PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

# Alternating Current

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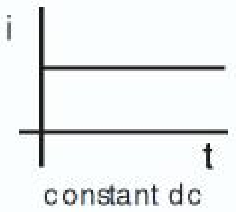
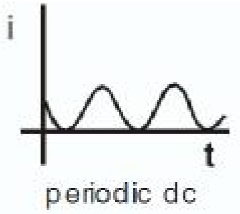
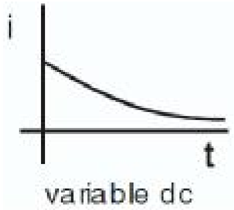
# AC and DC Current

<u>Direct Current (DC)</u>

If a current maintains its direction constant it is called direct current (DC).

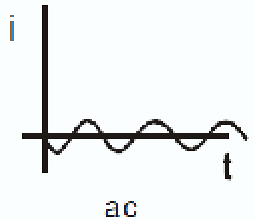


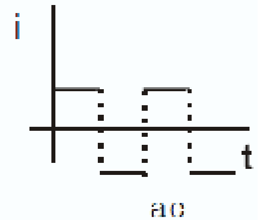




<u>Alternating Current (AC)</u>

A current that changes its direction periodically is called alternating current (AC).




If a function suppose current, varies with time as $i = I_m \sin(\omega t+\phi)$, it is called sinusoidally varying function.

Here $I_m$ is the peak current or maximum current and i is the instantaneous current.

The factor $(\omega t+\phi)$ is called phase. $\omega$ is called the angular frequency, its unit rad/s.

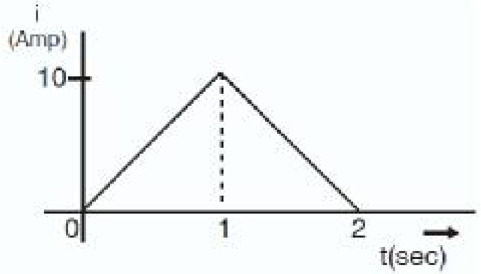
Also $\omega = 2\pi f$ where f is called the frequency, its unit $s^{-1}$ or Hz.

Also frequency $f = 1/T$ where T is called the time period.

# Average Value

Average value of a function, from $t_1$ to $t_2$, is defined as $\langle f \rangle = \dfrac{\int_{t_1}^{t_2} f\,dt}{t_2 - t_1}$. We can find the value of $\int_{t_1}^{t_2} f\,dt$ graphically if the graph is simple. It is the area of f-t graph from $t_1$ to $t_2$.

**Exercise .** Find the average value of current shown graphically, from t = 0 to t = 2 sec.




**Solution :** From the i – t graph, area from t = 0 to t = 2 sec

$= \dfrac{1}{2} \times 2 \times 10 = 10$ Amp. sec.

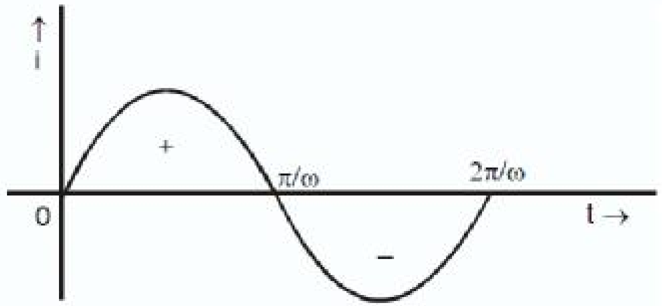
$\therefore$ Average Current $= \dfrac{10}{2} = 5$ Amp.

# Example

Find the average value of current from $t = 0$ to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

**Solution :**

$$\langle i \rangle = \frac{\int_0^{\frac{2\pi}{\omega}} I_m \sin \omega t\,dt}{\frac{2\pi}{\omega}} = \frac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0$$




It can be seen graphically that the area of i – t graph of one cycle is zero.

$\therefore$ $\langle i \rangle$ in one cycle = 0.

# Root Mean Square Value

Root Mean Square Value of a function, from $t_1$ to $t_2$, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$

**Example.** Find the rms value of current from t = 0 to $t = \dfrac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$ .

**Solution :**

$$i_{rms} = \sqrt{\dfrac{\int_0^{\frac{2\pi}{\omega}} I_m^2 \sin^2 \omega t dt}{\frac{2\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$$

**Note:**

- The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same.
- For sinusoidal functions **rms value** (Also called **effective value**)

$$= \dfrac{\text{peak value}}{\sqrt{2}} \quad \text{or} \quad I_{rms} = \dfrac{I_m}{\sqrt{2}}$$

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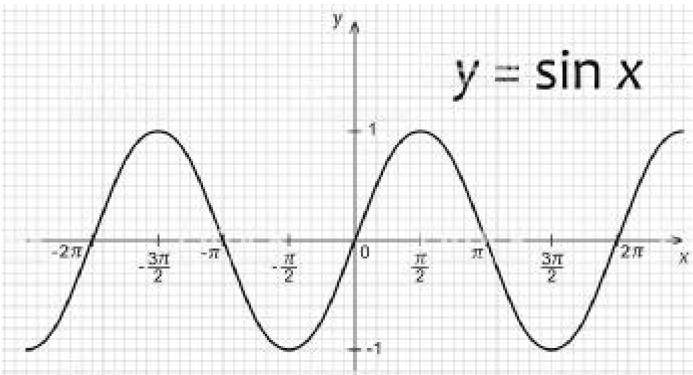
# Important Points

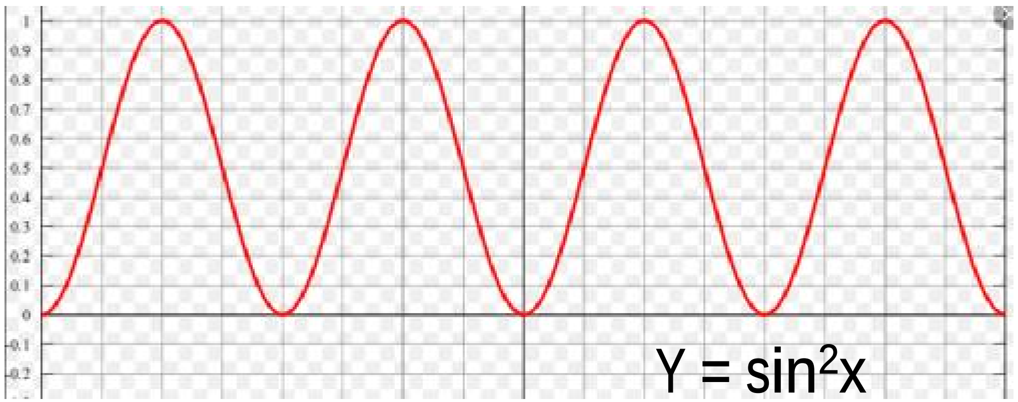
- Average value of any sin or cos function in one time period, integral multiple of time period or in large time interval is always zero.

- RMS value any $\sin^2$ or $\cos^2$ function in one time period, integral multiple of time period or in large time interval is 1/2.

- Time period of $\sin^2$ or $\cos^2$ function = Half of time period of sin or cos function.

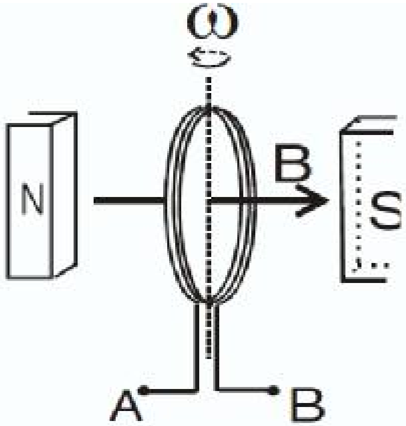
- Frequency of $\sin^2$ or $\cos^2$ function = Twice the frequency of sin or cos function.



# AC Sinusoidal Source

Figure shows a coil rotating in a magnetic field. The flux in the coil changes as $\varphi = NBA\cos(\omega t + \phi)$. Emf induced in the coil, from Faraday's law is $\frac{-d\varphi}{dt} = NBA\omega\sin(\omega t + \phi)$. Thus the emf between the points A and B will vary as $E = E_0 \sin(\omega t + \phi)$. The potential difference between the points A and B will also vary as $V = V_0 \sin(\omega t + \phi)$. The symbolic notation of the above arrangement is A—(~)—B. We do not put any + or – sign on the AC source.



# Power Consumed or Supplied in an AC Circuit

Consider an electrical device which may be a source, a capacitor, a resistor, an inductor or any combination of these. Let the potential difference be $v = V_A - V_B = V_m \sin\omega t$. Let the current through it be $i = I \sin(\omega t + \phi)$.

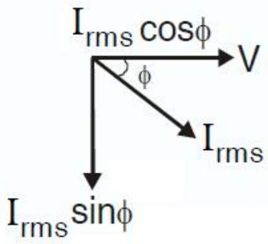

Instantaneous power P consumed by the device

$$P = v\,i = (V_m \sin \omega t)\,(I_m \sin(\omega t + \phi))$$

Average power consumed in a cycle $= V_{rms}\, I_{rms} \cos\phi$

Here $\cos\phi$ is called **power factor**.

**Note :** $I_{rms} \sin\phi$ is called "wattless current".



# Example

When a voltage $v_s = 200\sqrt{2}\ \sin(\omega t + 15^\circ)$ is applied to an AC circuit the current in the circuit is found to be $i = 2\sin(\omega t + \pi/4)$ then average power concsumed in the circuit is

(A) 200 watt (B) $400\sqrt{2}$ watt (C) $100\sqrt{6}$ watt (D) $200\sqrt{2}$ watt

**Solution :** $P_{av} = v_{rms}\ I_{rms}\ \cos\phi$

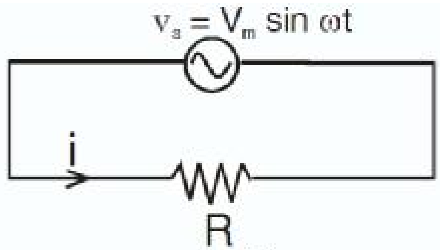
$$= \frac{200\sqrt{2}}{\sqrt{2}}\ \frac{2}{\sqrt{2}}\ .\ \cos(30^\circ) = 100\sqrt{6} \text{ watt}$$

# Purely Resistive Circuit

Writing KVL along the circuit,

$v_s - iR = 0$

or $i = \frac{v_s}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$

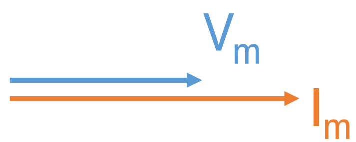



$\Rightarrow$ We see that the phase difference between potential difference across resistance, $v_R$ and $i_R$ is 0.

$I_m = \frac{V_m}{R} \quad \Rightarrow \quad I_{rms} = \frac{V_{rms}}{R}$

$$\langle P \rangle = V_{rms} I_{rms} \cos \phi = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

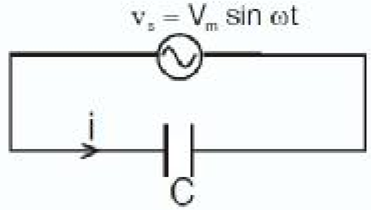
Phasor Diagram



# Purely Capacitive Circuit

Writing KVL along the circuit,

$$v_s - \frac{q}{C} = 0$$




$$\text{or } i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = \frac{d(CV_m \sin\omega t)}{dt} = CV_m\omega\cos\omega t = \frac{V_m}{1/\omega C}\cos\omega t = \frac{V_m}{X_C}\cos\omega t = I_m\cos\omega t = I_m\sin(\omega t + \frac{\pi}{2})$$

$X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is ohm $\Omega$.

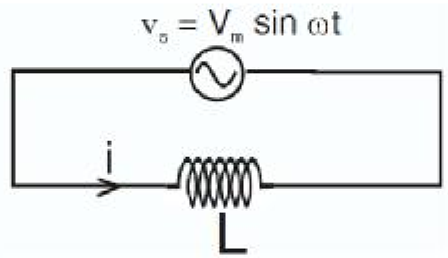
**Phasor Diagram**


Current leads voltage by $\frac{\pi}{2}$

Since $\phi = 90°$, $\langle P \rangle = V_{rms} I_{rms} \cos\phi = 0$

# Purely Inductive Circuit

Writing KVL along the circuit,



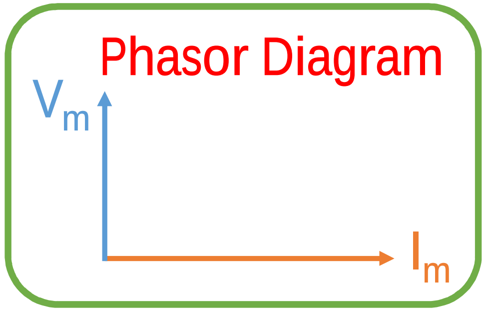

$$v_s - L\frac{di}{dt} = 0 \quad \Rightarrow \quad L\frac{di}{dt} = V_m \sin \omega t$$

$$\int L\,di = \int V_m \sin\omega t\, dt \qquad i = -\frac{V_m}{\omega L}\cos \omega t + C$$

$$< i > = 0 \quad \Rightarrow \quad C = 0$$

$$\therefore \quad i = -\frac{V_m}{\omega L}\cos\omega t = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$I_m = \dfrac{V_m}{X_L}$ where $X_L = \omega L$ and is called inductive reactance. Its unit is ohm $\Omega$.

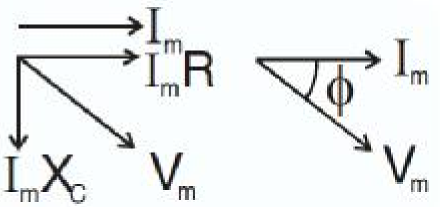


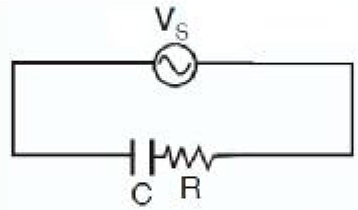

Current lags voltage by $\frac{\pi}{2}$

Since $\phi = 90°$, $<P> = V_{rms} I_{rms} \cos \phi = 0$

# RC series circuit with an AC source

Let $i = I_m \sin(\omega t)$



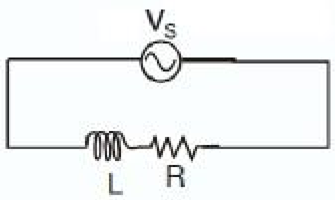



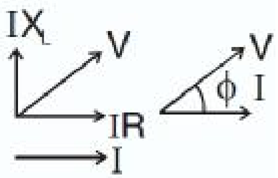

$$I_m = \frac{V_m}{\sqrt{R^2 + X_C^2}} \Rightarrow Z = \sqrt{R^2 + X_C^2}$$

Impedance

$$\tan\phi = \frac{I_m X_C}{I_m R} = \frac{X_C}{R}$$

# LR series circuit with an AC source

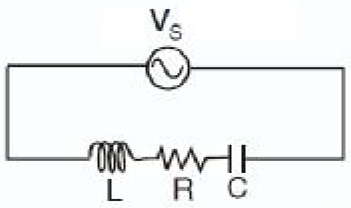


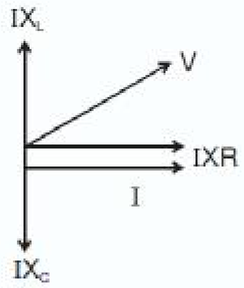



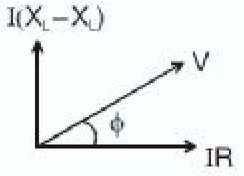

From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = I\,Z \quad \Rightarrow \quad \tan\phi = \frac{IX_L}{IR} = \frac{X_L}{R}$$

# LCR series circuit with an AC source




From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ$$

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

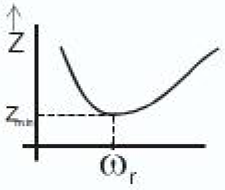
# Resonance

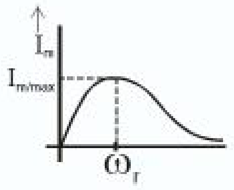
Amplitude of current (and therefore $I_{rms}$ also) in an RLC series circuit is maximum for a given value of $V_m$ and R , if the impedance of the circuit is minimum, which will be when $X_L - X_C = 0$. This condition is called **resonance**.

So at resonance:

$$X_L - X_C = 0.$$

or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$. Let us denote this $\omega$ as $\omega_r$.

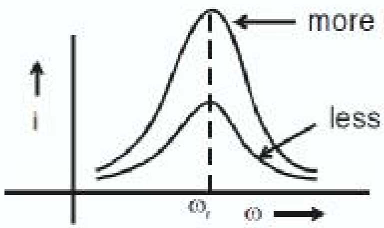


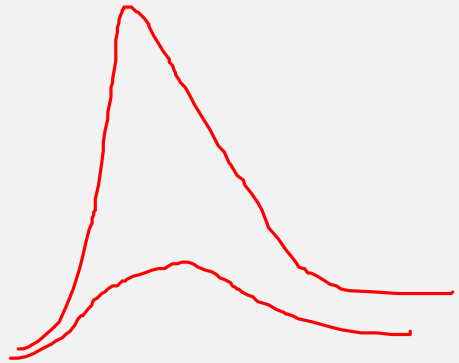



# Q Value (Quality Factor) of LCR Series Circuit

Q value is defined as $\frac{X_L}{R}$ where $X_L$ is the inductive reactance of the circuit, at resonance.

More Q value implies more sharpness of I Vs $\omega$ curve



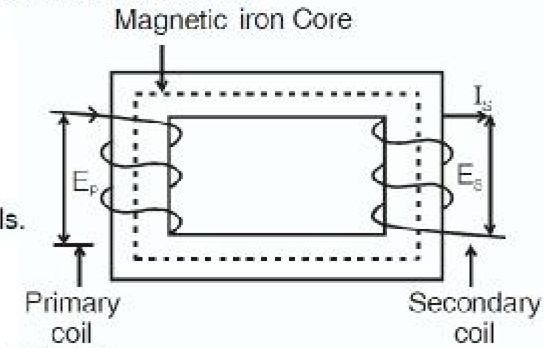
# Transformer

**A transformer** changes an alternating potential difference from one value to another of greater or smaller value using the principle of mutual induction .Two coils called the primary and secondary windings,which are not connected to one another in any way , are wound on a complete soft iron core.When an alternating voltage $E_P$ is applied to the primary , the resulting current produces a large alternating magnetic flux which links the secondary and induces an emf $E_S$ in it.It can be shown that for an ideal transformer

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S};$$

$\frac{N_S}{N_P}$ = turns ratio of the transformer.

$E_S$ N and I are the emf, number of turns and current in the coils.




$N_S > N_P \Rightarrow$ $E_S > E_P \rightarrow$ step up transformer.

$N_S < N_P \Rightarrow$ $E_S < E_P \rightarrow$ step down transformer.

**Note :** Phase difference between the primary and secondary voltage is $\pi$.

$P = VI$

$P$ = const

$E_p I_p = E_s I_s$

# Energy Losses in Transformer

Although transformers are very efficient devices, small energy losses do occur in them due to four main causes.

## RESISTANCE OF THE WINDINGS :

The copper wire used for the windings has resistance and so $I^2R$ heat losses occur.

## EDDY CURRENT :

Eddy current is induced in a conductor when it is placed in a changing magnetic field or when a conductor is moved in a magnetic field and/or both. Any imagined circuit within the conductor will change its magnetic flux linkage and the subsequent induced emf. will drive current around the circuit.Thus the alternating magnetic flux induces eddy currents in the iron core and causes heating.The effect is reduced by **laminating** the core, i.e, the core is made of this sheets of iron with insulating sheets between them so that the circuits for the eddy currents are broken.

## HYSTERESIS :

The magnetization of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

## FLUX LEAKAGE :

The flux due to the primary may not all link the secondary if the core is badly designed or has air gaps in it .Very large transformers have to be oil cooled to prevent overheating.