Problem Solving on Trigonometry

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1. If $\pi < 2\theta \leq \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals to

(A) $-2\cos\theta$ (B) $-2\sin\theta$

(C) $2\cos\theta$ (D) $2\sin\theta$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta \qquad \frac{\pi}{2} < \theta < \frac{3\pi}{4}$$

$$\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+\sqrt{2\times 2\cos^2 2\theta}}$$

$$= \sqrt{2+|2\cos 2\theta|}$$

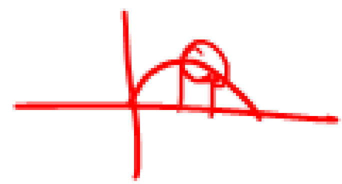
$$= \sqrt{2-2\cos 2\theta}$$

$$= \sqrt{2(1-\cos 2\theta)}$$

$$= \sqrt{2\times 2\sin^2\theta}$$

$$= |2\sin\theta|$$

$$= 2\sin\theta$$



(2) The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is -

(A) $\frac{3\pi}{5}$ (B) $\frac{7\pi}{5}$ (C) $\frac{\pi}{10}$ (D) $-\frac{\pi}{10}$

$$\sin^{-1}\left(\sin\frac{33\pi}{5}\right)$$

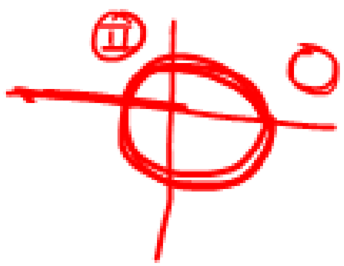
$$\sin^{-1}\left(\sin\left(7\pi - \frac{2\pi}{5}\right)\right)$$

$$\sin^{-1}(\sin 2\pi/5) = 2\pi/5$$

$$\sin^{-1}\left(\cos\frac{33\pi}{5}\right) = \frac{\pi}{2} - \cos^{-1}\left(\cos\frac{33\pi}{5}\right) \times$$

$$\sin^{-1}\left(\cos\left(6\pi + 3\pi/5\right)\right) =$$

$$\sin^{-1}\left(\cos 3\pi/5\right) = \frac{\pi}{2} - \cos^{-1}\left(\cos 3\pi/5\right)$$



$$= \pi/2 - 3\pi/5$$

$$= -\frac{\pi}{10}$$

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$[0, \pi]$ $[-\pi/2, \pi/2]$

The value of

$\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$ is equal to

(A) $\pi/4$ (B) $5\pi/12$

~~(C) $3\pi/4$~~ (D) $13\pi/12$

$\cos^{-1}(-1/2) = \theta$

$-\frac{1}{2} = \cos\theta$

$\frac{\pi}{4} + \underbrace{\frac{2\pi}{3} + \left(-\frac{\pi}{6}\right)}_{\pi/2}$

$\frac{\pi}{4} + \frac{\pi}{2}$

$\frac{3\pi}{4}$

$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

$-1 \le x \le 1$

$(-\pi/2, \pi/2)$

$\frac{\pi}{2} + \tan^{-1}x$

Q) $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

Range of $f(x)$?

$-\pi/4 \le \quad \le \pi/4$

A) $(0, \pi)$ B) $[0, \pi]$ C) $[\pi/4, \frac{3\pi}{4}]$

D) None

$$\frac{1}{\cos 0^\circ \cos 1^\circ} + \frac{1}{\cos 1^\circ \cos 2^\circ} + \frac{1}{\cos 2^\circ \cos 3^\circ} + \cdots + \frac{1}{\cos 59^\circ \cos 60^\circ} = \_\_\_\_$$

(A) 10 (B) 0

(C) $\frac{\sqrt{3}}{\sin 1^\circ}$ (D) $\frac{1}{\sqrt{3}\sin 1^\circ}$

$$\frac{1}{\sin 1^\circ}\left[\frac{\sin 1^\circ}{\cos 0^\circ \cos 1^\circ} + \frac{\sin 1^\circ}{\cos 1^\circ \cos 2^\circ} + - - - \frac{\sin 1^\circ}{\cos 59^\circ \cos 60^\circ}\right]$$

$$\frac{1}{\sin 1^\circ}\left[\frac{\sin(1^\circ - 0^\circ)}{\cos 0^\circ \cos 1^\circ} + \frac{\sin(2^\circ - 1^\circ)}{\cos 1^\circ \cos 2^\circ} - - - \frac{\sin(60^\circ - 59^\circ)}{\cos 59^\circ \cos 60^\circ}\right]$$

$$\left[\frac{\sin 1^\circ}{\cos 1^\circ} + \frac{\sin 2^\circ \cos 1^\circ - \cos 2^\circ \sin 1^\circ}{\cos 1^\circ \cos 2^\circ} - - - \frac{\sin 60^\circ \cos 59^\circ - \cos 60^\circ \sin 59^\circ}{\cos 59^\circ \cos 60^\circ}\right]$$

$$\frac{1}{\sin 1^\circ}\left[\cancel{\tan 1^\circ} + \cancel{\tan 2^\circ} - \cancel{\tan 1^\circ} + \cdots + \tan 60^\circ - \cancel{\tan 59^\circ}\right]$$

$$\frac{\sqrt{3}}{\sin 1^\circ}$$

The number of values of x in the interval [0, 5π] satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is

graphical

(A) 0 (B) 5

(C) 6 (D) 10

$$3\sin^2 x - 7\sin x + 2 = 0$$

$$3\sin^2 x - 6\sin x - \sin x + 2 = 0$$

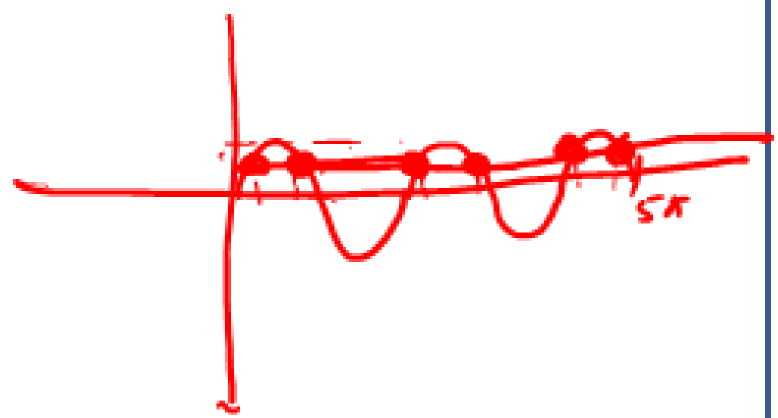
$$3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$$

$$(3\sin x - 1)\cancel{(\sin x - 2)} = 0$$

$$\boxed{\sin x = 1/3}$$

No of sol^n

$$\boxed{f(x) = g(x)}$$



9) Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians).

Then x lies in the interval

(A) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(B) $\left(-1, \frac{5\pi}{6}\right)$

(C) $(-1, 2)$

(D) $\left(\frac{\pi}{6}, 2\right)$

$x^2 - x - 2 < 0$

$x^2 - 2x + x - 2 < 0$

$x(x-2) + 1(x-2) < 0$

$(x+1)(x-2) < 0$

$\boxed{x \in (-1, 2)}$

$2\sin^2 x + 3\sin x - 2 > 0$

$2\sin^2 x + 4\sin x - \sin x - 2 > 0$

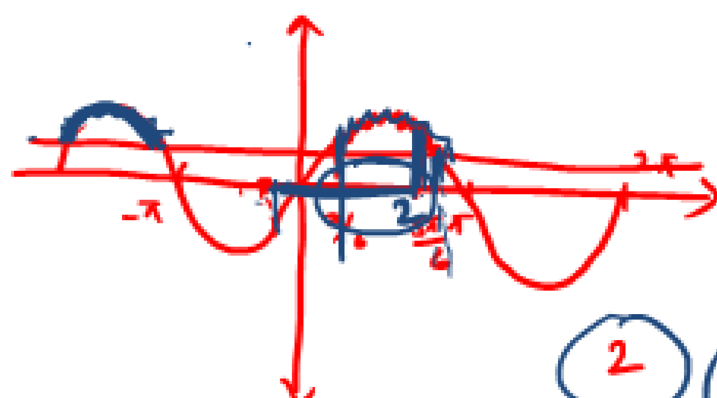
$2\sin x(\sin x + 2) - 1(\sin x + 2) > 0$

$(2\sin x - 1)\underbrace{(\sin x + 2)}_{+ve} > 0$

$2\sin x - 1 > 0$

$\boxed{\sin x > 1/2}$

$x \in (\pi/6, 5\pi/6)$



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$\frac{5\pi}{6}$

$= 0.8 \times 3.1$

$= 2.5$

$\left(\frac{\pi}{6}, 2\right)$

$$\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$$ is equal to ____

(A) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$

(B) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$

(C) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$

(D) $\tan^{-1}\left(\frac{n^2-n+2}{n^2-n}\right)$

$(\cancel{\tan^{-1}3} - \tan^{-1}1 + \cancel{\tan^{-1}7} - \cancel{\tan^{-1}3}$
$+ \cancel{\tan^{-1}13} - \cancel{\tan^{-1}7})$
$+ \tan^{-1}(n^2+n+1) - \cancel{\tan^{-1}(n^2-n+1)}$

$\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$

$$\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$$

$$= \tan^{-1}\left(\frac{2m}{1+(m^4+m^2+1)}\right)$$

$$= \tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$\tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(\ )(\ )}\right)$$

$$\sum_{m=1}^{n}\left[\tan^{-1}(m^2+m+1) - \tan^{-1}(m^2-m+1)\right]$$

$\cancel{\tan^{-1}(3)} - \tan^{-1}(1) + \cancel{\tan^{-1}7} - \cancel{\tan^{-1}3}$

$\tan^{-1}(n^2+n+1)$

$- \cancel{\tan^{-1}(n^2-n+1)}$

$\tan^{-1}(n^2+n+1) - \tan^{-1}(1)$

$$\tan^{-1}\left(\frac{n^2+n+\cancel{1}-\cancel{1}}{1+(n^2+n+1)}\right)$$

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The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\underbrace{\sum_{k=1}^{n}2k}\right)\right)$ is –

Previous

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

$2\cdot1+2\cdot2+2\cdot3 \_\_ 2\cdot n$

$2(1+2+3 \_\_ n)$

$\sum_{k=1}^{n}2k = \cancel{2}\frac{(n)(n+1)}{\cancel{2}}$

$\cot\left(\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))\right)$

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$

$\cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)\right)$

$\tan^{-1}\left(\frac{(n+1)-(n)}{1+n(n+1)}\right)$

$\sum_{n=1}^{23}\left[\tan^{-1}(n+1)-\tan^{-1}(n)\right]$

$(\cancel{\tan^{-1}2}-\tan^{-1}1)+(\cancel{\tan^{-1}(3)}-\cancel{\tan^{-1}2})$
$+\tan^{-1}(24)-\cancel{\tan^{-1}(23)}$

$\cot\left[\tan^{-1}(24)-\tan^{-1}(1)\right]$

$\cot\left(\tan^{-1}\left(\frac{23}{25}\right)\right)$

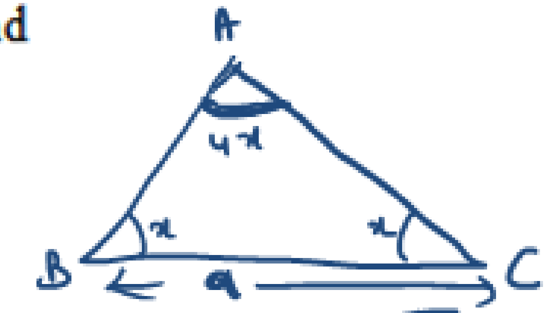
$\cot\left(\cot^{-1}(25/23)\right)$

$\frac{25}{23}$

If the angles of a triangle are in ratio 4 : 1 : 1 then the ratio of the longest side and perimeter of triangle is :

(A) $\frac{1}{2+\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}-2}$

(C) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (D) none of these



$4x + x + x = \pi$

$6x = \pi$

$x = \pi/6$

$\pi/6, \pi/6, 4\pi/6, 2\pi/3$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$

$a = K\sin A, \; b = K\sin B, \; c = K\sin C$

$a = K \times \frac{\sqrt{3}}{2}, \; b = \frac{K}{2}, \; c = \frac{K}{2}$

$\frac{a}{a+b+c}$

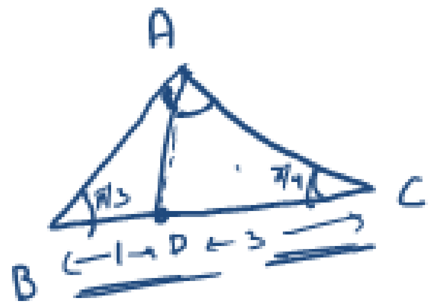
$\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{\sqrt{3}}{\sqrt{3}+2}$

Q) In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$,

Let D divide BC internally in the ratio 1 : 3.

Then $\frac{\sin \angle BAD}{\sin \angle CAD}$ (x) equal to-

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$



$$\frac{AD}{\sin B} = \frac{BD}{\sin\angle BAD} \quad \text{— (1)}$$

$$\frac{AD}{\sin C} = \frac{CD}{\sin\angle CAD} \quad \text{— (2)}$$

$$\frac{\sin C}{\sin B} = \frac{BD}{CD} \times \frac{\sin\angle CAD}{\sin\angle BAD}$$

$$\frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}} = \frac{1}{3} \times \left(\frac{1}{x}\right)$$

$$\frac{6}{\sqrt{6}} = \frac{1}{x}$$

$$\boxed{x = \frac{1}{\sqrt{6}}}$$