PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Photoelectric Effect, Dual Nature and X-Rays

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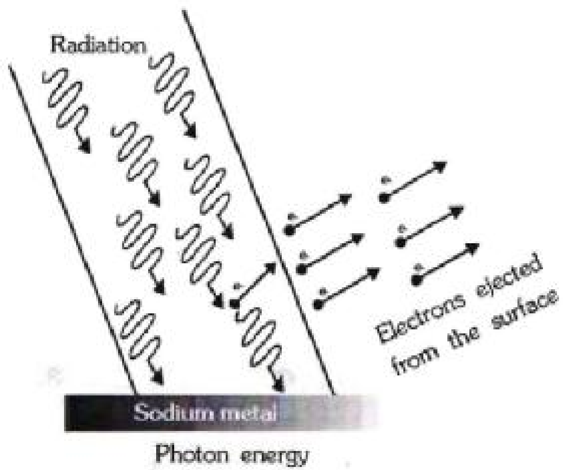
Photoelectric Effect

When radiation of suitable wavelength or frequency are incident on a metal, electrons are ejected. This effect is known as photo electric effect and electrons are known as photoelectron, current is known as photo electric current.

- Discovered by Hertz
- Laws of photo electric effect given by **Lenard**
- Explained by **Einstein** by Quantum theory of light



$E = mc^2$

Planck's Quantum Theory

Quantum Theory :- Energy radiated from a source is in the form of small packets and these are known as photons. According to Planck the energy of a photon is directly propotional to the frequency of the radiation.

$$E \propto \nu$$

$$E = h\nu$$

$$E = \frac{hc}{\lambda} \qquad (\because c = \nu\lambda)$$

$$E = \frac{hc}{\lambda} = \frac{12400}{\lambda}\ \text{eV} - \text{Å} \qquad \left[\because\ hc = 12400(\text{Å} - \text{eV})\right]$$

Here E = Energy of photon, c = Speed of light

h = Planck's constant, e = charge of electron

h = 6.62 × 10^{-34} J-s, ν = Frequency of photon, λ = Wavelength of photon

Linear Momentum, Energy and Effective Mass of Photon

Linear momentum of photon

Momentum of photon

$E = mc^2$

$p = mc$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

Energy of photon

$$E = h\nu = \frac{hc}{\lambda} = mc^2$$

Effective mass of photon :-

Effective mass of photon

$$m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$$

effective mass $m \propto \frac{1}{\lambda}$

So mass of violet light photon is greater than the mass of red light photon. $(\because \lambda_R > \lambda_V)$

Rest mass of photon = 0

U.V. VIBGYOR I.R.

danger

Energy, ν ←

→ λ

Intensity of Light

$$I = \frac{E}{At} = \frac{P}{A} \qquad ....(1)$$

Unit = $\frac{\text{joule}}{m^2 - s}$ or $\frac{\text{watt}}{m^2}$

Here P = power of source, A = Area, t = time taken
E = Energy incident in t time = Nhν
N = number of photon incident in t time

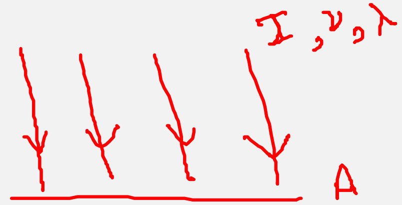
Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ....(2) $\left[\because \quad n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

from equantion (1) and (2),

$$\frac{P}{A} = \frac{n(h\nu)}{A}$$

$$n = \frac{P}{h\nu} = \frac{P\lambda}{hc}$$

$$n = 5 \quad 10^{24}\ J^{-1}\ m^{-1} \quad P \quad \lambda$$



$n = \frac{\text{Power}}{\text{Energy of one photon}}$

$n = \frac{P\lambda}{hc} = \frac{IA\lambda}{hc}$

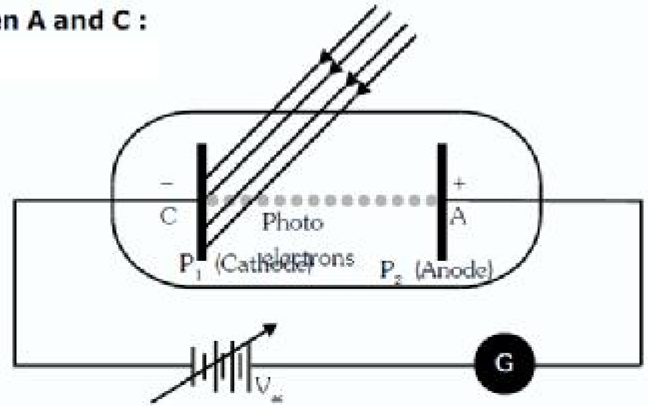
$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

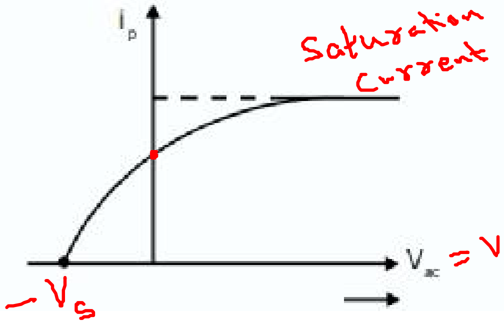
Experimental Observations of PEE

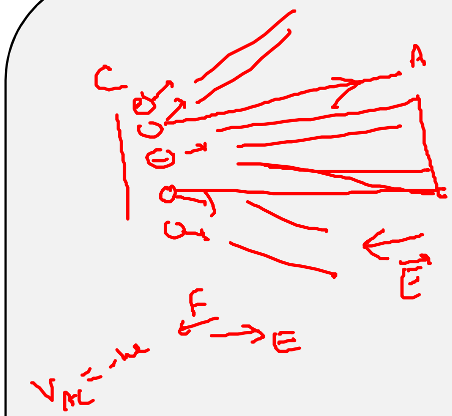
1. The effect of potential difference between A and C :

If V_{ac} is kept zero, a small current is observed. If V_{ac} is increased photo current increases but get saturated. When anode is kept at negative potential w.r.t. cathode, i_p decreases. At a certain negative potential, i_p becomes zero. **Magnitude of minimum negative potential at which i_p is zero is called stopping potential (V_0).** At stopping potential most energetic electrons are stopped. It means stopping potential is the measurement of maximum kinetic energy of photo electrons.

i.e. $K_{max.} = eV_0$



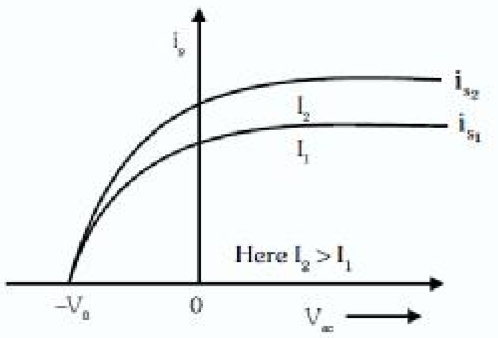


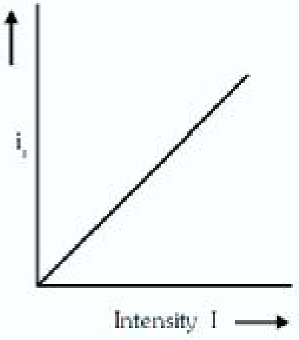


V_s or V_0 ⇒ stopping potential

Experimental Observations of PEE

2. Effect of intensity of light :- When intensity of given source is increased i_s increases but V_0 remains unchanged.





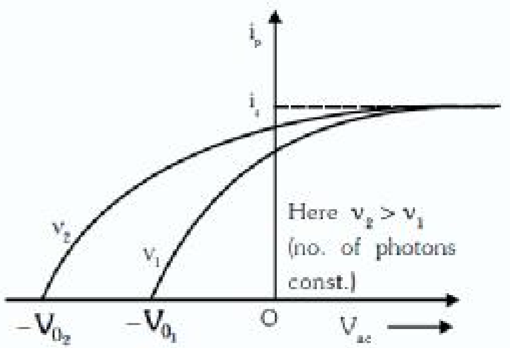
$I_p \propto I$

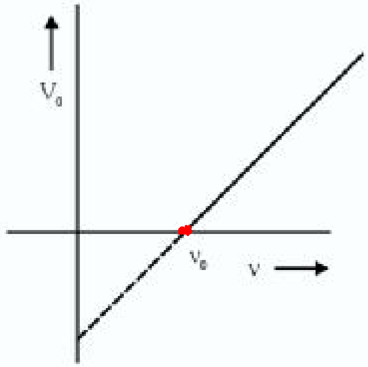
I_p → Photo current; I → Intensity

V_0 independant of intensity

Experimental Observations of PEE

3. **Effect of frequency :-** When frequency of radiations is increased, V_0 increases but i_s is almost unchanged. It is observed that below a certain frequency photo electron don't come out. **The minimum frequency which can eject the photo electrons is called cut off or threshold frequency**. Similarly corresponding maximum wave length is called **threshold wavelength.**





4. **Time lag :** There is no time lag between incident of radiations and emission of electron, even intensity is kept very low.

For P.E.E.

$\nu \geq \nu_{th}$

$\lambda \leq \lambda_{th}$

$\nu \uparrow \quad V_s \uparrow$

Important Points

key points

- Photo electric effect is an instantaneous process, as soon as light is incident on the metal, photo electrons are emitted.
- Stopping potential does not depend on the distance between cathode and anode.
- The work function represented the energy needed to remove the least tightly bound electrons from the surface. It depends only on nature of the metal and independent of any other factors.

Faliure of wave theory of light :

(1) According to wave theory when light incident on a surface, energy is distributed continuously over the surface. So that electron has to wait to gain sufficient energy to come out. But in experiment there is no time lag.

(2) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(3) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existance of threshold frequency.

Explanation by Einstein

(1) Radiations is absorbed by surface is in the form of quanta (photon). Energy of each photon depends on frequency. One photon can interact with one electron at a time. Interaction between photon and electron is an elastic collision and photon transfers its complete energy to the electron.

If energy is sufficient then electron come out without any time delay. It means photo electric effect is an instantaneous process.

(2) If intensity of the given source is increased then number of photon increases. So that, more number of electrons are emitted and greater saturation current is obtained. It means saturation current depends on intensity of the given source $i_s \propto I$

(3) At a time, only one photon can interact with one electron.

Energy of photon used by electron is

$h\nu$ = Kinetic energy of electron + Energy required to bring out electron (ϕ_0)
+ Energy lost in collision before emission (Q)

If Q = 0, means there is no heat loss. Then kinetic energy of electron is maximum.

Now

$$h\nu = (K.E._{max}) + \phi_0$$

It is known as Einstein's equation of P.E.E.

or $(K_{max}) = h\nu - \phi_0$

or $eV_0 = h\nu - \phi_0$

or $eV_0 = h\nu - h\nu_0$ Here ν_0 is threshold frequency for that $V_0 = 0$

It means maximum K.E. and stopping potential (V_0) depends on frequency. It is independent of intensity of the given source.

(4) Kinetic energy cannot be negative so that,

$h\nu \geq \phi_0$

$h\nu \geq h\nu_0$ $\left[\text{Here } \phi_0 = h\nu_0 = \frac{hc}{\lambda_0},\ \phi_0 = \frac{12400}{\lambda_0} eV - \mathring{A}\right]$

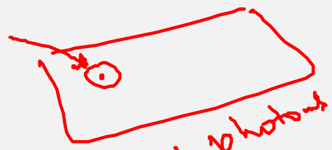
$\nu \geq \nu_0$

It means if frequency is less than 'ν_0', electron does not come out.

$0 \leq K.E_{cathode} \leq eV_s$

$E = h\nu$

W or $\phi = h\nu_0$



$I \propto$ no. of photons

$I_p \uparrow$

$K_{max} = eV_s = h\nu - \phi$
$= h\nu - h\nu_{th}$
$= \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$

Important Points

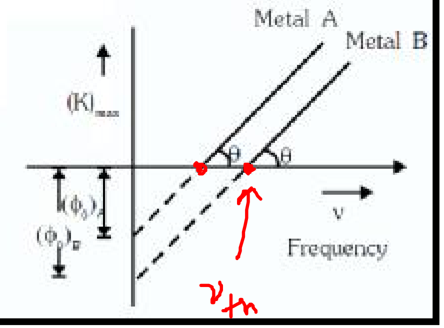
Graph between (K.E.)$_{max.}$ and frequency :-

$(K)_{max.} = h\nu - \phi_0$

$[\because \; Y = mx - c]$

slope = m = tanθ = h **(same for all metals)**

$(\phi_0)_B > (\phi_0)_A$

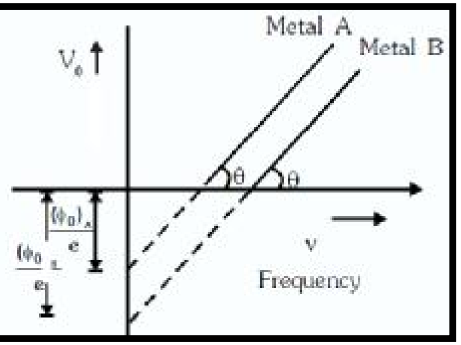


Graph between stopping potential (V_0) and frequency (ν) :-

$\because \quad eV_0 = h\nu - \phi_0$

$$V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{\phi_0}{e}\right)$$

slope = m = tanθ = $\frac{h}{e}$ **(same for all metals)**



Quantum efficiency

$$\text{Quantum efficiency} = \frac{\text{No. of electron emitted per second}}{\text{Total no. of photon incident per second}} = \frac{n_e}{n_{ph}}$$

$n = \frac{P\lambda}{hc}$

$n_e = 0.5\frac{P\lambda}{hc}$

Matter Wave

Dual Nature of Light :-

There are some experimental phenomena of light like reflection, refraction, interference, diffraction etc., which can be explained only on the basis of wave theory of light, i.e., these phenomena verify the wave nature of light. There are some experimental phenomena of light like photoelectric effect and compton effect etc., which can be explained only on the basis of the particle nature of light (i. e. quantum theory). These phenomena verify the particle nature of light. It is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both can not be possible simultaneously.

De Broglie Hypothesis :-

De Broglie imagined that as light (i. e. energy) possess both nature (i.e. wave and particle) similarly matter must also posses both nature, particle as well as wave. De Broglie imagined that despite particle nature of matter, waves must also be associated with material particles. These imaginary waves associated with material particles, are defined as matter waves.

De Broglie Wavelength Associated with Moving Particles :-

Kinetic energy of a particle of mass m and moving with velocity v.

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}, \quad p = \text{momentum of particle}$$

momentum of particle $\quad p = mv = \sqrt{2mE}$

According to De Broglie theory the wave length associated with the particles is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}, \text{ here E is kinetic energy}$$

i.e. $\quad \lambda \propto \frac{1}{p}, \lambda \propto \frac{1}{v}, \lambda \propto \frac{1}{\sqrt{E}}$

wave → λ

$p = \frac{h}{\lambda}$

particle → p

$\lambda = \frac{h}{p}$

Energy

wave (Light)

$E = h\nu = \frac{hc}{\lambda}$

$= mc^2$

Particle

$E = \frac{1}{2}mv^2$

$= \frac{p^2}{2m}$

De Broglie Wavelength Associated with the Charged Particles

The kinetic energy of a charged particle accelerated by potential difference V

$$E = \frac{1}{2}mv^2 = qV \quad q = \text{Charge on the particle}$$

Momentum of Particle : $p = mv = \sqrt{2mE} = \sqrt{2mqV}$

The De Broglie wavelength associated with charged particles $\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}}$

$\lambda_e = \frac{12.27}{\sqrt{V}}$ ← volt $= \sqrt{\frac{150}{V}}$

↑ Å

For an Electron :

$m = 9.1 \times 10^{-31}$ kg $\qquad q = 1.6 \times 10^{-19}$ Coulomb, $\qquad h = 6.62 \times 10^{-34}$ Joule - sec.

∴ De Broglie wavelength associated with electron

$$\therefore \quad \lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}} \quad \text{or} \quad \lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} m\sqrt{volt}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}} \mathring{A}\sqrt{volt}}$$

For Proton :

$m_p = 1.67 \times 10^{-27}$ kg.

$$\therefore \quad \lambda_p = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} V}} \quad \text{or} \quad \boxed{\lambda_p = \frac{0.286 \times 10^{-10}}{\sqrt{V}} m\sqrt{volt} = \frac{0.286}{\sqrt{V}} \mathring{A}\sqrt{Volt}}$$

Davisson Germer Experiment

The experimental arrangement is shown in fig.

There are three main parts of this experiment

(i) Electron gun

(ii) Nickle crystal

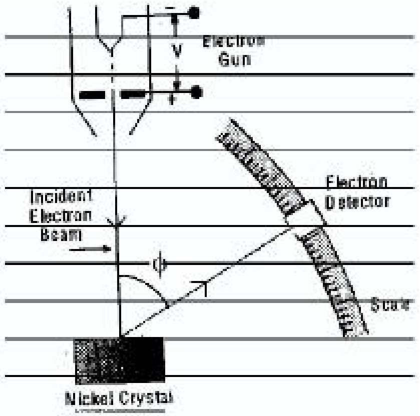
(iii) Ionisation chamber (Detector)

Electron gun : Electrons of desired energy are produced in it by the process of thermionic emission.

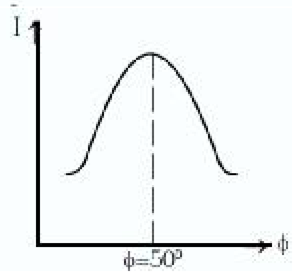
Nickle crystal diffracts the electrons beam obtained from electron gun.

Ionisation chamber : It detects the electron beam diffracted by the nickle crystal.

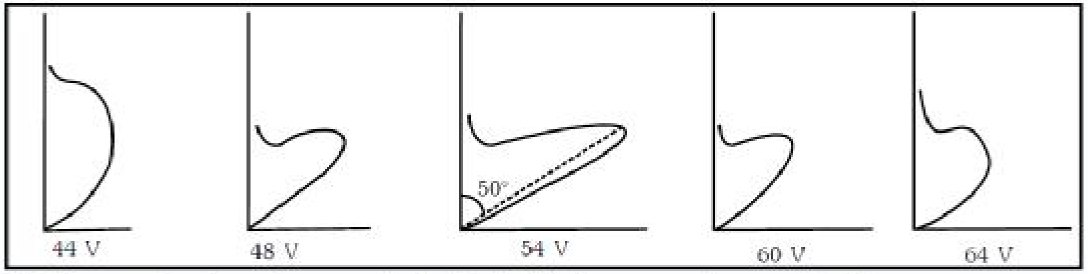
Nickle crystal behaves like a three dimensional diffraction grating

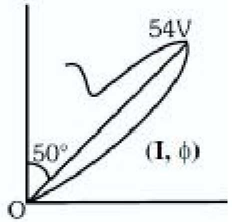


Curve between the intensity (I) of diffracted electrons and diffracting angle (ϕ)



Graph between I v/s ϕ for different accelerating potential V





In this experiment maximum peak is obtained in I–ϕ curve at an angle of diffraction of 50° and acceleration potential 54 volt.

In this experiment the experimental value of De Broglie wavelength associated with electron is obtained as 1.65 A° whereas according to De Broglie theory this wavelength comes out to be 1.66A°. In this experiment the wave nature of electron is verified due to their diffraction. Diagram representing the angle of diffraction (ϕ) and glancing angle (θ)

For maxima, path difference = $n\lambda$.

or $2d \sin\theta = n\lambda$.

Here d = distance between two consequtive crystal plane or interplanar distance.

n = order of diffraction

λ = De broglie wavelength associated with electron.

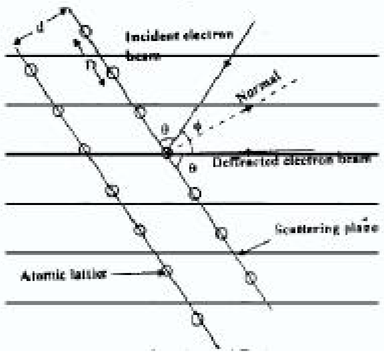
θ = glancing angle

ϕ = angle of diffraction

Relation between θ and ϕ :

$\phi = 180° - 2\theta$

or $\theta = 90° - \frac{\phi}{2}$



X Rays

It was discovered by **ROENTGEN**. The wavelength of x-rays is found between 0.1 Å to 10 Å.
These rays are invisible to eye.
They are electromagnetic waves and have speed $c = 3 \times 10^8$ m/s in vacuum.
Its photons have energy around 1000 times more than the visible light.
When fast moving electrons having energy of order of several KeV strike the metallic target then x-rays are produced.

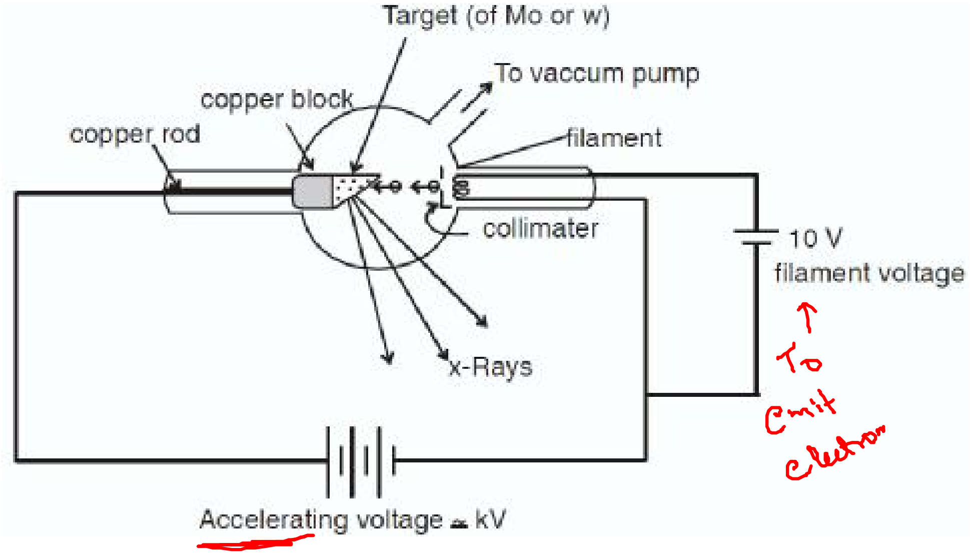
1Å

(E ≈ KeV)

Visible (λ) → 4000 Å to 7000 Å

(1.9 → 3.3) eV

Production of X-rays by Coolidge Tube



The melting point, specific heat capacity and atomic number of target should be high. When voltage is applied across the filament then filament on being heated emits electrons from it. Now for giving the beam shape of electrons, collimator is used. Now when electron strikes the target then x-rays are produced.

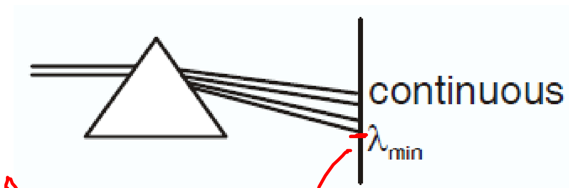
When electrons strike with the target, some part of energy is lost and converted into heat. Since, target should not melt or it can absorb heat so that the melting point, specific heat of target should be high.

Here copper rod is attached so that heat produced can go behind and it can absorb heat and target does not get heated very high.

For more energetic electron, accelerating voltage is increased.

For more no. of photons voltage across filament is increased.

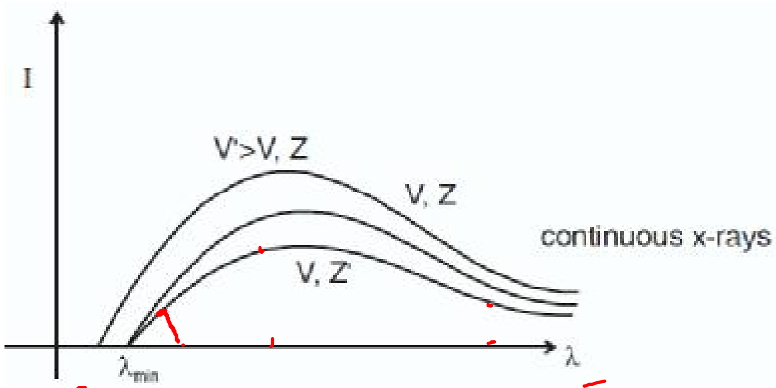
The x-ray were analysed by mostly taking their spectrum



Continuous X-Rays

Max energy of e^-

$E = eV$ ← acc. vol.



The minimum wavelength corresponds to the maximum energy of the x-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V(\text{involts})} \text{ Å.}$$

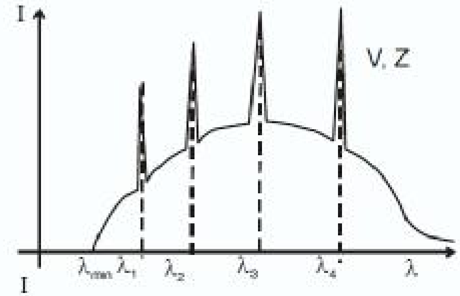
Acc. voltage

We see that cutoff wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

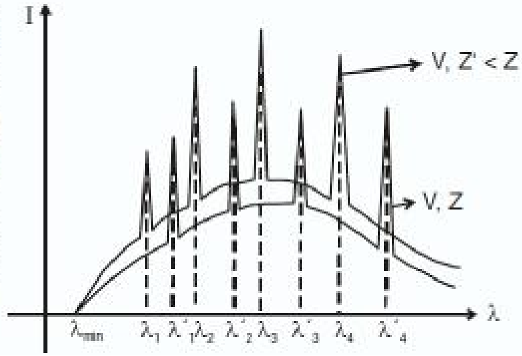
Characteristic X-Rays

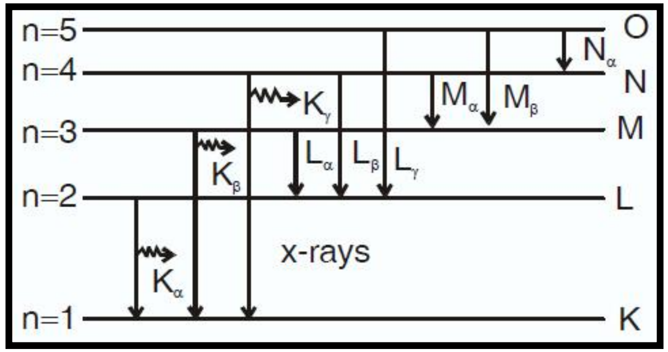
The sharp peaks obtained in graph are known as characteristic x-rays because they are characteristic of target material.

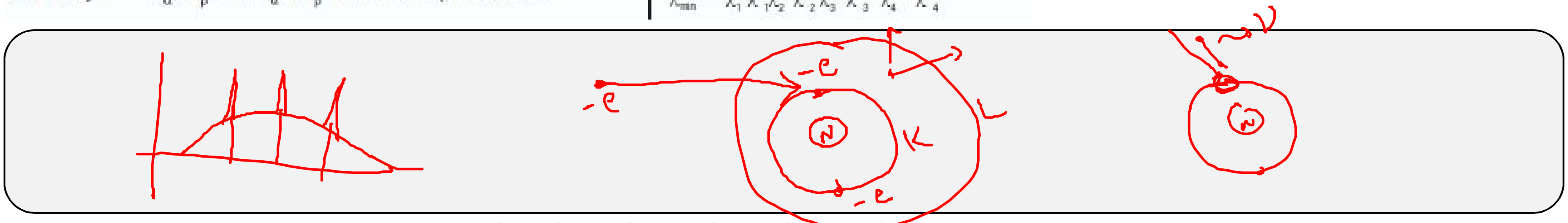
$\lambda_1, \lambda_2, \lambda_3, \lambda_4, ...$ = charecteristic wavelength of material having atomic number Z are called **characteristic x-rays** and the spectrum obtained is called **characteristic spectrum**. If target of atomic number Z' is used then peaks are shifted.



Characteristic x-ray emission occurs when an energetic electron collides with target and remove an inner shell electron from atom, the vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost K-shell is filled by an electron droping from next higher level L-shell then K_α characteristic x-ray is obtained. If vaccany in K-shell is filled by an electron from M-shell, K_β line is produced and so on similarly L_α, L_β,.....M_α, M_β lines are produced.

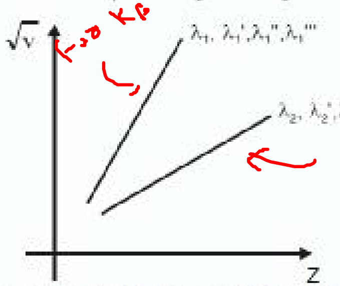






Moseley's Law

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the sqaure root of frequency against position number in periodic table. He discovered that plot is very closed to a straight line not passing through origin.



Z_1	λ_1	λ_2
Z_2	λ_1'	λ_2'
Z_3	λ_1''	λ_2''
Z_4	λ_1'''	λ_2'''

Wavelength of charactristic wavelengths.

Moseley's observations can be mathematically expressed as

$$\sqrt{\nu} - a(Z-b)$$

a and b are positive constants for one type of x-rays & for all elements (independent of Z).

Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \cdot (Z-b)$$ by using the formula $\frac{1}{\lambda} = R z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ for multi electron system.

b → known as screening constant or shielding effect, and $(Z-b)$ is effective nuclear charge.

for K_α line $n_1 = 1$, $n_2 = 2$

$$\therefore \sqrt{\nu} = \sqrt{\frac{3RC}{4}}\,(Z-b)$$

$$\sqrt{\nu} = a(Z-b)$$ Here $a = \sqrt{\frac{3RC}{4}}$, [b = 1 for K_α lines]