Problem Solving on Complex, QE, S&S and Trigo

By
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The value of $\sum_{k=1}^{10}\left(i \sin\frac{2k\pi}{11} + \cos\frac{2k\pi}{11}\right)$ is

(A) i

(B) 1

(C) –1

(D) –i

De Moivre's Theorem

11th root of unity

$\frac{\cos 2k\pi}{11} + i\sin\frac{2k\pi}{11}$

$k = \{0, 1, \ldots n-1\}$

nth root of unity

$1, \alpha, \alpha^2 \ldots \alpha^{n-1}$

$1 + \alpha + \alpha^2 + \ldots \alpha^{n-1} = 0$

$\boxed{1 + \alpha + \alpha^2 \ldots \alpha^{10} = 0}$

$\alpha + \alpha^2 \ldots \alpha^{10} = -1$

9. If $|z + 4| \le 3$ then the maximum value of $|z + 1|$

(A) 4 (B) 10

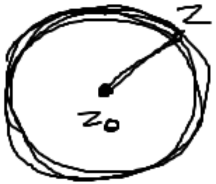
(C) 6 (D) 0

$|z - (-1)|$

$|z_1 - z_2|$

$|z - z_0| = r$

circle

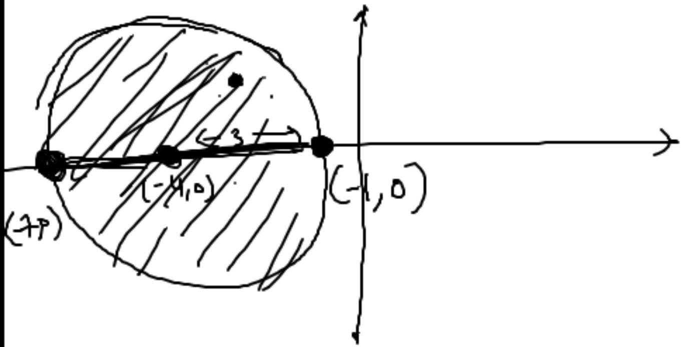


$|z + z_0|$ $|z - (-z_0)|$

$|z - z_0|$

$|z+4| \le 3$

$|z - (-4)| \le 3$



Q) The conjugate of a complex number is $\frac{1}{i-1}$, then that complex number is

(A) $\frac{-1}{i-1}$

(B) $\frac{1}{i+1}$

(C) $\frac{-1}{i+1}$

(D) $\frac{1}{i-1}$

$z = (x + iy)$

$(\bar{z} = x - iy)$

$\bar{\bar{z}} = x + iy$

$\bar{z} = \frac{1}{i-1}$

$\frac{1}{-i-1}$

$z = \frac{-1}{i+1}$

If $\omega (\neq 1)$ is a cube root of unit, and $(1+\omega)^7 = A + B\omega$. Then (A,B) equals :

(A) (1, 0) (B) (−1,1)

(C) (0, 1) (D) (1, 1)

$$\boxed{1 + \omega + \omega^2 = 0} \qquad \boxed{\omega^3 = 1}$$

(ω^3)

$$(1+\omega)^7 = A + B\omega$$

$$(-\omega^2)^7$$

$$-\omega^{14}$$

$$-\omega^2 = A + B\omega$$

$$1 + \omega = A + B\omega$$

Zoom Group Chat

From Roshan Yadav to Me: (Privately)
pichle que ka ans deya tha ise solve kar raha hu

From Ayush Singh to Me: (Privately)
D

From viraj rao to Me: (Privately)
d

From Siddharth Gupta to Me: (Privately)
d
d

From aryandogra221 to Me: (Privately)
D

From Divyanshu Singh to Me: (Privately)
sir pichle waale me -iota-1 k bajaaye -iota +1 hona chahiye tha

To: Roshan Yadav (Privately)

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z and w are two non zero complex no. s such that $|z| = |w|$ and $\text{Arg } z + \text{Arg } w = \pi$ then z equals.

(A) $\bar{w}$ (B) $-\bar{w}$

(C) w (D) $-w$

Sol^n

$|z| = |w|$

$\arg(z) + \arg(w) = \pi$

$re^{i\theta}$

$\theta + \phi = \pi$

$\theta = (\pi - \phi)$

$z = re^{i\theta}$

$w = re^{i\phi}$

$w = r[\cos\phi + i\sin\phi]$

$\bar{w} = r[\cos\phi - i\sin\phi]$

$z = re^{i(\pi - \phi)}$

$= r[\cos(\pi - \phi) + i\sin(\pi - \phi)]$

$z = r[-\cos\phi + i\sin\phi]$

$= -r[\cos\phi - i\sin\phi] \Rightarrow z = -\bar{w}$

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If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$

then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

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(A) $3x^2 - 19x + 3 = 0$ (B) $3x^2 + 19x - 3 = 0$

(C) $3x^2 - 19x - 3 = 0$ (D) $x^2 - 5x + 3 = 0$

$\alpha^2 = 5\alpha - 3$

$\beta^2 = 5\beta - 3$

$x^2 = 5x - 3$

$\boxed{x^2 - 5x + 3 = 0} \Rightarrow \alpha, \beta$

$\alpha + \beta = 5$, $\alpha\beta = 3$

$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 25 - 6$

$\alpha^2 + \beta^2 = 19$

$(P = 1)$

$S = \left(\frac{19}{3}\right)$, $P = 1$

$x^2 - \frac{19}{3}x + 1 = 0$

Participants (65)

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Aditi Rai

1

yes no go

Invite Mute

If $(1-p)$ is a root of quadratic equation $x^2 + px + 1(1-p) = 0$, then its roots are

(A) 0,1 (B) -1,2

(C) 0,-1 (D) -1,1

$$x^2 + px + (1-p) = 0$$

$$(1-p)^2 + p(1-p) + (1-p) = 0$$

$$1 + \cancel{p^2} - 2p + \cancel{p} - \cancel{p^2} + 1 - \cancel{p} = 0$$

$$2 - 2p = 0$$

$$\boxed{p = 1}$$

$$x^2 + x = 0$$

$$x(x+1) = 0$$

$$x = 0,\ x = -1$$

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The value of a for which the sum of the squares of the roots of the equation

$x^2 - (a-2)x - a - 1 = 0$ assume the least valu

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α β

~~(A)~~ 1

(B) 0

(C) 3

(D) 2

$$x^2 - (a-2)x - (a+1) = 0$$

$$\alpha+\beta = (a-2)$$

$$\alpha\beta = -(a+1)$$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$\alpha^2+\beta^2 = (a-2)^2 - 2(-(a+1))$$

$$= (a-2)^2 + 2(a+1)$$

$$= a^2 - 4a + 4 + 2a + 2$$

$$\alpha^2+\beta^2 = a^2 - 2a + 6$$

$$2a - 2 = 0$$

$$a = 1$$

$$= (a-1)^2 + 5$$

$$\boxed{a=1}$$

Sum of infinite number of terms in GP is 20 and sum of their square is 100.

The common ratio of GP is

(A) 5 ~~(B)~~ 3/5

(C) 8/5 (D) 1/5

$$a + ar + ar^2 \quad - \quad -$$

$$S_\infty = \frac{a}{1-r} = 20 \Rightarrow \frac{a^2}{(1-r)^2} = 400$$

$$a^2 + a^2r^2 + a^2r^6 \quad - \quad - \quad -$$

$$S_\infty = \frac{a^2}{1-r^2} = 100$$

$$\frac{\cancel{(1-r)}(1+r)}{(1-r)\cancel{(1+r)}} = 4 \qquad \frac{1-r^2}{(1-r)^2} = 4 \Rightarrow 1-r^2 = 4(1+r^2-2r)$$

$$= 5r^2 - 8r - 3$$

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The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative then the first term is

(A) 4 (B) −4

(C) −12 (D) 12

$-12, 24, -$

$a, ar, ar^2, ar^3 \ldots$

$a + ar = 12$ — (1)

$a(1+r) = 12$

$ar^2 + ar^3 = 48$

$ar^2(1+r) = 48$

$\frac{1}{r^2} = \frac{1}{4} \Rightarrow r = \pm 2$

$r = -2$

$\Rightarrow a(1-2) = 12$

$-a = 12$

$a = -12$

$1^3 - 2^3 + 3^3 - 4^3 + \cdots + 9^3 =$

(A) ~~425~~ (B) −425

(C) 475 (D) −475

$(2 \times 3)^3$

$2^3 \, 3^3$

$S = 1^3 - 2^3 + 3^3 - 4^3 - - - + 9^3$

$= (1^3 + 3^3 + 5^3 + 7^3 + 9^3) - (2^3 + 4^3 + 6^3 + 8^3)$

$= (1^3 + 2^3 - \cdots - 9^3) - 2 \times 2^3 (1^3 + 2^3 + 3^3 + 4^3)$

$= \left(\frac{n(n+1)}{2}\right)^2 - 16 \cdot \left(\frac{n(n+1)}{2}\right)^2$

$= (45)^2 - 16 \times 100$

$= 2025 - 1600$

$= 425$

$$\begin{array}{r} 45 \\ 45 \\ \hline 225 \\ 180\times \\ \hline 2025 \end{array}$$

The sum to infinity of the series $\frac{1}{2.4}+\frac{1}{4.6}+\frac{1}{6.8}+\frac{1}{8.10}+\;...+$ is :

(A) 1/4 (B) 1/8

(C) 1/2 (D) 1/16

$$S = \frac{1}{2\cdot 4} + \frac{1}{4\cdot 6} + \frac{1}{6\cdot 8} - - -$$

$$S = \frac{1}{2r(2r+2)}$$

$$= \frac{1}{4}\left[\frac{1}{r(r+1)}\right]$$

$$= \frac{1}{4}\left[\frac{(r+1)-r}{r(r+1)}\right] = \frac{1}{4}\left[\frac{1}{r} - \frac{1}{r+1}\right]$$

$$= \frac{1}{4}\left[1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} - \cancel{\;\;} - \cancel{\frac{1}{\infty}}\right.$$

The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is –

(A) $\frac{1}{2}$

(B) $\cos\frac{\pi}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

$$\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\pi/8\right)\right)$$

$$\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\pi/8\right)$$

$$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\left(1-\cos^2(\pi/8)\right)\left(1-\cos^2 3\pi/8\right)$$

$$\left[1-\left(\frac{1+\cos\pi/4}{2}\right)\right]\left[1-\left(\frac{1+\cos 3\pi/4}{2}\right)\right]$$

$$\left[\frac{1}{2}-\frac{1}{2\sqrt{2}}\right]\left[\frac{1}{2}+\frac{1}{2\sqrt{2}}\right] \Rightarrow \left(\frac{1}{4}-\frac{1}{8}\right) = \frac{1}{8}$$

The maximum value of the expression $\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is......

(A) 0 (B) 1

(C) 4 (D) 2

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha =$ ________

(A) $\cot\alpha$

(B) $\tan\alpha$

(C) 0

(D) $\tan 2\alpha$