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PHYSICS

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Time varying Magnetic Field, L-R Circuit and L-C Oscillations

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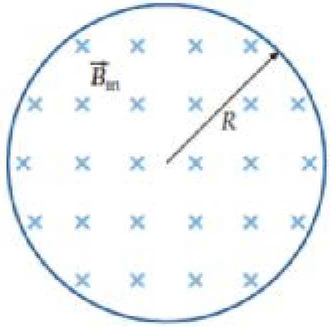
Induced Electric Field

Consider a cylindrical region having uniform and time varying magnetic field.



In the region r < R

the rate of change of magnetic flux is :

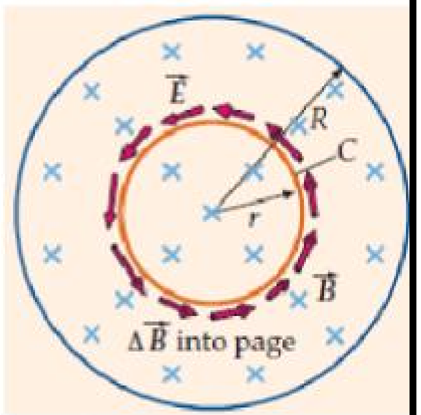
$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(-BA) = -\left(\frac{dB}{dt}\right)\pi r^2$$

Using equation, we have

$$\oint \vec{E}_{nc}.d\vec{s} = E_{nc}(2\pi r) = -\frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)\pi r^2$$

which implies

$$\boxed{E_{nc} = \frac{r}{2}\frac{dB}{dt}}$$



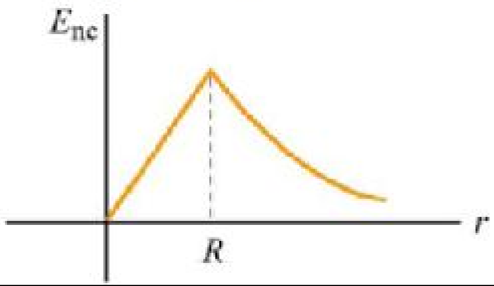
Similarly, for r > R

the induced electric field may be obtained as

$$E_{nc}(2\pi r) = \frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)\pi R^2$$

or $$\boxed{E_{nc} = \frac{R^2}{2r}\frac{dB}{dt}}$$

A plot of E_{nc} as a function of r is shown in figure.



Example

The current in an ideal, long solenoid is varied at a uniform rate of 0.01 A/s. The solenoid has 2000 turns/m and its radius is 6.0 cm. (a) Consider a circle of radius 1.0 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 2.0 seconds. (b) Find the electric field induced at a point on the circumference of the circle. (c) Find the electric field induced at a point outside the solenoid at a distance 8.0 cm from its axis.

Sol. (a) $B = \mu_0 nI$

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt} = \pi(1 \times 10^{-2})^2 \times \mu_0 \times \frac{2000}{1} \times \frac{dI}{dt}$$

$$= \pi\mu_0 \times 10^{-4} \times 2000 \times 0.01$$

$$\Delta\phi = 2 \times \frac{d\phi}{dt} = 4\pi \times 10^{-3} \times \mu_0$$

$$= 16\pi^2 \times 10^{-10} \text{ weber.}$$

(b) $$E = \frac{\varepsilon}{2\pi r} = \frac{2\pi \times 10^{-3} \times \mu_0}{2\pi \times 1 \times 10^{-2}}$$

$$= 0.1\,\mu_0 = 4\pi \times 10^{-6} \text{ V/m.}$$

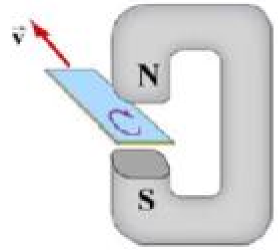
(c) $$E' \times 2\pi r' = A'\frac{dB}{dt}$$

$$E' \times 2\pi \times 8 \times 10^{-2} = \pi \times (6 \times 10^{-2})^2 \frac{dB}{dt}$$

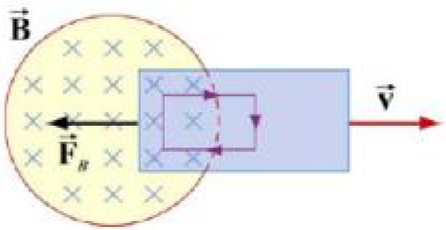
$$E' = \frac{36}{8}.E = \frac{18}{4}E = \frac{18}{4} \times 4\pi \times 10^{-6}$$

$$= 18\pi \times 10^{-6} \text{ V/m.}$$

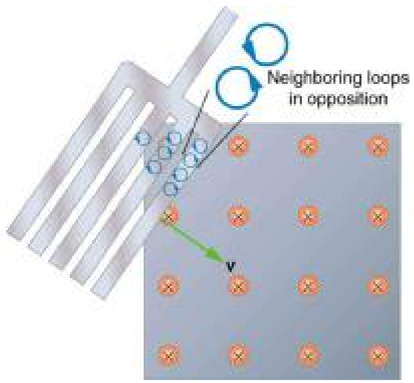
Eddy Currents

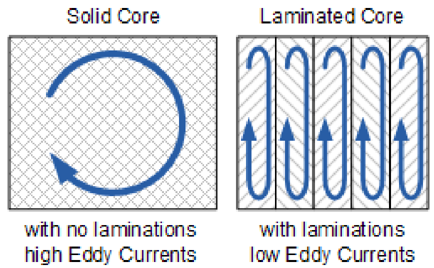


We have seen that when a conducting loop moves through a magnetic field, current is induced as the result of changing magnetic flux. If a solid conductor were used instead of a loop, as shown in Figure, currents can also be induced along any closed loop in the conductor. The induced current are called an eddy current.



The induced eddy currents also generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field.





Eddy currents losses can be minimized using,

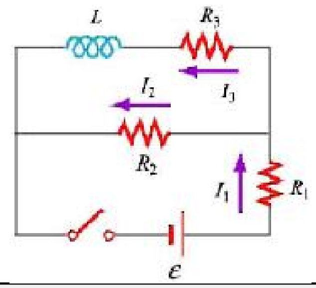
1. Constructing the slab by using gluing together thin strips that are insulated from one another.
2. By making cuts in the slab, thereby disrupting the conducting path.

There are important applications of eddy currents. For example, the currents can be used to suppress unwanted mechanical oscillations. Another application is the magnetic braking systems in high-speed transit cars.

L-R Circuit

1. Current does not change instantaneously in the branch containing inductor.
2. At steady state, current becomes constant in the branch containing the inductor. And potential difference across the inductor becomes zero. For calculation purposes it can be assumed as wire of zero resistance.

Example : Consider the circuit shown in figure below.
Determine the current through each resistor
(a) immediately after the switch is closed.
(b) a long time after the switch is closed.



Answer

(a) $I_1 = I_2 = \dfrac{\varepsilon}{R_1 + R_2}$

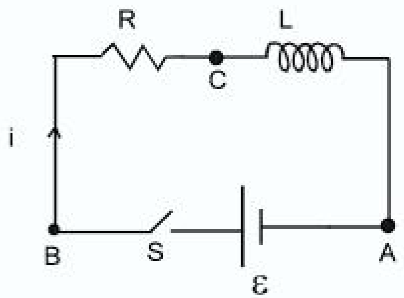
(b) $I_1 = \dfrac{(R_2 + R_3)\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$,

$I_2 = \dfrac{R_3\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$,

$I_3 = \dfrac{R_2\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$

Growth of Current in Series L-R Circuit

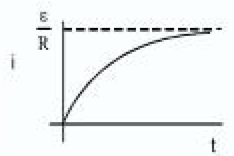
Figure shows a circuit consisting of a cell, an inductor L and a resistor R ,connected in series. Let the switch S be closed at t=0.Suppose at an instant current in the circuit be i which is increasing at the rate di/dt.



Writing KVL along the circuit , we have $\varepsilon - L\frac{di}{dt} - iR = 0$

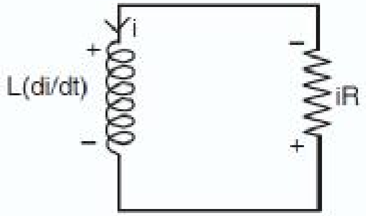
On solving we get, $i = \boxed{\frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})}$

The quantity L/R is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.



Decay of Current in Series L-R Circuit

Let the initial current in the circuit be I_0. At any time t, let the current be i and let its rate of change at this instant be $\frac{di}{dt}$.



$$L.\frac{di}{dt} + iR = 0$$

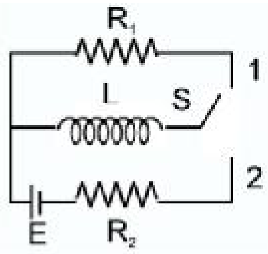
$$\frac{di}{dt} = -\frac{iR}{L}$$

$$\int_{I_0}^{i} \frac{di}{i} = -\int_{0}^{t} \frac{R}{L}.dt$$

$$\ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L} \quad \text{or} \quad \boxed{i = I_0 e^{\frac{-Rt}{L}}}$$

Example

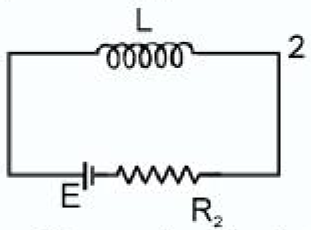
In the circuit shown switch S is connected to position 2 for a long time and then joined to position 1.
The total heat produced in resistance R_1 is :



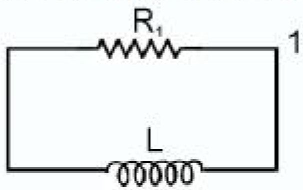
(A) $\frac{LE^2}{2R_2^{\,2}}$ (B) $\frac{LE^2}{2R_1^{\,2}}$ (C) $\frac{LE^2}{2R_1R_2}$ (D) $\frac{LE^2(R_1+R_2)^2}{2R_1^{\,2}R_2^{\,2}}$

Sol. When the key is at position (2) for a long time ; the energy stored in the inductor is :

$$U_B = \frac{1}{2}Li_0^{\,2} = \frac{1}{2}.L.\left(\frac{E}{R_2}\right)^2 = \frac{L.E.^2}{2R_2^{\,2}}$$



This whole energy will be dissipated in the form of heat when the inductor is connected to R_1 and no source is connected.



Hence (A).

Example

A coil of resistance 40 Ω is connected across a 4.0 V battery, 0.10 s after the battery is connected, the current in the coil is 63 mA. Find the inductance of the coil. [$e^{-1} \simeq 0.37$]

Sol. $I = I_0(1 - e^{-t/\tau})$

$$e^{-t/\tau} = 1 - \frac{I}{I_0} = 0.37 \quad = e^{-1}$$

$$\frac{tR}{L} = 1$$

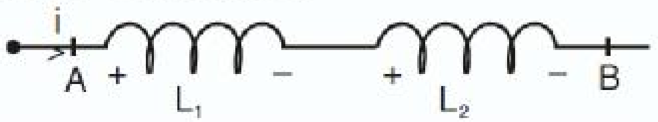
$L = tR$

$= 0.10 \times 40$

$L = 4H.$

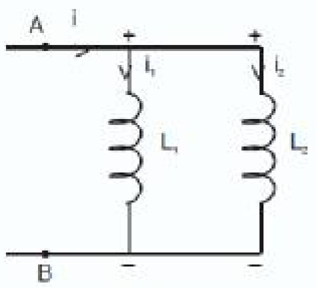
Inductors in Series and Parallel

Series combination



$L = L_1 + L_2$ (neglecting mutual inductance)

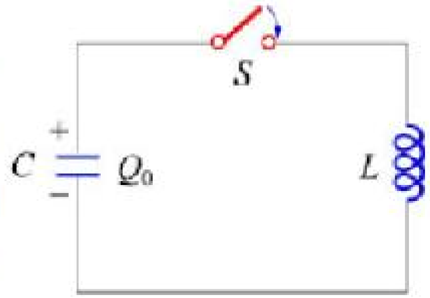
Parallel Combination :



$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$ (neglecting mutual inductance)

LC Oscillations

Consider an LC circuit in which a capacitor is connected to an inductor, as shown in Figure. Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.



The total energy in the LC circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where $I = \frac{dQ}{dt}$ and $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

The general solution to equation is

$$Q = Q_0 \cos(\omega_0 t + \phi)$$

where Q_0 is the amplitude of the charge and ϕ is the phase.

The angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Example

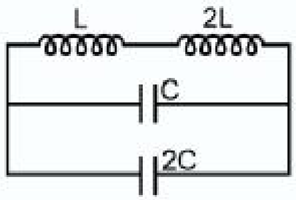
The frequency of oscillation of current in the inductor is:

(A) $\frac{1}{3\sqrt{LC}}$

(B) $\frac{1}{6\pi\sqrt{LC}}$

(C) $\frac{1}{\sqrt{LC}}$

(D) $\frac{1}{2\pi\sqrt{LC}}$



Maxwell's Equations

1. $\oint \mathbf{E} \cdot d\mathbf{A} = Q / \varepsilon_0$ (Gauss's Law for electricity)

2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ (Gauss's Law for magnetism)

3. $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\Phi_B}{dt}$ (Faraday's Law)

4. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ (Ampere – Maxwell Law)

displacement current	$i = \varepsilon_0 \left(\frac{d\Phi_E}{dt} \right)$

Electromagnetic Waves

- Stationary charges produces only electrostatic fields
- Charges in uniform motion (steady currents) produces magnetic fields
- Accelerated charges radiate electromagnetic waves
- Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on.
- The oscillating electric and magnetic fields thus regenerate each other, so to speak, as the wave propagates through the space. The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge.
- The energy associated with the propagating wave comes at the expense of the energy of the source – the accelerated charge.

Nature of Electromagnetic Waves

- The electric and magnetic fields in an electromagnetic wave are perpendicular to each other, and to the direction of propagation.
- Figure shows a typical example of a plane electromagnetic wave propagating along the z direction (the fields are shown as a function of the z coordinate, at a given time t).

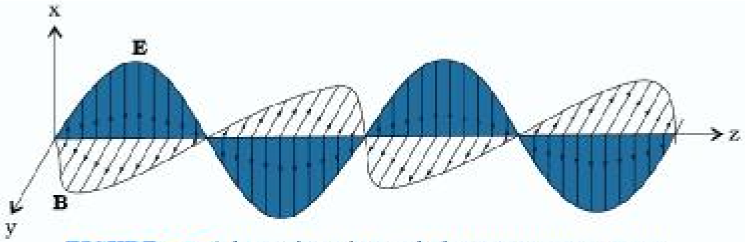


FIGURE ⇨ A linearly polarised electromagnetic wave, propagating in the z-direction with the oscillating electric field **E** along the x-direction and the oscillating magnetic field **B** along the y-direction.

- We can write E_x and B_y as follows:

$$E_x = E_0 \sin(kz - \omega t)$$
$$B_y = B_0 \sin(kz - \omega t)$$

where, $k = \frac{2\pi}{\lambda}$

ω is the angular frequency

Speed of propagation $= \omega / k$

$\omega = ck$, where, $c = 1/\sqrt{\mu_0 \varepsilon_0}$

Speed of light in medium is $v = \frac{1}{\sqrt{\mu\varepsilon}}$

It is also seen from Maxwell's equations that

$B_0 = (E_0/c)$

Energy Density of EM Waves

$$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}}$$

In electric field; $U_E = \frac{1}{2} e_0 E^2$

In magnetic field; $U_B = \frac{B^2}{2m_0}$

Total energy density of electric and magnetic field in free space

$$U = U_E + U_B = \frac{1}{2} e_0 E^2 + \frac{B^2}{2m_0}$$

Average Energy Density of EM Waves

$$U_{avg} = \frac{1}{2}\epsilon_0 E_{rms}^2 + \frac{B_{rms}^2}{2\mu_0}$$

where, $E_{rms} = \dfrac{E_0}{\sqrt{2}}$ *and* $B_{rms} = \dfrac{B_0}{\sqrt{2}}$

$$\langle U_E \rangle = \frac{1}{2}\epsilon_0 E_{rms}^2 = \frac{\epsilon_0 E^2}{4}$$

and

$$\langle U_B \rangle = \frac{B_{rms}^2}{2\mu_0} = \frac{B^2}{4\mu_0}$$

$$\because B_0 = \frac{E_0}{c}$$

$$\therefore \langle U_B \rangle = \frac{B^2}{4\mu_0} = \frac{E_0^2}{4\mu_0 c^2} = \frac{\mu_0 \epsilon_0 E_0^2}{4\mu_0} \quad \because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{\epsilon_0 E^2}{4}$$

Hence

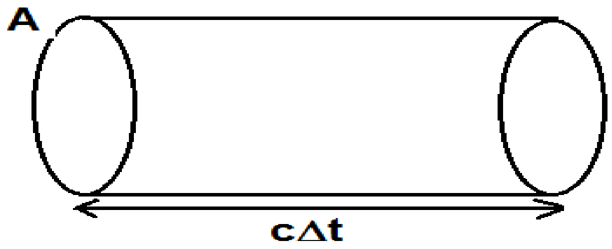
$$\langle U_E \rangle = \langle U_B \rangle$$

$$U_{avg} = 2\langle U_E \rangle = 2\langle U_B \rangle = \frac{1}{2}\epsilon_0 E_0^2 = \epsilon_0 E_{rms}^2$$

Intensity of EM Waves

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{time}}$$

Total energy contained in the cylinder



$$U = \text{Avg. energy density} \times \text{Volume}$$

$$U = \frac{1}{2}\varepsilon_0 E_0^2 \times (c\Delta t) \times A$$

$$\boxed{\text{Intensity} = \frac{U}{A\Delta t} = \frac{1}{2}c\varepsilon_0 E_0^2}$$

$$\boxed{I = c\varepsilon_0 E_{rms}^2}$$

Energy, Momentum and Pressure of EM Waves

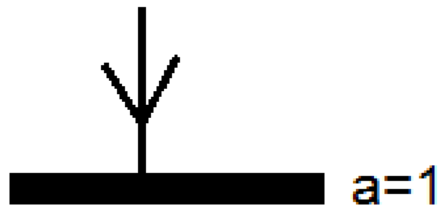
$$E = mc^2 \quad and \quad p = mc = \frac{E}{c} = \frac{h}{\lambda}$$

\ Momentum carried by an e.m. wave

$$p = \frac{\text{Energy of the wave}}{\text{speed of the wave}} = \frac{U}{c}$$

$p_i = \frac{U}{c}$

$p_f = 0$



$$\text{Force} = \frac{dp}{dt} = \frac{\Delta p}{\Delta t} = \frac{U/c}{\Delta t} = \frac{U}{c\Delta t} \quad and \quad \text{Pressure} = \frac{F}{A} = \frac{U}{c\Delta t \times A} = \frac{I}{c}$$

Note: If $r = 1$, then

$$p_f = \frac{U}{c} \quad and \quad \Delta p = \frac{2U}{c}$$

$$\text{Pressure} = \frac{2I}{c}$$