$$T = \sqrt{\frac{2H}{g}} \Rightarrow T \propto \sqrt{H}$$

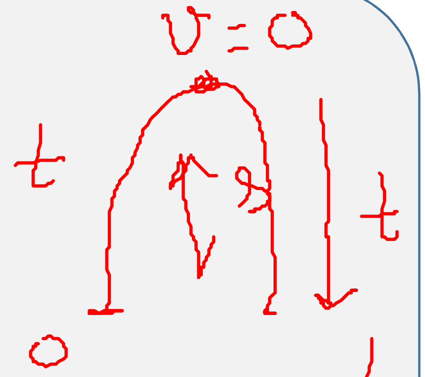
$$\frac{T_1}{T_2} = \sqrt{\frac{h}{2h}} = \frac{1}{\sqrt{2}}$$

Example

If a ball is thrwon vertically upwards with speed u, the distance covered during the last t seconds of its ascent is : **[2003]**

(1) $\frac{1}{2}gt^2$ (2) $ut - \frac{1}{2}gt^2$ (3) $(u - gt)t$ (4) ut

$$s = vt - \frac{1}{2}at^2$$
$$= 0 - \frac{1}{2}(-g)t^2$$
$$= \frac{1}{2}gt^2$$



t
+ve

For downward motion
$u = 0$
$a = g$

$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2}gt^2$$

Example

5 15 25 35 45 50 50

A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower? (take g = 10 m/s^2)

Sol. Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n-1)$

$\Rightarrow$ 45 = 0 + $\frac{10}{2}(2n-1)$

n = 5 sec

Height of tower; h = $\frac{1}{2}$gt^2

= $\frac{1}{2} \times 10 \times 5^2$ = 125 m

Let n^{th} second be the last second

$S_n = u + \frac{a}{2}(2n-1)$

$45 = 0 + 5(2n-1)$

$2n-1 = 9 \Rightarrow n = 5$

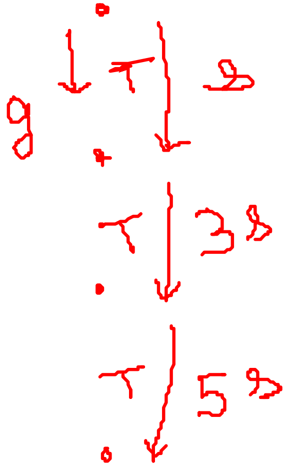
$H = \frac{1}{2}gt^2 = \frac{1}{2} \times \cancel{10}^{5} \times 25$

$= 125m$

Galileo's Law of Odds

If initial velocity is zero and acceleration is constant then ratio of displacements (distance travelled) in successive equal time intervals is in the ratio 1:3:5:7…
i.e. ratio of odd numbers

$u = 0$



If $T = 1$

$s = \frac{1}{2} g T^2$

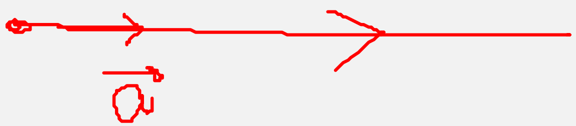
$= 5\,m$

If $u = 0$

and $\vec{a} = \text{constant}$

$s_T : s_{2T-T} : s_{3T-2T}$

$= 1 : 3 : 5 \ldots$



Example

A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is :- **[2013]**

(1) $h_1 = h_2 = h_3$ (2) $h_1 = 2h_2 = 3h_3$ ✓(3) $h_1 = h_2/3 = h_3/5$ (4) $h_2 = 3h_1$ and $h_3 = 3h_2$

$u = 0$

$g \downarrow$

$h_1 = x$

$h_2 = 3x$

$h_3 = 5x$

$h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

Example

From the top of a tower, a particle is thrown vertically downwards with a velocity of 10 m/s. The ratio of the distances, covered by it in the 3rd and 2nd seconds of the motion is (Take g = 10 m/s^2)

(1) 5 : 7 (2) 7 : 5 (3) 3 : 6 (4) 6 : 3 **[2002]**

$$S_n = u + \frac{g}{2}(2n-1)$$

$$S_3 = 10 + \frac{10}{2}(2\times3-1) = 10 + 5(5) = 35\ m$$

$$S_2 = 10 + \frac{10}{2}(2\times2-1) = 10 + 5(3) = 25\ m$$

$$S_3 : S_2 = 35 : 25 = 7 : 5$$

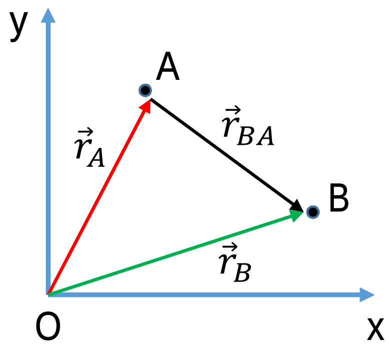
Relative Motion

Position vector of one particle w.r.t. other

$\vec{r}_A$ = position vector of point A

$\vec{r}_B$ = position vector of point B

$\vec{r}_{BA}$ = position vector of point B w.r.t. point A



$$\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$$

Velocity vector of one particle w.r.t. other

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Acceleration vector of one particle w.r.t. other

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

$$\vec{r}_A + \vec{r}_{BA} = \vec{r}_B$$

If a_{rel} = const

then

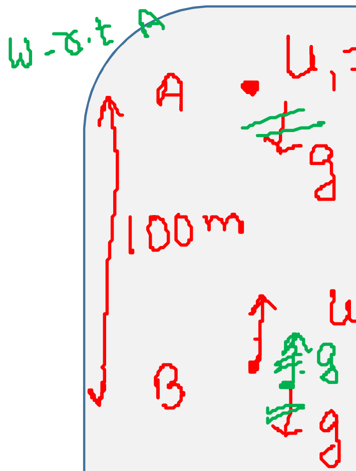
$$v_{rel} = u_{rel} + a_{rel} t$$

$$s_{rel} = u_{rel} t + \frac{1}{2} a_{rel} t^2$$

$$v_{rel}^2 = u_{rel}^2 + 2 a_{rel} \cdot s_{rel}$$

Example

A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet?



$u_1 = 0$

$u_2 = 50\ m/s$

$a_{rel} = 0$

$u_{rel} = 50\ m/s$

$s_{rel} = 100\ m$

$s_{rel} = u_{rel}\, t$

$100 = 50t$

$\boxed{t = 2s}$

$s_B = u_B t + \frac{1}{2} a_B t^2$

$= 50 \times 2 - \frac{1}{\cancel{2}} \times 10 \times \cancel{4}^{2}$

$= 100 - 20$

$= 80\ m$

Example

A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s^2.

Sol.

$$S = ut + \frac{1}{2}at^2$$

$$-60 = 5(t) + \frac{1}{2}(-10)t^2$$

$$-60 = 5t - 5t^2$$

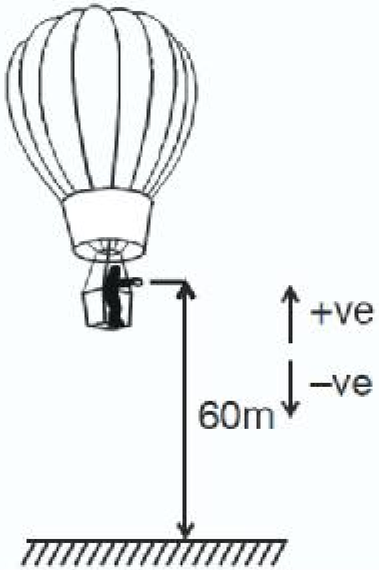
$$5t^2 - 5t - 60 = 0$$

$$t^2 - t - 12 = 0$$

$$t^2 - 4t + 3t - 12 = 0$$

$$(t - 4)(t + 3) = 0$$

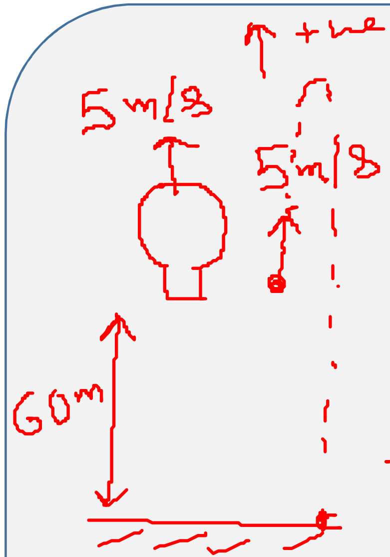
$$\therefore \quad t = 4$$



Height of balloon from ground at this instant

$$= 60 + 4 \times 5$$

$$= 80 \text{ m}$$



For stone

$u = 5$ m/s

$a = -10$

$s = -60$

$$s = ut + \frac{1}{2}at^2$$

$$-\overset{12}{\cancel{60}} = \cancel{5}t - \cancel{5}t^2$$

$$t^2 - t - 12 = 0$$

$$(t-4)(t+3) = 0$$

$t = 4$ s

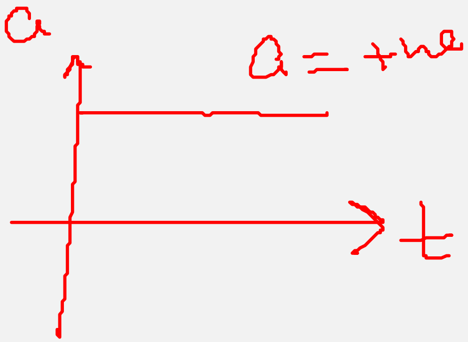
Height of balloon $= 60 + 20$

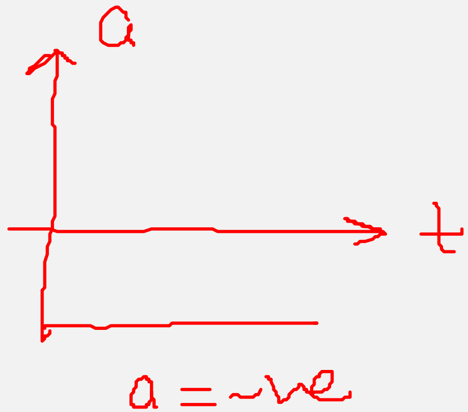
$= 80$ m

Graphs in uniformly accelerated motion

a-t graph

$a = \text{constant}$





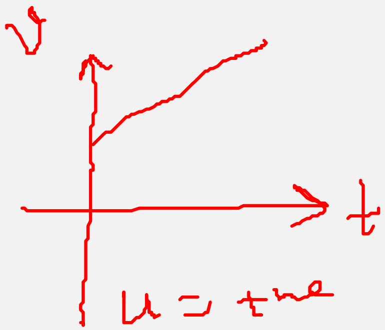
Graphs in uniformly accelerated motion

$v - t$ graph

$v = u + at$

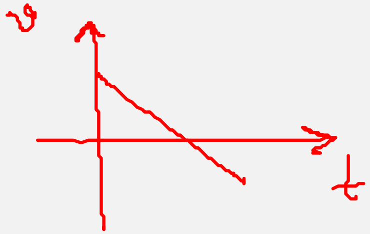
$y = mx + c$

Graph will be st. line with slope $= a$ and y intercept u.



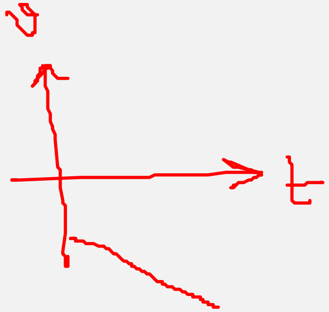
$u = +ve$

$a = +ve$



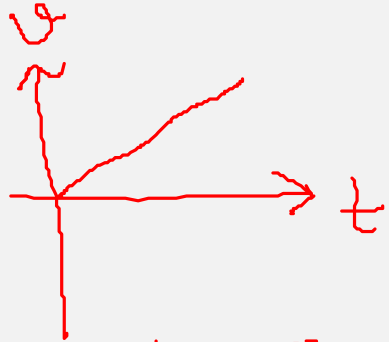
$u = +ve$

$a = -ve$



$u = -ve$

$a = -ve$



$u = 0$

$a = +ve$

Graphs in uniformly accelerated motion

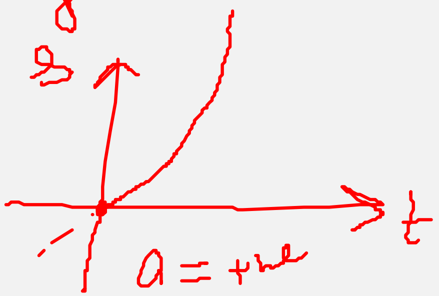
<u>s – t graph</u>

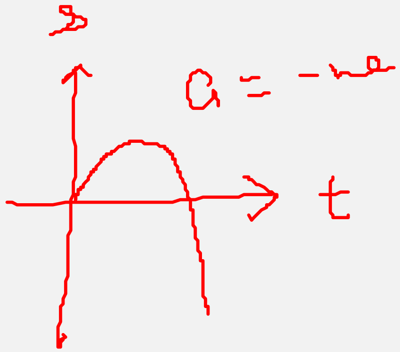
$s = ut + \frac{1}{2}at^2$

Graph will be parabolic

If $a = +ve$, then graph will be upward parabola

If $a = -ve$, " " " " downward " .

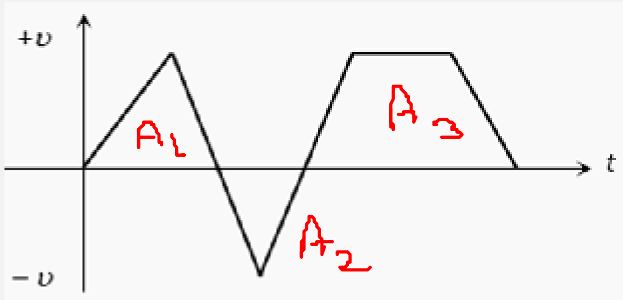




$v = \frac{ds}{dt}$

Slope of s-t or x-t graph gives velocity

Distance and displacement from v-t graph

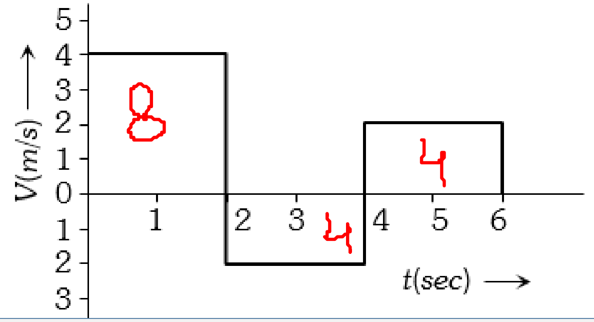


Distance $= A_1 + A_2 + A_3$

Displacement $= A_1 - A_2 + A_3$

Example

The velocity-time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 *sec* are respectively



(a) 8 m, 16 m (b) 16 m, 8 m

(c) 16 m, 16 m (d) 8 m, 8 m

disp = 8 − 4 + 4 = 8

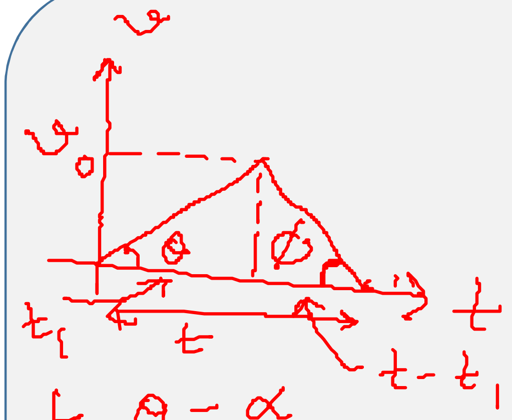
distance = 8 + 4 + 4 = 16 m

Example

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β and comes to rest. If the total time elapsed is t, then the maximum velocity acquired by the car is

[IIT 1978; CBSE PMT 1994]

(a) $\left(\dfrac{\alpha^2+\beta^2}{\alpha\beta}\right)t$ (b) $\left(\dfrac{\alpha^2-\beta^2}{\alpha\beta}\right)t$ (c) $\dfrac{(\alpha+\beta)\,t}{\alpha\beta}$ (d) $\dfrac{\alpha\beta\, t}{\alpha+\beta}$



$\tan\theta = \alpha$

$\tan\phi = \beta$

$$\boxed{s = \frac{1}{2} t \cdot v_0}$$

$$\frac{v_0}{t_1} = \alpha \Rightarrow t_1 = v_0/\alpha \quad -(1)$$

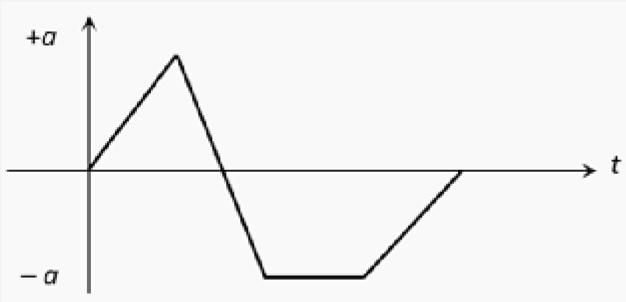
$$\frac{v_0}{t-t_1} = \beta \Rightarrow t - t_1 = \frac{v_0}{\beta} \quad -(2)$$

(1) + (2)

$$t = \frac{v_0}{\alpha} + \frac{v_0}{\beta} = v_0\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

$$v_0 = \frac{\alpha\beta t}{\alpha+\beta}$$

Change in velocity from a-t graph



$$a = \frac{dv}{dt}$$

$$\int_u^v dv = \int_0^t a\,dt$$

$$\Delta v = v - u = \int_0^t a\,dt$$

Area under a-t graph gives change in velocity

Example

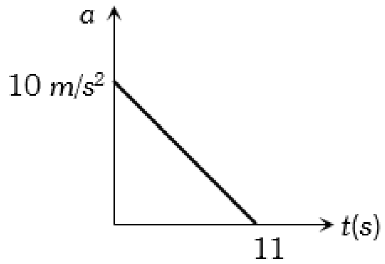
A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be **[IIT-JEE (Screening) 2004]**

(a) 110 m/s

(b) 55 m/s

(c) 550 m/s

(d) 660 m/s



$\Delta v = \text{Area}$

$v - 0 = \frac{1}{\cancel{2}} \times 11 \times \cancel{10}^{5}$

$\boxed{v = 55\ m/s}$

Motion with non-uniform acceleration

① $a = \frac{dv}{dt}$ If $a(t)$, then $\int dv = \int a(t)\,dt$

② $v = \frac{ds}{dt} \Rightarrow \int ds = \int v\,dt \Rightarrow s = \int v\,dt$

③ $a = v\frac{dv}{dx}$ If $a(x)$, then $\int a(x)\,dx = \int v\,dv$

Example

Particles acceleration is given as a = 4t m/s^2. Find velocity at t = 2 s, if initial velocity is 5 m/s.

$$a = \frac{dv}{dt}$$

$$4t = \frac{dv}{dt}$$

$$\int_5^v dv = \int_0^2 4t\,dt$$

$$[v]_5^v = 4\left[\frac{t^2}{2}\right]_0^2$$

$$v - 5 = 2[4 - 0]$$

$$v - 5 = 8$$

$$\boxed{v = 13 \text{ m/s}}$$