Problem Solving on Vector & 3D

By
Ankush Garg(B. Tech, IIT Jodhpur)

If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and

$|\vec{a}| = 7$, $|\vec{b}| = 5$, $|\vec{c}| = 3$ then angle Between vector $\vec{b}$ and $\vec{c}$ is

(A) 60° (B) 30°

(C) 45° (D) 90°

$\vec{a} + \vec{b} + \vec{c} = 0 \qquad |\vec{a}| = 7, |\vec{b}| = 5, |\vec{c}| = 3$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c} = |\vec{a}|^2$

$25 + 9 + 2|\vec{b}||\vec{c}|\cos\theta = 49$

$34 + 2 \times 5 \times 3 \cos\theta = 49$

$30\cos\theta = 15$

$\cos\theta = \cancel{\frac{15}{30}}\ \frac{1}{2}$

2 If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 4$ then $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] =$

(A) ~~16~~ (B) 64

(C) 4 (D) 8

$[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}]$

$\vec{a} \times \vec{b} \cdot [(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$

$(\vec{a} \times \vec{b})[(\vec{b} \times \vec{c} \cdot \vec{a})\vec{c} - 0]$

$(\vec{a} \times \vec{b})[\vec{a}\vec{b}\vec{c}] \cdot \vec{c})$

$[\vec{a}\vec{b}\vec{c}]^2$

$= (4)^2$

$= 16$

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

3 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non – zero vectors such that no two of these are collinear .If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ (λ being some non – zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals

(A) $\lambda\vec{a}$

(B) $\lambda\vec{b}$

(C) $\lambda\vec{c}$

(D) 0

$(\vec{a} + 2\vec{b}) = k_1\vec{c}$ — 1

$2((\vec{b} + 3\vec{c}) = k_2\vec{a})$

$\vec{a} - 6\vec{c} = k_1\vec{c} - 2k_2\vec{a}$

$\vec{a}(1+2k_2) = (k_1+6)\vec{c}$

$\vec{a} = \left(\frac{k_1+6}{1+2k_2}\right)\vec{c}$

$\boxed{k_1 = -6}$, $1+2k_2 = 0$

$\boxed{k_2 = -1/2}$

$\boxed{\vec{a} = \alpha\vec{c}}$ ✗

$\vec{a} + 2\vec{b} = -6\vec{c}$

$\boxed{\vec{a} + 2\vec{b} + 6\vec{c} = 0}$

9) Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\bar{u}| = 1$, $|\bar{v}| = 2$, $|\bar{w}| = 3$. If the projection $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other then $|\bar{u} - \bar{v} + \bar{w}|$ equals

(A) 2

(B) $\sqrt{7}$

(C) $\sqrt{14}$

(D) 14

$|\vec{u}| = 1, \ |\vec{v}| = 2, \ |\vec{w}| = 3$

$$\frac{\vec{v}\cdot\vec{u}}{\cancel{|\vec{u}|}} = \frac{\vec{w}\cdot\vec{u}}{\cancel{|\vec{u}|}} \Rightarrow \boxed{\vec{v}\cdot\vec{u} = \vec{w}\cdot\vec{u}}$$

$$\boxed{\vec{v}\cdot\vec{w} = 0}$$

$|\vec{u} - \vec{v} + \vec{w}| = K$

$$|\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 \cancel{- 2\vec{u}\cdot\vec{v}} \cancel{+ 2\vec{u}\cdot\vec{w}} \cancel{- 2\vec{v}\cdot\vec{w}} = K^2$$

$1 + 4 + 9 = K^2$

$14 = K^2 \Rightarrow K = \sqrt{14}$

9) The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

(A) $\frac{10}{9}$

(B) $\frac{10}{3\sqrt{3}}$

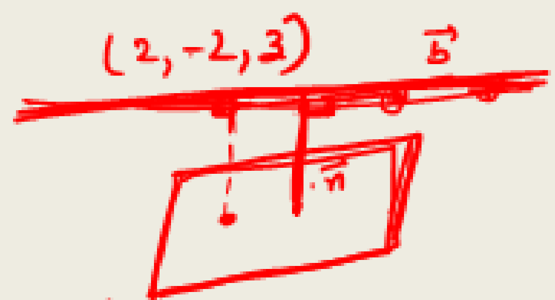
(C) $\frac{3}{10}$

(D) $\frac{10}{3}$

$\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$

$\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$

$(1 - 5 + 4 = 0)$



$\boxed{x + 5y + z = 5}$

(2,-2,3)

$\left|\frac{2 - 10 + 3 - 5}{\sqrt{1 + 25 + 1}}\right| = \frac{10}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$

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Q) Let $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$.

Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on

(A) Only y

(B) Only x

(C) Both x and y

(D) Neither x nor y

$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$

$\vec{a} = \hat{i} - \hat{k}$

$\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$

$\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & (1-x) \\ y & x & (1+x-y) \end{vmatrix}$$

$$= 1\cdot(1+\cancel{x}-y-\cancel{x}+x^2) - 1\cdot(x^2-y)$$

$$= 1\cancel{+y}+\cancel{x^2}-\cancel{x^2}+\cancel{y}$$

$$= 1$$

9 The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(A) $k = 1$ or -1

(B) $k = 0$ or -3

(C) $k = 3$ or -3

(D) $k = 0$ or -1

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$(x_2, y_2, z_2) = (1, 4, 5)$

$(x_1, y_1, z_1) = (2, 3, 4)$

$$\begin{vmatrix} -1 & 1 & +1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$-1(1+2k) - 1(1+k^2) + 1(2-k)$

$\cancel{-1} - 2k \cancel{-1} - k^2 + \cancel{2} - k = 0$

$-k^2 - 3k = 0$

$k^2 = -3k$

$k = 0, k = -3$

Q) Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(A) $\frac{7}{2}$ (B) $\frac{5}{2}$

(C) $\frac{3}{2}$ (D) $\frac{9}{2}$

$P_1 : 2x + y + 2z - 8 = 0$

$P_2 : 2x + y + 2z + \frac{5}{2} = 0$

$$\left| \frac{-8 - 5/2}{\sqrt{4+1+4}} \right| = \left| \frac{\cancel{21}\,7}{2 \times \cancel{3}} \right|$$

$$= 7/2$$

9) The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(A) 90°
(B) 0°
(C) 30°
(D) 45°

$$\frac{x-0}{1/2} = \frac{y-0}{1/3} = \frac{z-0}{-1} = L_1$$

$$\frac{x-0}{1/6} = \frac{y-0}{-1} = \frac{z-0}{-1/4} = L_2$$

$$\underline{\underline{a_1a_2 + b_1b_2 + c_1c_2 = 0}}$$

$$\cos\theta = \frac{1}{2}\times\frac{1}{6} + \left(\frac{1}{3}\times -1\right) + \left(-1\times -\frac{1}{4}\right)$$

$$= \frac{1}{12} - \frac{1}{3} + \frac{1}{4}$$

$$= \cancel{\frac{1}{12}} - \cancel{\frac{1}{12}}$$

$$= \boxed{0}$$

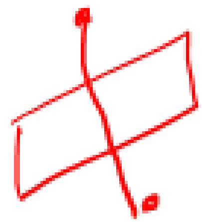
The image of the point (–1, 3, 4) in the plane x – 2y = 0 is

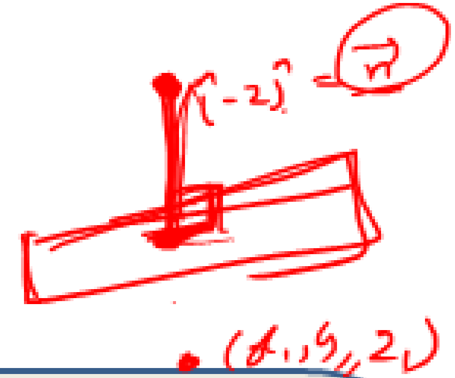
(A) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$

(B) (15, 11, 4)

(C) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$

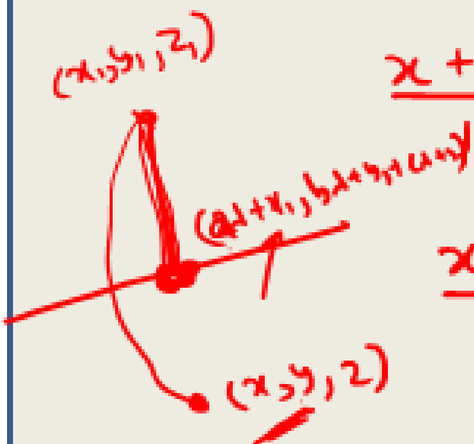
(D) $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$





Sol^n

$$\boxed{\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=-2\frac{(ax_1+by_1+cz_1)}{a^2+b^2+c^2}}$$



$$\frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}=-2\left(\frac{1\times -1+(-2)\times 3+0}{5}\right)$$

$$\frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}=\frac{2}{5}(7)$$

$$=\frac{14}{5}$$

$$\frac{x+1}{1}=\frac{14}{5}$$

$x=\frac{9}{5}$, $\frac{y-3}{-2}=\frac{14}{5}$, $y-3=-\frac{28}{5}$

$y=3-\frac{28}{5}=-13/5$