JEE and NEET CRASH COURSE

Rectilinear Motion

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Displacement and Distance

Displacement

- Vector joining the initial and final position of the particle.
- Can be negative, positive or zero.

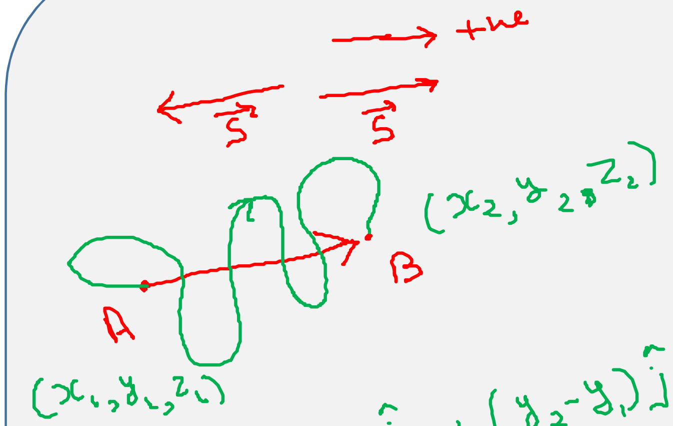
Distance

- It is the length of the actual path travelled by a particle.
- Scalar quantity

m

[L]

Distance travelled ≥ |Displacement|



$$\vec{S} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$S = |\vec{S}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Average velocity and speed

Average Velocity

$$\text{Average Velocity} = \frac{\text{Displacement}}{\text{Time Interval}} = \frac{\text{Change in position}}{\text{Time Interval}}$$

$$= \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

m/s

$[LT^{-1}]$

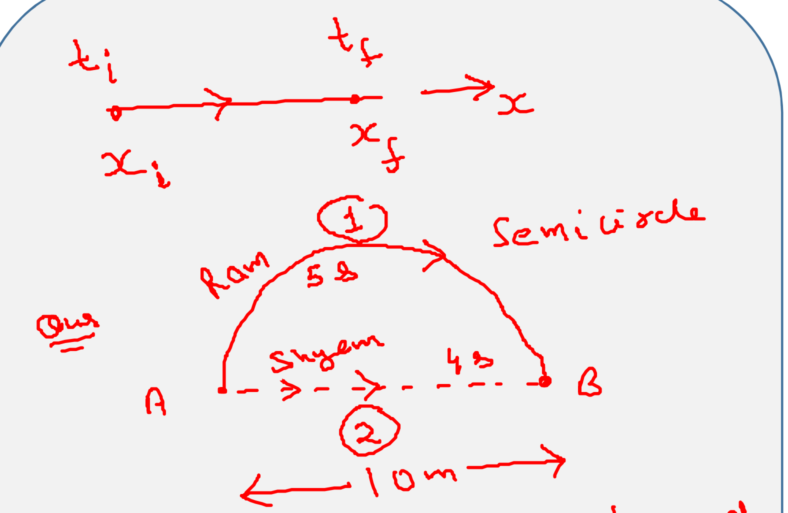
- It is vector in the direction of displacement.

Average Speed

$$\text{Average Speed} = \frac{\text{Distance travelled}}{\text{Time Interval}}$$

- Scalar quantity

Average Speed ≥ |Average Velocity|



#	Dist.	Disp.	Avg speed	Avg. vel.
Ram	$\pi R = 5\pi$	10 m	π m/s	$\frac{10}{5} = 2$ m/s
Shyam	10 m	10 m	$\frac{10}{4} = 2.5$ m/s	2.5 m/s

If, speed = constant
then, distance = speed × time

Example

A 150 m long train is moving with a uniform velocity of 45 km / h. The time taken by the train to cross a bridge of length 850 metres is : **[2001]**

(1) 56 sec (2) 68 sec (3) 80 sec (4) 92 sec

$$v = 45\ \text{Km/h} = \cancel{45}^{5} \times \frac{5}{\cancel{18}_{2}} = \frac{25}{2}\ \text{m/s}$$

$$1\ \text{Km/h} = \frac{5}{18}\ \text{m/s}$$

$$S = 850 + 150 = 1000\text{m}$$

$$\text{dist.} = \text{speed} \times \text{time}$$

$$\Rightarrow \text{time} = \frac{\cancel{1000}^{40}}{\cancel{25}} \times 2 = 80\ \text{s}$$

Example

A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

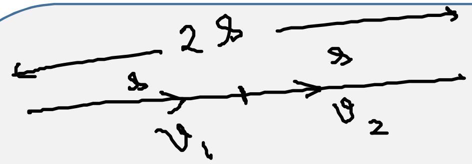
Sol.

Let total distance travelled by the particle be 2s.

Time taken to travel first half $= \dfrac{s}{v_1}$

Time taken to travel next half $= \dfrac{s}{v_2}$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$



$$= \frac{2s}{s\left[\frac{1}{v_1}+\frac{1}{v_2}\right]}$$

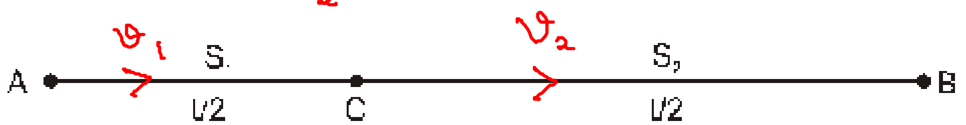
$$= \frac{2}{\frac{v_2+v_1}{v_1v_2}}$$

$$\boxed{v_{avg} = \frac{2v_1v_2}{v_1+v_2}}$$

Example

A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by

Sol. $S_1 = v_1 \frac{t}{2}$ $\qquad$ $S_2 = v_2 \frac{t}{2}$



As shown, the person travels from A to B through a distance S, where first part S_1 is travelled in time t/2 and next S_2 also in time t/2.

So, according to the condition : $v_1 = \frac{S_1}{t/2}$ and $v_2 = \frac{S_2}{t/2}$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}} = \frac{S_1 + S_2}{t} = \frac{\frac{v_1 t}{2} + \frac{v_2 t}{2}}{t} = \frac{v_1 + v_2}{2}$$

Total disp. $= s_1 + s_2$

$= (v_1 + v_2)\frac{t}{2}$

$$\text{Avg. velocity} = \frac{\text{Total disp.}}{\text{Total time}}$$

$$= \frac{(v_1 + v_2)\frac{t}{2}}{t}$$

$$= \frac{v_1 + v_2}{2}$$

Instantaneous Velocity and Speed

Instantaneous Velocity

- The velocity at a particular instant of time is known as instantaneous velocity.

$$v = \frac{dx}{dt}$$

m/s

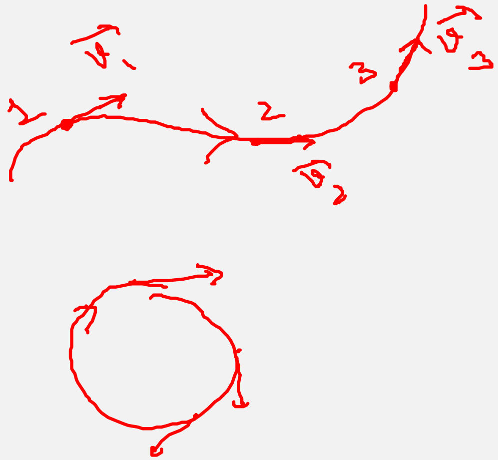
$[LT^{-1}]$

- **It is always tangential to the path.**

Instantaneous Speed

- The magnitude of instantaneous velocity is called instantaneous speed.

Instantaneous Speed = |Instantaneous Velocity|



$$\vec{v} = \frac{d\vec{r}}{dt}$$

Average and Instantaneous Acceleration

Average Acceleration

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

Ø It is a vector in the direction of change in velocity.

Instantaneous Acceleration

Ø It is acceleration at a particular instant of time.

$$a = \frac{dv}{dt} = v\frac{dv}{ds}$$

$$= \frac{d^2x}{dt^2}$$

m/s^2

$[LT^{-2}]$

$$\boxed{a = \frac{dv}{dt}}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$\boxed{a = v\frac{dv}{dx}}$$

Example

Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at $t = 2$ s?

Sol.

Velocity; $\quad v = \dfrac{dx}{dt} = 10t + 4$

At $\quad t = 2$ s

$v = 10(2) + 4$

$v = 24$ m/s

Acceleration; $\quad a = \dfrac{d^2x}{dt^2} = 10$

Acceleration is constant, so at $t = 2$ s, $a = 10$ m/s^2

$a = \dfrac{d^2x}{dt^2} = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right)$

$= \dfrac{d}{dt}(10t + 4)$

$= 10$

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$\dfrac{d}{dt}(t^n) = nt^{n-1}$

$\dfrac{d}{dt}(\text{const}) = 0$

Example

Equation of displacement for any particle is s = $3t^3 + 7t^2 + 14t + 8$m. Its acceleration at time t = 1 sec is : **[2000]**

(1) 10 m/s^2 (2) 16 m/s^2 (3) 25 m/s^2 (4) 32 m/s^2

$v = \frac{ds}{dt} = 9t^2 + 14t + 14$

$a = \frac{dv}{dt} = 18t + 14$

At $t = 1s$

$a = 18(1) + 14$

$= 32 \, m/s^2$

Example

The motion of a particle along a straight line is described by equation : $x = 8 + 12t - t^3$, where x is in metre and t in second. The retardation of the particle when its velocity becomes zero, is : **[2012]**

(1) $24\ ms^{-2}$ (2) zero (3) $6\ ms^{-2}$ (4) $12\ ms^{-2}$

$x = 8 + 12t - t^3$

$v = \frac{dx}{dt} = 0 + 12 - 3t^2$

$a = \frac{dv}{dt} = -6t$

when $v = 0$

$\Rightarrow 12 - 3t^2 = 0$

$\Rightarrow \cancel{3}t^2 = \cancel{12}\ 4$

$\Rightarrow t = 2s$

$a = -6(2)$

$= -12\ m/s^2$

retardation $= 12\ m/s^2$

UNIFORM MOTION

Motion with constant velocity

$\vec{v}$ = constant

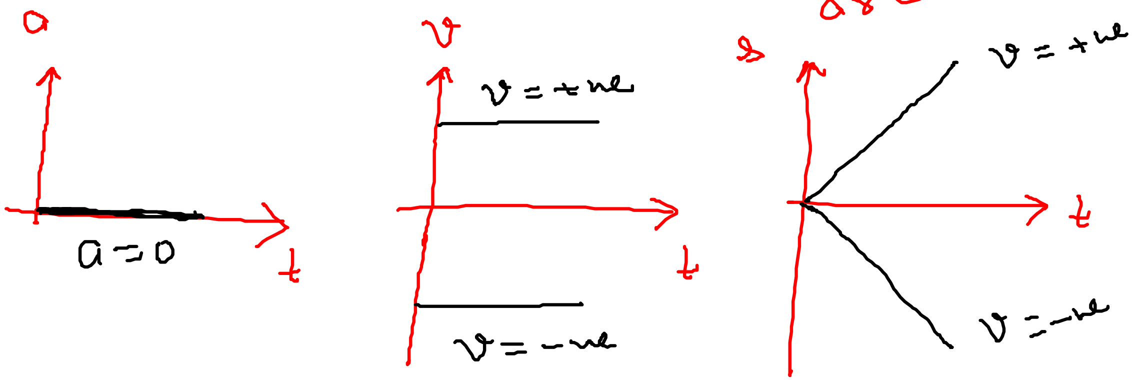
a = zero

$s = x_f - x_i = vt$

$\vec{v}$ = constant

↓

Magnitude and direction both are constant



$\vec{v}$ = const.

Avg. velocity = $\vec{v}$

$\frac{\vec{s}}{t} = \vec{v}$

$\boxed{\vec{s} = \vec{v}\,t}$

y

$s = v\,t$ (c, m, x)

$y = mx$

St. line

$y = mx + c$

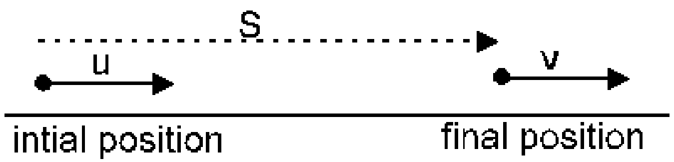
slope

y-intercept

SAFALTA.COM

Uniformly Accelerated Motion

Motion with constant acceleration

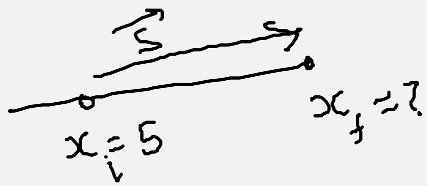


(a)	$v = u + at$
(b)	$s = ut + 1/2\ at^2$
	$s = vt - 1/2\ at^2$
	$x_f = x_i + ut + 1/2\ at^2$
(c)	$S = \left(\frac{v+u}{2}\right)t$
(d)	$V_{av} = \frac{v+u}{2}$
(e)	$v^2 = u^2 + 2as$
(f)	$s_n = u + a/2\ (2n - 1)$

$x_f - x_i$

u = initial velocity
a = acceleration
v = final velocity
s = displacement ($x_f - x_i$)
x_f = final coordinate (position)
x_i = initial coordinate (position)
s_n = displacement during the n^{th} sec

$\vec{a}$ = constant



Note: In case of motion under gravity, constant acceleration is g downwards.

Example

A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

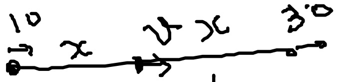
Sol.

Let the total distance be 2x.

$\therefore$ distance upto midpoint = x

Let the velocity at the mid point be v

and acceleration be a.



From equations of motion

$v^2 = 10^2 + 2ax$ ... (1)

$30^2 = v^2 + 2ax$ ... (2)

(1)−(2) ~~(2) − (1)~~ gives

$v^2 - 30^2 = 10^2 - v^2 \;\Rightarrow\; 2v^2 = 10^2 + 30^2 = 100 + 900$

$\Rightarrow \quad v^2 = 500 \quad \Rightarrow \quad v = 10\sqrt{5}$ m/s

$v_m = \sqrt{\dfrac{u^2+v^2}{2}}$