PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Electromagnetic Induction

By,
Ritesh Agarwal, B. Tech. IIT Bombay

Magnetic Flux

The number of lines of induction passing through a unit area normal to the area, measures the magnitude of magnetic induction or magnetic flux density $\vec{B}$. Obviously in a region, smaller is the relative spacing of lines of induction, greater is the value of magnetic induction.

The tangent to the line of induction at any point gives the direction of magnetic induction at that point.

The magnetic flux 'ϕ_B' through a surface of area A is the total number of magnetic lines of induction passing through that area normally.

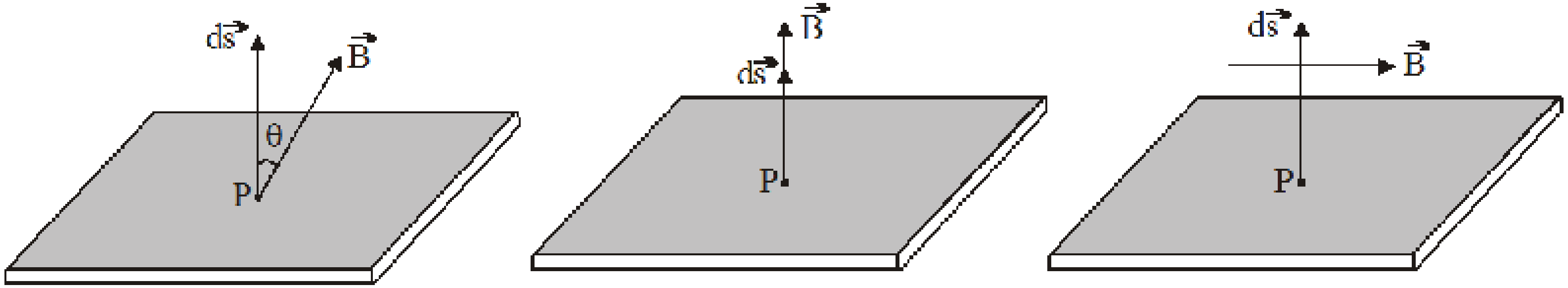
Mathematically, magnetic flux, $\phi_B = \int \vec{B}.d\vec{A} = BA\cos\theta$ if B is uniform

The unit of magnetic field induction in SI system is Wb/m^2 or tesla (T).

$1 \text{ gauss} = 10^{-4}$ T

$\therefore$ 1 maxwell 10^{-8} Wb

Magnetic Flux



$d\phi_B = B\,ds\cos\theta$

(A)

$(d\phi_B)_{max} = B\,ds$

(B)

$(d\phi_B)_{min} = 0$

(C)

Faraday's Laws of Electromagnetic Induction

(i) When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf.
If the circuit is closed then the current will be called induced current.

$$\text{magnetic flux} = \int \vec{B}.d\vec{s}$$

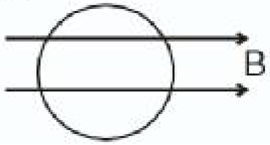
(ii) The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic lines of force are cut by a wire

$$E = -\frac{d\phi}{dt}$$

(–) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.
SI unit of magnetic flux = Weber.

Example

A coil is placed in a constant magnetic field .The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil .



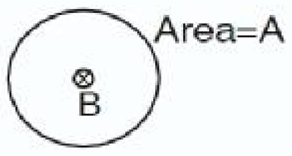
Solution :

$\phi = 0$ (always) since area is perpendicular to magnetic field.

$\therefore$ emf = 0

Example

Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



Solution :

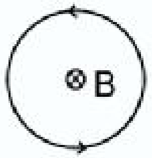
ϕ = BA (always)

= const.

$\therefore$ emf = 0

Example

Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.

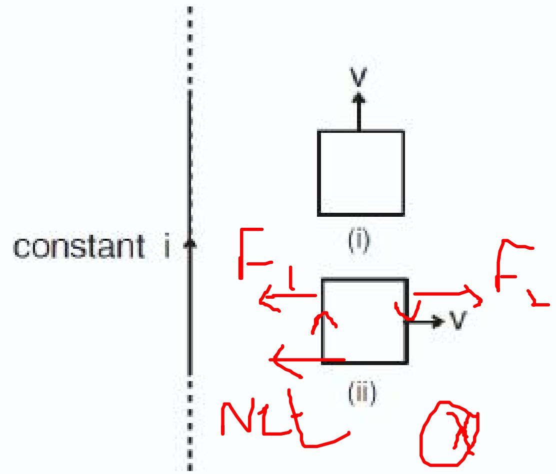


Solution :

Inward flux is increasing with time. To opposite it outward magnetic field should be induced. Hence current will flow anticlockwise.

Example

Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



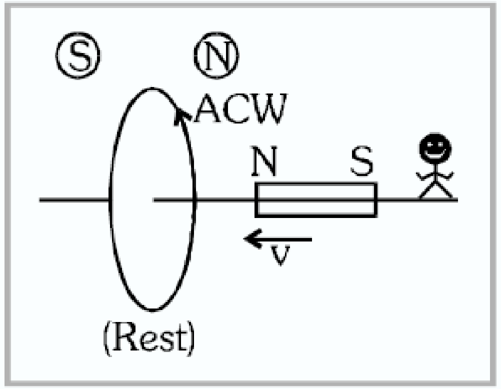
Solution :

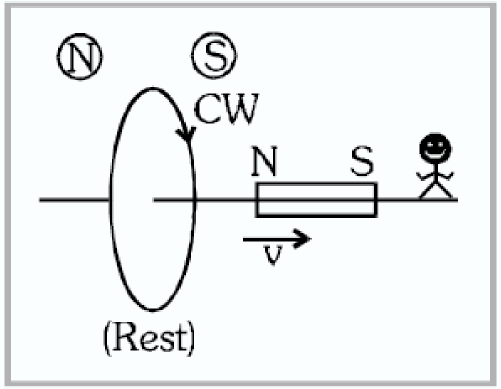
In loop (i) no emf will be induced because there is no flux change.

In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

Lenz's Law (Conservation of Energy Principle)

According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.





Important Formulas

Average induced. emf. $e_{av} = \frac{-\Delta\phi}{\Delta t}$

Instantaneous **induced emf** $e = -\frac{d\phi}{dt}$

Induced current flow at this instant $I = \frac{e}{R}$

$$I = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$$

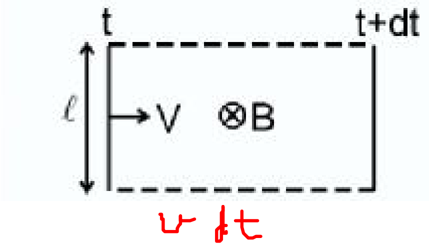
In time interval dt, **induced charge** $dq = Idt$

$$dq = \frac{d\phi}{R}$$

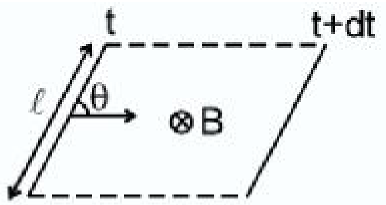
Induced heat :- $H = \int_0^t I^2 R dt = \int_0^t \frac{e^2}{R} dt$

Motional EMF

emf induced between the ends of the rod $= Bv\ell$

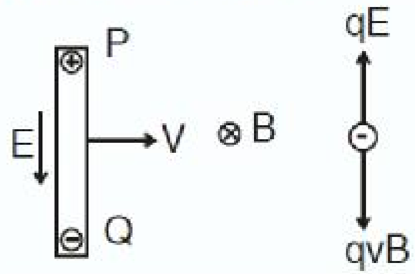


$\text{emf} = Bv\ell\sin\theta$



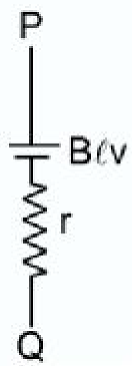
Explanation of EMF induced in the rod

If a rod is moving with velocity v in a magnetic field B ,as shown , the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end. These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other. Thus E = v B.



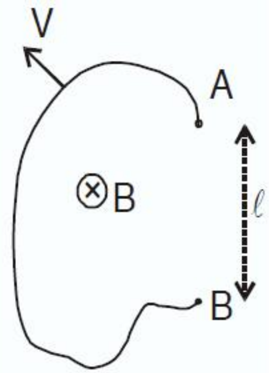
$V_P - V_Q = V B \ell$

The moving rod is equivalent to the following diagram, electrically.



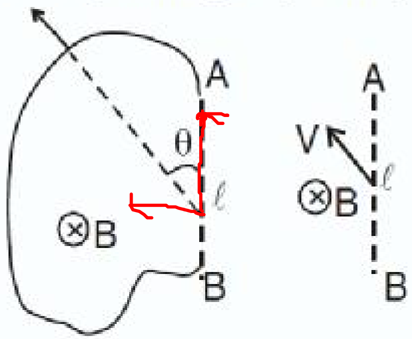
Example

Figure shows an irregular shaped wire AB moving with velocity v, as shown. Find the emf induced in the wire.

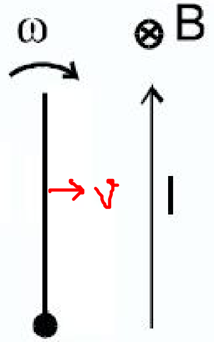


Solution :

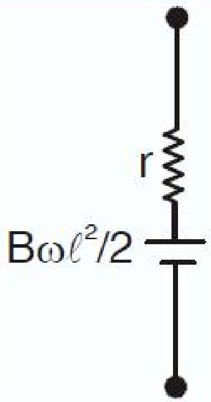
The same emf will be induced in the straight imaginary wire joining A and B , which is $Bv\ell \sin\theta$



Induced EMF due to Rotation of Rod



Emf induced in the rod $\varepsilon = \frac{1}{2}B\omega\ell^2$

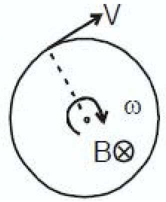


Equivalent of this rod is as following

Induced EMF due to Rotation of Ring/Coil

If coil rotates about it's axis

Flux passing through the ring ϕ = B.A is a constant
therefore emf induced in the coil is zero
Every point of this ring is at the same potential, by symmetry.



If coil rotates about it's diameter

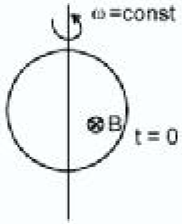
At any time t, $\phi = BA\cos\theta = BA\cos\omega t$
Now induced emf in the loop

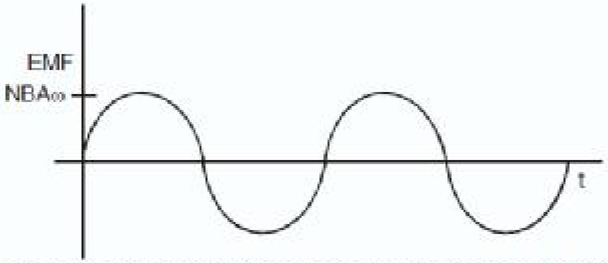
$$e = \frac{-d\phi}{dt} = BA\,\omega\sin\omega t$$

If there are N turns
emf = $BA\omega N\sin\omega t$
$BA\,\omega N$ is the amplitude of the emf
$e = e_m\sin\omega t$

$$i = \frac{e}{R} = \frac{e_m}{R}\sin wt = i_m\sin wt$$

$$i_m = \frac{e_m}{R}$$





The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

Self Induction and Self Inductance

When a current flowing through a coil is changed the flux linking with its own winding changes & due to the change in linking flux with the coil an emf is induced which is known as self induced emf & this phenomenon is known as self induction. This induced emf opposes the causes of Induction. The property of the coil or the circuit due to which it opposes any change of the current coil or the circuit is known as **SELF - INDUCTANCE**. It's unit is Henry.

Coefficient of Self inductance $\boxed{L = \frac{\phi_s}{i}}$ or $\phi_s = Li$

L depends only on ;

(i) shape of the loop &

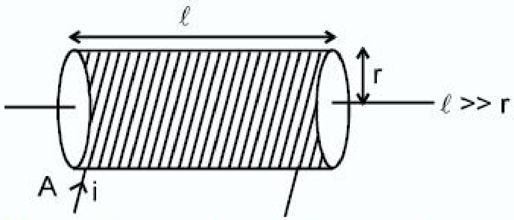
(ii) medium

If i = current in the circuit.

ϕ_s = magnetic flux linked with the circuit due to the current i.

self induced emf $e_s = -\frac{d\phi_s}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$ (if L is constant)

Self Inductance of Solenoid



Let the volume of the solenoid be V , the number of turns per unit length be n.

Let a current I be flowing in the solenoid.Magnetic field in the solenoid is given as $B=\mu_0 nI$.The magnetic flux through one turn of solenoid $\phi = \mu_0 n I \ A$.

The total magnetic flux through the solenoid = $N\phi = N\mu_0 nI A = \mu_0 n^2 I A l$

$\therefore \quad L = \mu_0 n^2 l \ A = \mu_0 n^2 V$

$\phi = \mu_0 \ n \ i \ \pi r^2 \ (n\ell)$

$$L = \frac{\phi}{i} = \mu_0 \ n^2 \ \pi r^2 \ell.$$

Inductance per unit volume = $\mu_0 n^2$.

Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant.Current can not change suddenly in the inductor.

Inductor

It is represent by

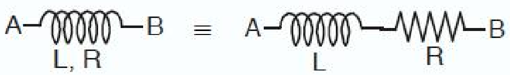
electrical equivalence of loop

If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current.If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.

Over all result

$$V_A - L\frac{di}{dt} = V_B$$

Note : If there is a resistance in the inductor (resistance of the coil of inductor) then :



Example

A B is a part of circuit. Find the potential difference $v_A - v_B$ if

(i) current i = 2A and is constant

(ii) current i = 2A and is increasing at the rate of 1 amp/sec.

(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Solution :

$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B $\quad V_A - 1\frac{di}{dt} - 5 - 2i = V_B$

(i) Put $i = 2, \frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B$

$\therefore V_A - V_B = 9$ volt

(ii) Put $i = 2, \frac{di}{dt} = 1$; $V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10\ V_0$

(iii) Put $i = 2, \frac{di}{dt} = -1$; $V_A + 1 - 5 - 2 \times 2 = V_B$ or $V_A = 8$ volt.

Energy Stored in Inductor

$$U = \frac{1}{2} L I^2$$

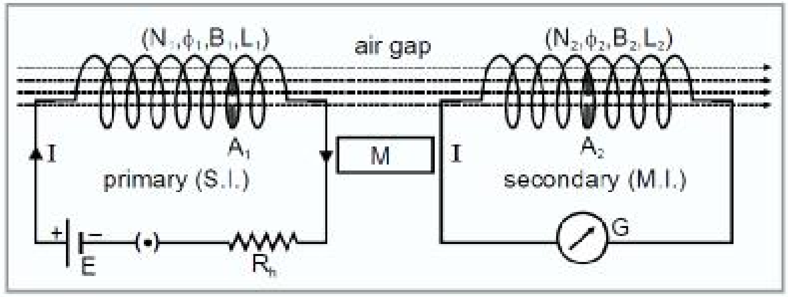
Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

$$\text{Total energy } U = \int \frac{B^2}{2\mu_0\mu_r} dV$$

Mutual Induction

Whenever current passing through primary coil or circuit change with respect to time then magnetic flux in neighbouring secondary coil or circuit will also changes with respect to time. According to Lenz Law for opposition of flux change an emf and a current induced in the neighbouring coil or circuit. This phenomenon called as 'Mutual induction'.



Due to Air gap always $\phi_2 < \phi_1$

Total flux of secondary is directly proportional to current flow through the primary coil

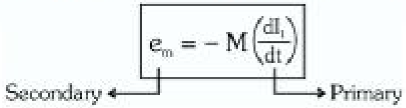
$$\phi_2 \propto I_1$$

$$\phi_2 = MI_1$$

The units and dimension of M are same as 'L'.

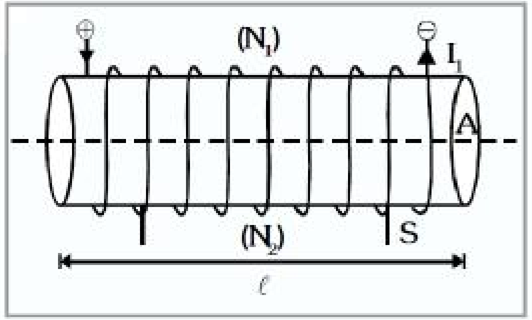
The mutual inductance does not depends upon current through the primary and it is constant for both circuits.

total mutual induced emf of secondary coil $e_m = -\dfrac{d\phi_2}{dt}$



Mutual Inductance of two Solenoids

Two co–axial solenoids ($M_{S_1S_2}$):-



$$M_{s_1s_2} = \frac{N_2 B_1 A}{I_1}$$

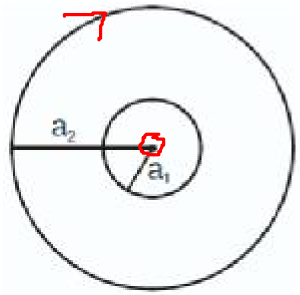
$$= \frac{N_2}{I_1}\left(\frac{\mu_0 N_1 I_1}{\ell}\right)A, \text{ where } B_1 = \frac{\mu_0 N_1 I_1}{\ell} \Rightarrow \boxed{M_{s_1s_2} = \left(\frac{\mu_0 N_1 N_2 A}{\ell}\right)} = \sqrt{L_1 L_2}$$

$$B_1 = \mu_0 \frac{N_1 I_1}{l}$$

$$\phi_{2m} = B_1 N_2 l A$$

Example

Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Solution :

Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil = $\frac{\mu_0 i}{2a_2}\,\pi a_1^2$

or $\quad Mi = \frac{\mu_0 i}{2a_2}\,\pi a_1^2 \quad$ or $\quad M = \frac{\mu_0 \pi a_1^2}{2a_2}$