Mathematical Reasoning

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STATEMENT :

A sentence which is either true or false but cannot be both are called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.

If a statement is true then its truth value is T and if it is false then its truth value is F

For ex.

(i) "New Delhi is the capital of India", a true statement

(ii) "3 + 2 = 6", a false statement

(iii) "Where are you going ?" not a statement beasuse it connot be defined as true or false

Note : A statement cannot be both true and false at a time

SIMPLE STATEMENT :

Any statement whose truth value does not depend on other statement are called simple statement

For ex. (i) "$\sqrt{2}$ is an irrational number" T (ii) "The set of real number is an infinite set" T

COMPOUND STATEMENT :

A statement which is a combination of two or more simple statements are called compound statement

Here the simple statements which form a compound statement are known as its sub statements

For ex. q → True p → True

(i) "If x is divisible by 2 then x is even number"

(ii) "ΔABC is equilatral if and only if its three sides are equal"

LOGICAL CONNECTIVES

The words or phrases which combined simple statements to form a compound statement are called logical connectives.

In the following table some possible connectives, their symbols and the nature of the compound statement formed by them

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	→ conjunction
2.	or	$\vee$	$p \vee q$	→ disjunction
3.	not	~ or '	~p or p'	→ negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	→ Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	→ Equivalence or Bi-conditional

Double Implication

and $\wedge$ or $\vee$

If (x is divisible by 2) then (x is even)

Explanation :

(i) $p \wedge q \equiv$ statement p and q

($p \wedge q$ is true only when p and q both are true otherwise it is false)

(ii) $p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true i.e. $p \vee q$ is false only when p and q both are false)

(iii) $\sim p \equiv$ not statement p

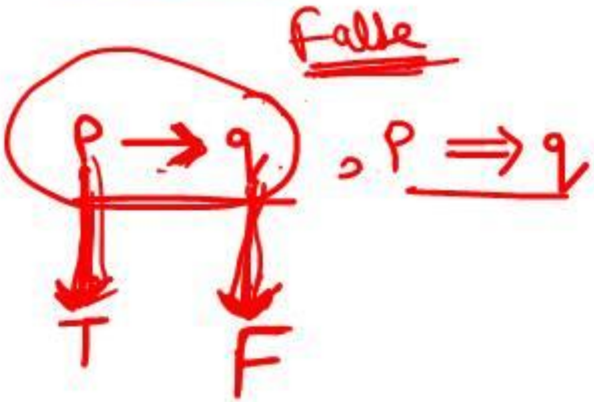
($\sim p$ is true when p is false and $\sim p$ is false when p is true)

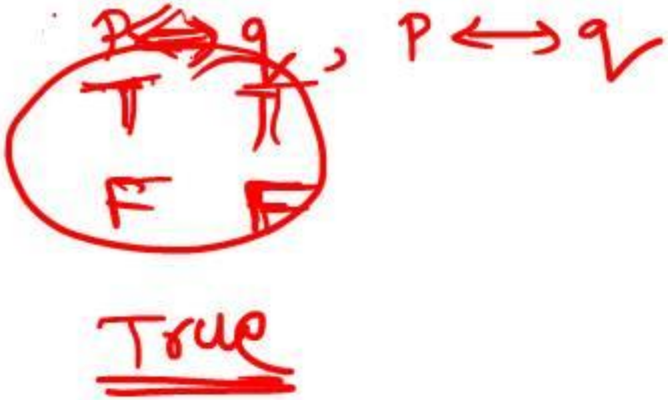
(iv) $p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false only when p is true and q is false otherwise it is true for all other cases)

(v) $p \Leftrightarrow q \equiv$ statement p if and only if statement q

($p \Leftrightarrow q$ is true only when p and q both are true or false otherwise it is false)





Conjunction (AND)

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Negation (NOT)

p	(~ p)
T	F
F	T

q ~q

T F → F

Conditional (P→q)

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional (P↔q)

p	q	p → q	q → p	(p → q) ∧ (q → p) or p ↔ q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$$P \longleftrightarrow q = \underline{(P \rightarrow q)} \wedge (\underline{q \longrightarrow} P)$$

Venn Diagram

Regions → 1, 2, 3, 4

① AND $(p \wedge q) \longrightarrow 2$ ✓

② OR $(p \vee q) \longrightarrow 1, 2, 3$ ✓

③ NOT $(\sim p) \longrightarrow 3, 4$

$(\sim q) \longrightarrow 1, 4$

④ Implication $(\underset{T}{\underset{\downarrow}{p}} \rightarrow \underset{F}{\underset{\downarrow}{q}}) \longrightarrow \underline{\underline{2, 3, 4}}$ ✓

⑤ Double Implication $(p \leftrightarrow q) \longrightarrow \underline{\underline{2, 4}}$

9 If $\overset{T}{\overset{\downarrow}{p}} \Rightarrow \overset{F}{\overset{\downarrow}{(q \vee r)}}$ is false, then the truth values of p, q, r are respectively (2019 Main, 9 April II)

(a) T, T, F [✓ (b) T, F, F]

(c) F, F, F (d) F, T, T

$p \rightarrow T$

T, F, F

$\underset{F}{\underset{\downarrow}{q}} \vee \underset{F}{\underset{\downarrow}{r}} \longrightarrow F$

⇒ LOGICAL EQUIVALENCE

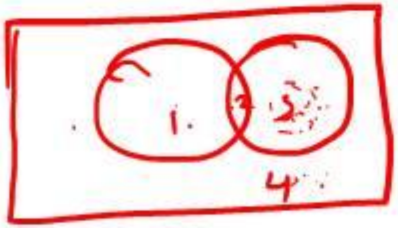
Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

For ex. The truth table for $(p \to q)$ and $({\sim}p \vee q)$ given as below

	p	q	(~p)	$p \to q$	$\sim p \vee q$
2 →	T	T	F	T	T
1 →	T	F	F	F	F
3 →	F	T	T	T	T
4 →	F	F	T	T	T

$\boxed{{\sim}p \vee q}$

3, 4 . 2, 3



$\boxed{2, 3, 4}$

We observe that last two columns of the above truth table are identical hence compound statements $(p \to q)$ and $({\sim}p \vee q)$ are equivalent

i.e. $\boxed{p \to q \equiv {\sim} p \vee q}$

$\boxed{p \to q \equiv {\sim} p \vee q}$

Q). $({\sim}p) \wedge q \longrightarrow ③$

TAUTOLOGY AND CONTRADICTION :

(i) **Tautology** : A statement is said to be a tautology if it is true for all logical possibilities i.e. its truth value always T. it is denoted by t.

(ii) **Contradiction** : A statement is a contradiction if it is false for all logical possibilities. i.e. its truth value always F. It is denoted by c.

Q) Which one of the following statements is not a tautology? (2019 Main, 8 April II)

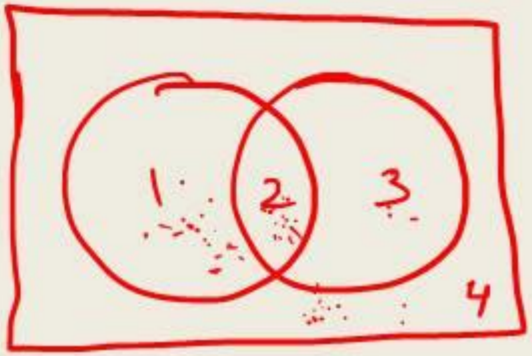
T F

(a) $(p \wedge q) \rightarrow (\sim p) \vee q$

(b) $(p \wedge q) \rightarrow p$

(c) $p \rightarrow (p \vee q)$

(d) $(p \vee q) \rightarrow (p \vee (\sim q))$



(a) $2 \rightarrow (2, 3, 4) \rightarrow$ True

True False

(b) $2 \rightarrow 1, 2 \rightarrow$ True

(c) $1, 2 \rightarrow (1, 2, 3) \rightarrow$ True

(d) $1, 2, 3 \rightarrow 1, 2, 4 \rightarrow$ False

T

① **Distributive laws :** (a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$\rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\rightarrow A \cup (B \cap C) = (A \cap B) \cap (A \cup C)$

② **De Morgan Laws :** (a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ $\leftarrow$

(b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim$

NEGATION OF COMPOUND STATEMENTS :

If p and q are two statements then

(i) **Negation of conjunction :** $\sim(p \wedge q) \equiv \sim p \vee \sim q$

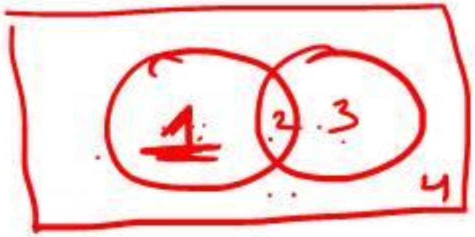
~~(ii)~~ **Negation of disjunction :** $\sim(p \vee q) \equiv \sim p \wedge \sim q$

~~(iii)~~ **Negation of conditional :** $\sim(p \to q) \equiv \underline{p \wedge \sim q}$

(iv) **Negation of biconditional :** $\sim(p \leftrightarrow q) \equiv \underset{①}{\underline{\overset{p \to \sim}{(p \wedge \sim q)}}} \vee \underset{①}{\underline{\overset{q \to p}{(q \wedge \sim p)}}}$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\boxed{\sim(p \to q) \equiv p \wedge \sim q}$



$p \to q$ $\underline{2,3,4}$

1. The boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to

(2019 Main, 12 April II)

(a) $p \wedge q$ (b) $q \Rightarrow \sim p$

(c) $p \vee q$ (d) $(\sim p) \Rightarrow q$

4 Marks

$$\sim(p \rightarrow q) \equiv \underline{p \wedge \sim q}$$

$$\sim(p \rightarrow (\sim q)) \equiv p \wedge \sim(\sim q)$$

$$\equiv \boxed{\underline{p \wedge q}}$$

8) The negation of the boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to **(2019 Main, 10 April II)**

~~(a)~~ $s \wedge r$ (b) $\sim s \wedge \sim r$

(c) $s \vee r$ (d) r

$$\boxed{\sim s \vee (\sim r \wedge s)}$$

$$\sim(\sim s \vee (\sim r \wedge s))$$

$$s \wedge \sim(\sim r \wedge s)$$

$$s \wedge (r \vee \sim s)$$

$$(s \wedge r) \vee \underbrace{(s \wedge \sim s)}_{\text{False}}$$

For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is **(2019 Main, 9 April I)**

(a) $\sim p \wedge \sim q$

(b) $\sim p \vee \sim q$

(c) $p \wedge q$

(d) $p \leftrightarrow q$

If p and q are two statements then

Let p $\Rightarrow$ q Then

(i) (Contrapositive of p $\Rightarrow$ q)is($\sim q \Rightarrow \sim p$)

(ii) (Contradiction of p $\Rightarrow$ q)is($q \Rightarrow \sim p$)

(iii) (Converse of p $\Rightarrow$ q)is($q \Rightarrow p$)

$p \rightarrow q$

$\boxed{\sim q \rightarrow \sim p}$

Write the contrapositive of the following statement : "If Mohan is poet, then he is poor"

$(p) \rightarrow q$

If He is not Poor then Mohan is not poet