PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Atomic Physics and Nuclear Physics

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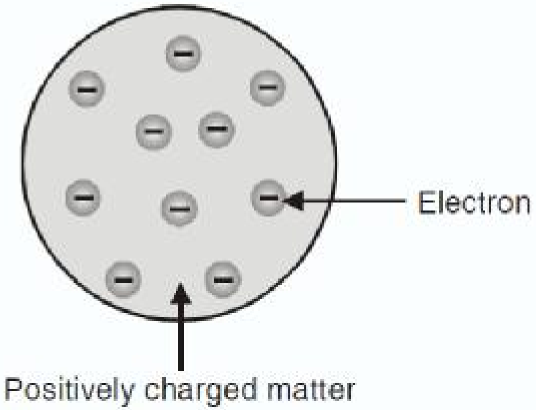
Thomson's Atomic Model

J.J. Thomson suggested that atoms are just positively charge lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's model called the 'plum pudding' model is illustrated in figure.



Rutherford's Atomic Model

Rutherford suggested that; " All the positive charge and nearly all the mass were concentrated in a very small volume of nucleus at the centre of the atom. The electrons were supposed to move in circular orbits round the nucleus (like planets round the sun). The electrostatic attraction between the two opposite charges being the required centripetal force for such motion.

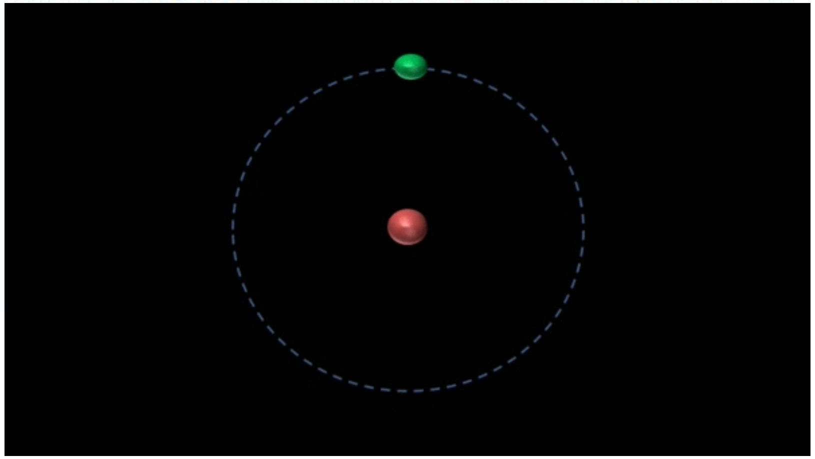
Hence $$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

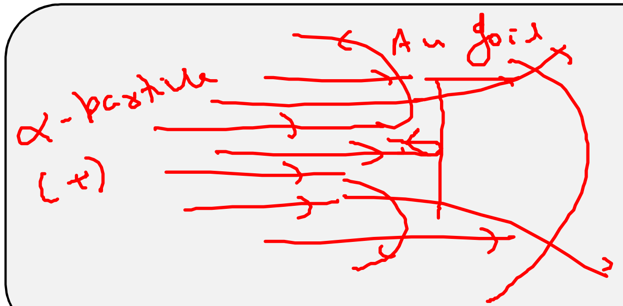
and total energy = potential energy + kinetic energy

$$= \frac{-kZe^2}{2r}$$

$\frac{K(Ze)(-e)}{r} = -\frac{KZe^2}{r}$

$\frac{1}{2}mv^2 = \frac{KZe^2}{2r}$





T.E. = −K.E. = $\frac{P.E}{2}$

Defects of Rutherford's Model

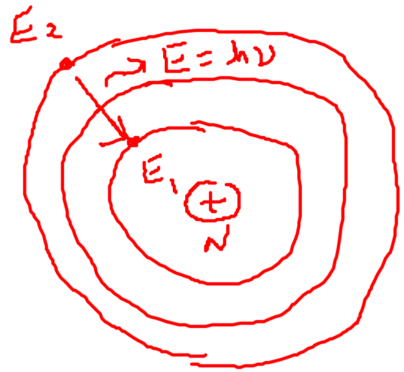
Regarding stability of atom : An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should therefore, emit radiation continuously and thereby lose energy. If total energy decreases then radius increases as given by above formula. If this happened the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second. But atoms do not collapse. In 1913 an effort was made by Neils Bohr to overcome this paradox.

Regarding explanation of line spectrum : In Rutherford's model, due to continuously changing radii of the circular orbits of electrons, the frequency of revolution of the electrons must be chaning. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these waves will be 'continuous' in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

Bohr's Atomic Model

In 1913, Prof. Niel Bohr removed the difficulties of Rutherford's atomic model by the application of Planck's quantum theory. For this he proposed the following postulates

(1) An electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory.



(2) According to Bohr, there is a definite energy associated with each stable orbit and an atom radiaties energy only when it makes a transition from one of these orbits to another. If the energy of electron in the higher orbit be E_2 and that in the lower orbit be E_1, then the frequency ν of the radiated waves is given by

$$h\nu = E_2 - E_1$$

or

$$\nu = \frac{E_2 - E_1}{h}$$

(3) Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \qquad (n = 1, 2, 3....)$$

Radius, Speed and Energy of electron in n^{th} orbit

FOR HYDROGEN ATOM : (Z = atomic number = 1)

(i) L_n = angular momentum in the n^{th} orbit $= n\dfrac{h}{2\pi}$.

(ii) r_n = radius of n^{th} circular orbit $= (0.529\ A^o)\, n^2$; $(1A^o = 10^{-10}\ m)$; $r_n \propto n^2$.

(iii) E_n Energy of the electron in the n^{th} orbit $= \dfrac{-13.6\ ev}{n^2}$ i.e. $E_n \propto \dfrac{1}{n^2}$.

Note : Total energy of the electron in an atom is negative , indicating that it is bound .

$$\text{Binding Energy } (BE)_n = -E_n = \frac{13.6\ ev}{n^2}\ .$$

$$\frac{KZe^2}{r^2} = \frac{mv^2}{r} \quad -(1)$$

$$mvr = \frac{nh}{2\pi} \quad -(2)$$

$$E = -\frac{KZe^2}{2r} \quad -(3)$$

$$r_n = \frac{n^2h^2}{4\pi^2 mKZe^2} = \left(\frac{h^2}{4\pi^2 mKe^2}\right)\frac{n^2}{Z}$$

$$\boxed{r_n \propto \frac{n^2}{Z}}$$

Radius, Speed and Energy of electron in nth orbit

(iv) $E_{n2} - E_{n1}$ = Energy emitted when an electron jumps from n_2^{th} orbit to n_1^{th} orbit $(n_2 > n_1)$.

$$\Delta E = (13.6\ ev)\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right].$$

$\Delta E = h\nu$; ν = frequency of spectral line emitted.

$$\frac{1}{\lambda} = \bar{\nu} = \text{wave no. [no. of waves in unit length (1m)]} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right].$$

$$\frac{1}{R} \approx 911\ \mathring{A}$$

Where R = Rydberg's constant for hydrogen = $1.097 \times 10^7\ m^{-1}$. $\approx 1.1 \times 10^7\ m^{-1}$

(v) For hydrogen like atom/spicies of atomic number Z :

$$r_{nz} = \frac{\text{Bohr radius}}{Z}\ n^2 = (0.529\ A^o)\frac{n^2}{Z} \quad ; \quad E_{nz} = (-13.6)\frac{Z^2}{n^2}\ ev$$

$R_z = RZ^2$ – Rydberg's constant for element of atomic no. Z.

$r_n \propto \frac{n^2}{Z}$ $E_n \propto \frac{Z^2}{n^2}$ $v \propto \frac{Z}{n}$

Definitions valid for single electron system

(1) **Ground state :** Lowest energy state of any atom or ion is called ground state of the atom. (n = 1)

Ground state energy of H atom = –13.6 eV | Ground state energy of He^+ Ion = –54.4 eV

(2) **Excited State :** State of atom other than the ground state are called its excited states.

n = 2	first excited state	n = 4	third excited state
n = 3	second excited state	$n = n_0 + 1$	n_0^{th} excited state

(3) **Ionisation energy (I.E.) :** Minimum energy required to move an electron from ground state to $n = \infty$ is called ionisation energy of the atom or ion

Ionisation energy of H atom = 13.6 eV | Ionisation energy of He^+ Ion = 54.4 eV

(4) **Ionisation potential (I.P.) :** Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called ionisation potential of the atom.

I.P of H atom = 13.6 V | I.P. of He^+ Ion = 54.4 V

$I.P. = \frac{I.E}{e}$

(5) **Excitation energy :** Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.

Energy in ground state of H atom = –13.6 eV

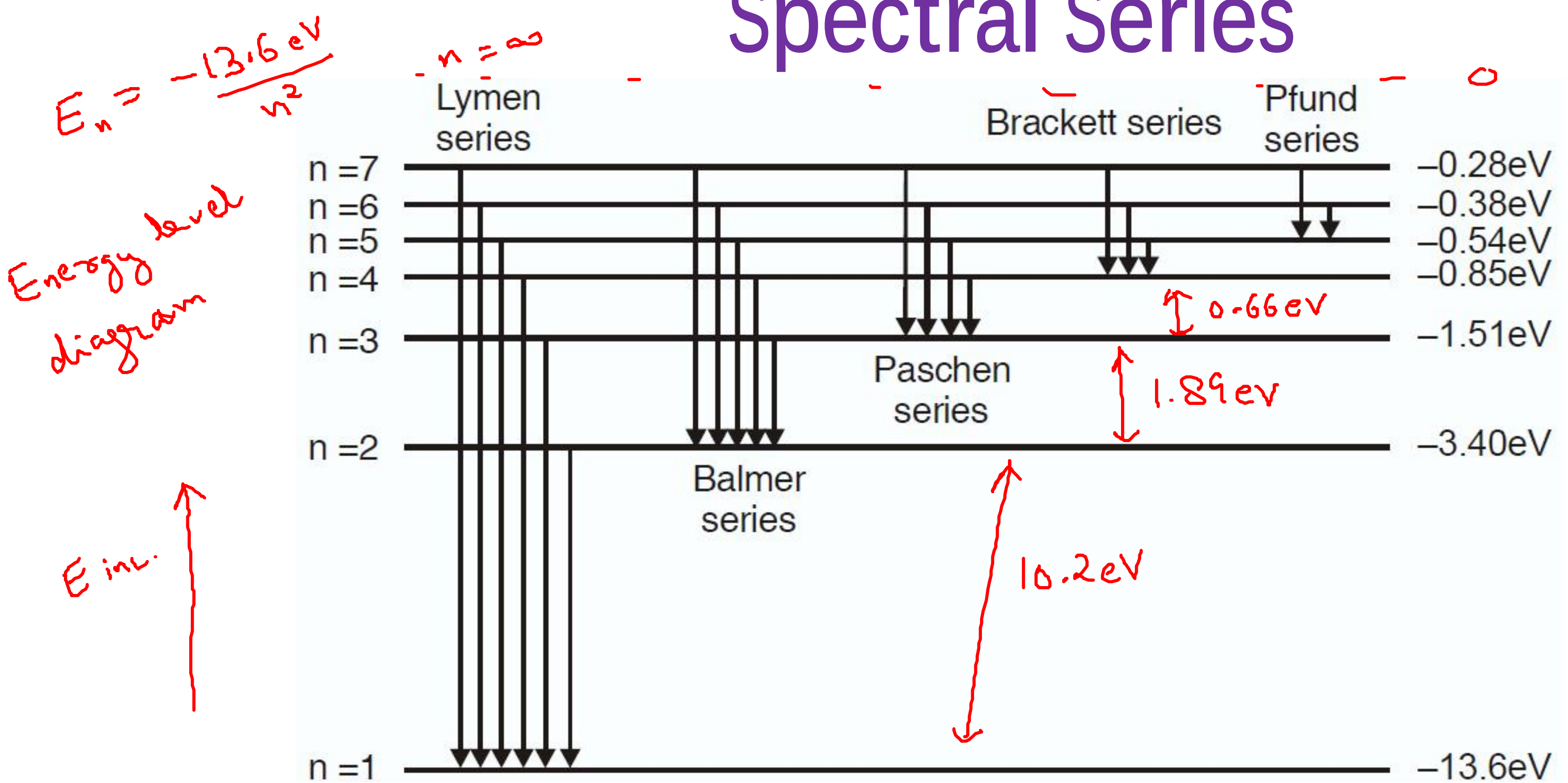
Energy in first excited state of H-atom = –3.4 eV

I^{st} excitation energy = 10.2 eV.

(6) **Binding energy or Seperation energy :** Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state. or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular n is called binding energy of that state.

Binding energy of ground state of H-atom = 13.6 eV

Spectral Series



Spectral Series

(i) **Lyman Series :** (Landing orbit n = 1) .

Ultraviolet region $\bar{v} = R\left[\frac{1}{1^2} - \frac{1}{n_2^{\,2}}\right]$; $n_2 > 1$

(ii) **Balmer Series :** (Landing orbit n = 2)

Visible region $\bar{v} = R\left[\frac{1}{2^2} - \frac{1}{n_2^{\,2}}\right]$; $n_2 > 2$

(iii) **Paschan Series :** (Landing orbit n = 3)

In the near infrared region $\bar{v} = R\left[\frac{1}{3^2} - \frac{1}{n_2^{\,2}}\right]$; $n_2 > 3$

(iv) **Bracket Series :** (Landing orbit n = 4)

In the mid infrared region $\bar{v} = R\left[\frac{1}{4^2} - \frac{1}{n_2^{\,2}}\right]$; $n_2 > 4$

(v) **Pfund Series :** (Landing orbit n = 5)

In far infrared region $\bar{v} = R\left[\frac{1}{5^2} - \frac{1}{n_2^{\,2}}\right]$; $n_2 > 5$

In all these series n_2 $= n_1 + 1$ is the α line

$= n_1 + 2$ is the β line

$= n_1 + 3$ is the γ line .... etc . where n_1 = Landing orbit

3 6 ——— $\frac{3(3-1)}{2}$

2 5 ———

1 4 ———

1st 4 lines lie in visible region. rest in UV region.

No. of different photons/spectral lines emitted if electron is in nth state $= \frac{n(n-1)}{2}$

nearest transition ⇒ 1st line of series (E min., λ max)

farthest transition ⇒ last line of series or series limit (E max, λ min.)

Nucleus

(a) ***Discoverer*** **:** Rutherford

(b) ***Constituents*** **:** neutrons (n) and protons (p) [collectively known as nucleons]

1. **Neutron :** It is a neutral particle. It was discovered by J. Chadwick.
   Mass of neutron, $m_n = 1.6749286 \times 10^{-27}$ kg.

2. **Proton :** It has a charge equal to +e. It was discovered by Goldstein.
   Mass of proton, $m_p = 1.6726231 \times 10^{-27}$ kg

$$m_p \lesssim m_n$$

Nucleus

(c) ***Representation*** :

$$_ZX^A \quad \text{or} \quad {}^A_ZX$$

where X ⇒ symbol of the atom

Z ⇒ Atomic number = number of protons

A ⇒ Atomic mass number = total number of nucleons.
= no. of protons + no. of neutrons.

Atomic mass number :

It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

$$1 \text{ a.m.u.} = \frac{1}{12} [\text{mass of one atom of } {}_6C^{12} \text{ atom at rest and in ground state}]$$

1.6603×10^{-27} kg ; 931.478 MeV/c^2

mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

Some definitions :

(1) ***Isotopes*** : $_6C^{12}$ $_6C^{14}$

The nuclei having the same number of protons but different number of neutrons are called isotopes.

(2) ***Isotones :*** $_6C^{14}$ $_8O^{16}$

Nuclei with the same neutron number N but different atomic number Z are called isotones.

(3) ***Isobars*** : $_7N^{14}$ $_6C^{14}$

The nuclei with the same mass number but different atomic number are called isobars.

Nucleus

(d) ***Size*** of nucleus : Order of 10^{-15} m (fermi)

Radius of nucleus ; $R = R_0 A^{1/3}$

where $R_0 = 1.1 \times 10^{-15}$ m (which is an empirical constant)

A = Atomic mass number of atom.

(e) ***Density*** : $\text{density} = \dfrac{\text{mass}}{\text{volume}} \cong \dfrac{Am_p}{\frac{4}{3}\pi R^3} = \dfrac{Am_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3} = \dfrac{3m_p}{4\pi R_0^3}$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.1 \times 10^{-15})^3} = 3 \times 10^{17}\ \text{kg/m}^3$$

Nuclei of almost all atoms have almost same density as nuclear density is independent of the mass number (A) and atomic number (Z).

Mass Defect

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{expected} = Z\,m_p + (A - Z)m_n$$

$$M_{observed} = M_{atom} - Zm_e$$

It is found that

$$M_{observed} < M_{expected}$$

Hence, mass defect is defined as

Mass defect = $M_{expected} - M_{observed}$

$\Delta m = [Zm_p + (A - Z)m_n] - [M_{atom} - Zm_e]$

Binding Energy

It is the minimum energy required to break the nucleus into its constituent particles.

or

Amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

Binding Energy (B.E.) = Δmc^2

$$BE = \Delta m \text{ (in amu)} \times 931.5 \text{ MeV/amu}$$

$$= \Delta m \times 931.5 \text{ MeV}$$

$$= \Delta m \times 931 \text{ MeV}$$

Note : If binding energy per nucleon is more for a nucleus then it is more stable.

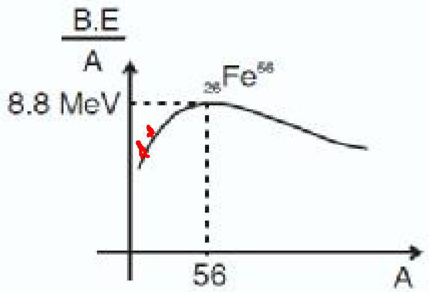
For example

If $\left(\frac{B.E_1}{A_1}\right) > \left(\frac{B.E_2}{A_2}\right)$

then nucleus 1 would be more stable.

Variation of Binding Energy per Nucleon with Mass Number

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.7 MeV for A : 50 → 80. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases. Binding energy per nucleon is maximum for $_{26}Fe^{56}$, which is equal to 8.8 MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.



Radioactivity

It was dscovered by Henry Becquerel.

Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called ***radioactivity****. Substances which shows radioactivity are known as* ***radioactive substance****.*

Radioactivity was studied in detail by Rutherford.

In radioactive decay, an unstable nucleus emits α particle or β particle. After emission of α or β the remaining nucleus may emit γ-particle, and converts into more stable nucleus.

Alpha, beta and gamma particles

α-particle :

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of α-particle = Mass of $_2He^4$ atom $- 2m_e = 4\ m_p$

Charge of α-particle = + 2 e

β-particle :

(a) **β^- (electron) :**

Mass = m_e ; Charge = $-e$

(b) **β^+ (positron) :**

Mass = m_e ; Charge = $+e$

γ-particle : They are energetic photons of energy of the order of Mev and having rest mass zero.

αβγ by Becquerel

Curie

1 Curie ≈ 3.7×10^{10} αβγ

↑

1 gm radium

Alpha Decay

$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$ + photon (γ particle)

Q value : *It is definied as energy released during the decay process.*

Q value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in K.E. of the products.

Let, M_x = mass of atom $_ZX^A$

M_y = mass of atom $_{Z-2}Y^{A-4}$

M_{He} = mass of atom $_2He^4$.

$$Q \text{ value} = [(M_x - Zm_e) - \{(M_y - (Z-2)\,m_e) + (M_{He} - 2m_e)\}]c^2$$

$$= [M_x - M_y - M_{He}]\,c^2$$

Kinetic Energy of products

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q - E);\quad T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q - E)$$

E is the energy of emitted γ-particles

Beta (minus) Decay

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + Q + \bar{\nu}$$

Antineutrino

$_{-1}\beta^{0}$

Let, M_x = mass of atom $_{Z}X^{A}$

M_y = mass of atom $_{Z+1}Y^{A}$

m_e = mass of electron

Q value = $[(M_X - Zm_e) - \{(M_Y - (z+1)m_e) + m_e\}]\,c^2 = [M_X - M_Y]\,c^2$

Considering actual number of electrons.

Q value = $[M_X - \{(M_Y - m_e) + m_e\}]\,c^2 = [M_X - M_Y]\,c^2$

During β^- - decay, inside the nucleus a neutron is converted to a proton with antineutrino.

$$n \rightarrow p + {}_{-1}e^{0} + \bar{\nu}$$

Beta (plus) Decay

$_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + {}_{+1}e^{0} + \nu + Q$

In β^+ decay, inside a nucleus a proton is converted into a neutron, positron and neutrino.

$p \rightarrow n + {}_{+1}e^{0} + \nu$ ← neutrino

As mass increases during conversion of proton to a neutron, hence it requires energy for β^+ decay to take place, ∴ β^+ decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-1)\,m_e) + m_e\}]\,c^2$$
$$= [M_X - M_Y - 2m_e]\,c^2$$

Considering actual number of electrons.

$$Q \text{ value} = [M_X - \{(M_Y + m_e) + m_e\}]\,c^2$$
$$= [M_X - M_Y - 2m_e]\,c^2$$

Radioactive Decay (Statistical Law)

Rate of radioactive decay $\propto$ N

where N = number of active nuclei

$$= \lambda N$$

where λ = decay constant of the radioactive substance.

Decay constant is different for different radioactive substances, but it does not depend on amount of substance and time.

SI unit of λ is s^{-1}

If $\lambda_1 > \lambda_2$ then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ

	A $\xrightarrow{\lambda}$	B
t = 0	N_0	0
t = t	N	N'

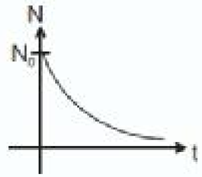
where N_0 = number of active nuclei of A at t = 0

where N = number of active nuclei of A at t = t

Rate of radioactive decay of A = $-\dfrac{dN}{dt} = \lambda N$

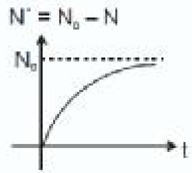
$$-\int_{N_0}^{N} \frac{dN}{N} = \int_0^t \lambda dt \Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$

(it is exponential decay)



Number of nuclei decayed (i.e. the number of nuclei of B formed)

$$N' = N_0 - N$$
$$= N_0 - N_0 e^{-\lambda t}$$
$$N' = N_0(1 - e^{-\lambda t})$$



Half Life

It is the time in which number of active nuclei becomes half.

$$N = N_0 e^{-\lambda t}$$

After one half life, $N = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda t} \Rightarrow t = \frac{\ln 2}{\lambda} \Rightarrow \frac{0.693}{\lambda} = t_{1/2}$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$ **(to be remembered)**

Number of nuclei present after n half lives i.e. after a time $t = n\, t_{1/2}$

$$N = N_0 e^{-\lambda t} \qquad = N_0 e^{-\lambda n t_{1/2}} \qquad = N_0\, e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0\, e^{\ln 2^{(-n)}} \qquad = N_0 (2)^{-n} = N_0 (1/2)^n \qquad = \frac{N_0}{2^n}$$

$\{n = \frac{t}{t_{1/2}}$. It may be a fraction, need not to be an integer$\}$

or $$N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3 \ldots\ldots\ldots\ldots\ldots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

Activity

Activity is defined as rate of radioactive decay of nuclei

It is denoted by A or R $\quad A = \lambda N$

If a radioactive substance changes only due to decay then

$$A = -\frac{dN}{dt}$$

As in that case, $N = N_0 e^{-\lambda t}$

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

SI Unit of activity : becquerel (Bq) which is same as 1 dps (disintegration per second)

The popular unit of activity is curie which is defined as

1 curie = 3.7×10^{10} dps (which is activity of 1 gm Radium)

Average Life

$$T_{avg} = \frac{\text{sum of ages of all the nuclei}}{N_0} = \frac{\int_0^{\infty} \lambda N_0 e^{-\lambda t}\, dt \,.\, t}{N_0} = \frac{1}{\lambda}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = 0.693/\lambda$$

$$T_{avg} = \frac{1}{\lambda}$$

$$T_{avg} > T_{1/2}$$

Nuclear Fission

(i) Heavy nuclei of A, above 200, break up onto two or more fragments of comparable masses.

(ii) The total B.E. increases and excess energy is released .

(iii) The man point of the fission energy is leberated in the form of the K.E. of the fission fragments

. eg. $^{235}_{92}U + {}_{o}n^{1} \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_{o}n^{1}$ + energy

Nuclear Fusion (Thermo nuclear reaction)

(i) Light nuclei of A below 20 , fuse together , the B.E. per nucleon increases and hence the excess energy is released .

(ii) These reactions take place at ultra high temperature ($\simeq 10^7$ to 10^9)

(iii) Energy released exceeds the energy liberated in the fission of heavy nuclei .

eg . $4\,{}^1_1P \rightarrow {}^4_1He + {}^{0}_{+1}e$. (Positron)

(iv) The energy released in fusion is specified by specifying Q value .
i.e. Q value of reaction = energy released in a reaction .