PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Capacitance

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Capacitance of an Isolated Conductor

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor

$q \propto V$

$\Rightarrow \quad q = CV$

$V = \frac{Kq}{r} \Rightarrow V \propto q$

Where C is proportionality constant called capacitance of the conductor.

$q \propto V$

$$\boxed{q = CV}$$

Capacitance of an Isolated Spherical Conductor

(i) If the medium around the conductor is vacuum or air.

$\boxed{C_{Vacuum} = 4\pi\varepsilon_0 R}$

R = Radius of spherical conductor. (may be solid or hollow.)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity.

$C_{medium} = 4\pi\varepsilon_0 KR$

(iii) $\dfrac{C_{medium}}{C_{air\,/\,vaccum}} = K$ = dielectric constant.

$V = \frac{Kq}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$

$C = \frac{q}{V} = 4\pi\varepsilon_0 R$

Ques → Find capacitance of earth assuming it to be perfect conductor

Sol $C = 4\pi\varepsilon_0 R = \frac{R}{K} = \frac{6400\,\text{Km}}{9\times10^9} \approx 711\,\mu F$

Important Points

$U = \frac{q^2}{2C}$

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^2 T^4$

(iii) **1 Farad :** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$1\ \mu F = 10^{-6}$ F, $1 nF = 10^{-9}$ F or $1 pF = 10^{-12}$ F

(iv) **Capacitance of an isolated conductor depends on following factors :**

(a) Shape and size of the conductor :

On increasing the size, capacitance increases.

(b) On surrounding medium :

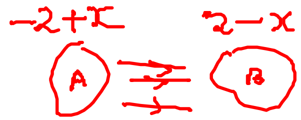
With increase in dielectric constant K, capacitance increases.

(c) Presence of other conductors :

When a neutral conductor is placed near a charged conductor capacitance of conductors increases.

(v) Capacitance of a conductor do not depend on

(a) Charge on the conductor (b) Potential of the conductor (c) Potential energy of the conductor.



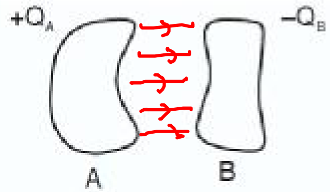
Capacitor

A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) In capacitor two conductors have equal but opposite charges.

(ii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iii) Formulae related with capacitors



(a) $Q = CV$

$$\Rightarrow \quad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.
V = Potential difference between positive and negative plates of capacitor
C = Capacitance of capacitor.

(b) Energy stored in the capacitor

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

This energy is stored inside the capacitor in its electric field with energy density

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon E^2 \text{ or } \frac{1}{2}\varepsilon_0\varepsilon_r E^2$$

Capacitor

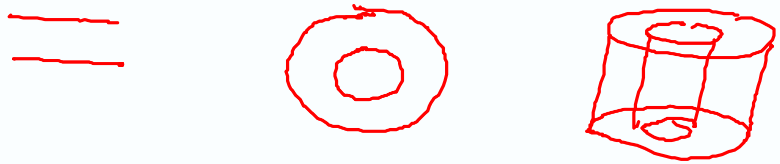
(v) The capacitor is represented as following:

(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor.

(b) Spherical capacitor.

(c) Cylindrical capacitor.



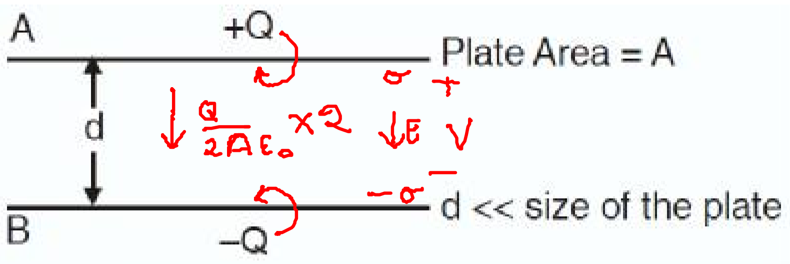
(vii) Capacitance of a capacitor depends on

(a) Area of plates.

(b) Distance between the plates.

(c) Dielectric medium between the plates.

Parallel Plate Capacitor



$$E = \frac{Q}{A\varepsilon_0}$$

$$V_A - V_B = E.d. = \frac{Qd}{A\varepsilon_0} = \cancel{\frac{Q}{C}}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

$$C = \frac{Q}{V_A - V_B} = \frac{\varepsilon_0 A}{d}$$

where A = area of the plates.

d = distance between plates.

Electric field intensity between the plates of capacitors (air filled)

$$\boxed{E = \sigma/\varepsilon_0 = V/d} = \frac{Q}{A\varepsilon_0}$$

Force experienced by any plate of capacitor

$$\sigma = \frac{q}{A}$$

$$F = qE = q\frac{\sigma}{2\varepsilon_0} = \frac{q^2}{2A\varepsilon_0}$$

$$\boxed{F = \frac{q^2}{2A\varepsilon_0}}$$

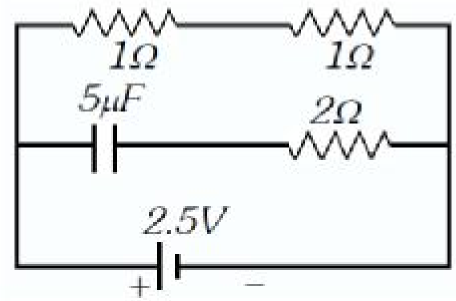
Note: In steady state, current through branch containing the capacitor will become zero.

Example

A capacitor of capacitance 5 μF is connected as shown in the fig. The internal resistance of the cell is 0.5 Ω. The amount of charge on the capacitor plate is :-

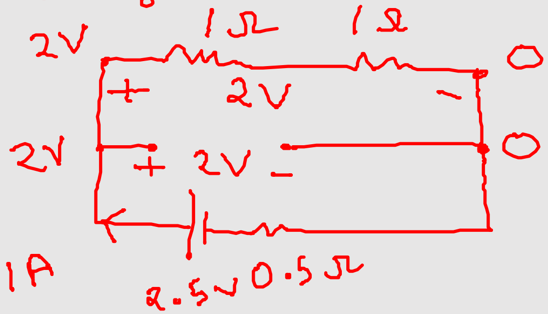
(A) 0 μC (B) 5 μC

~~(C)~~ 10 μC (D) 25 μC



Sol.

Finally



$V = I R_{eq.}$

$Q = CV$

$= 10 \mu C$

Distribution of Charges on Joining two Capacitor

When likely charged plates are connected together

(a) Common potential :

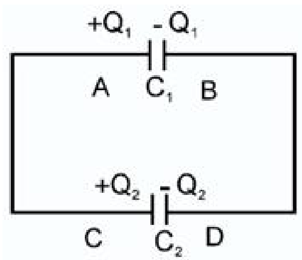
$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1 V = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)$ $\quad Q_2' = C_2 V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$

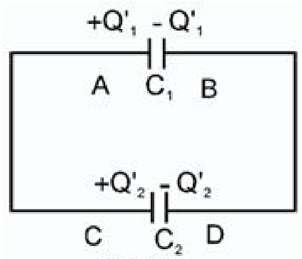
(c) Heat loss during redistribution :

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

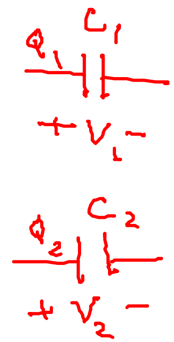
The loss of energy is in the form of Joule heating in the wire.



Initially



Finally



When oppositely charged plates are connected together

$V_2 \rightarrow -V_2$

$$V = \frac{C_1V_1 \quad C_2V_2}{C_1 + C_2}$$

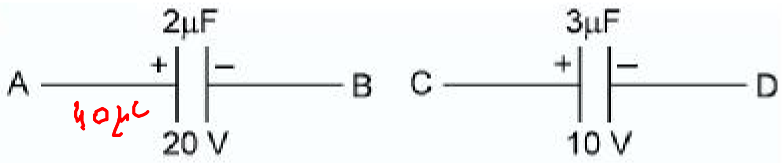
$$H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$$

Example

Find out the following if A is connected with C and B is connected with D.

(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.



Sol.

(i) Common potential

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{40 + 30}{5}$$

$$= 14 \text{ volt}$$

Charge flow = 40 – 28 = 12 μC

(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2 - \frac{1}{2} \times 5 \times (14)^2$

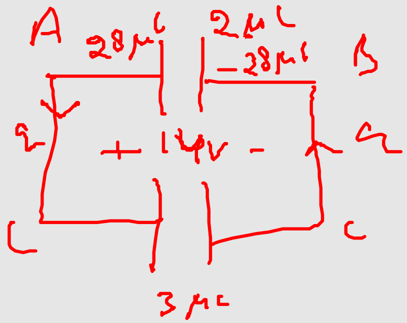
$= 400 + 150 - 490$

$= 550 - 490$

$= 60\ \mu J$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{40\mu + 30\mu}{5\mu}$$

$$= 14V$$



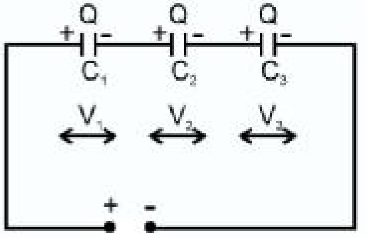
$q = 40 - 28$

$= 12\mu C$

$$U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

Combination of Capacitors

Series Combination :



$Q = C_1V_1 = C_2V_2$
$= C_3V_3$

$V_1 = \frac{Q}{C_1}$

(i) When initially uncharged capacitors are connected as shown in the combination is called series combination.

(ii) All capacitors will have same charge but different potential difference across then.

(iii) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

(iv) Equivalent Capacitance :

Equivalent capacitance of any combination is that capacitance which when connected in place of the combination stores same charge and energy that of the combination.

In series :

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots\ldots}$$

Example

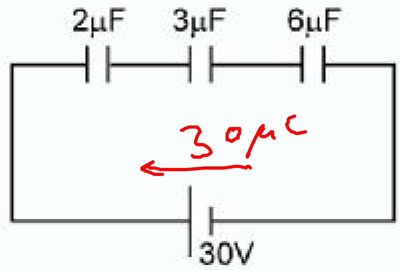
Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following:-

(i) charge flow through the battery,

(ii) potential energy in 3 μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit



Sol.

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$$

$C_{eq} = 1\mu F.$

(i) $Q = C_{eq} V = 30\mu C.$

(ii) charge on $3\mu F$ capacitor $= 30\mu C$

$$\text{energy} = \frac{Q^2}{2C} = \frac{30 \times 30}{2 \times 3} = 150\mu J$$

(iii) $$U_{total} = \frac{30 \times 30}{2}\ \mu J$$

$= 450\ \mu J$

(iv) Heat produced $= (30\ \mu C)(30) - 450\ \mu J$

$= 450\ \mu J.$

$$\frac{1}{C_{eq.}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} \Rightarrow C_{eq} = 1\mu F$$

$Q = C_{eq}.V = 30\mu C$

$$U_{3\mu C} = \frac{Q^2}{2C} = \frac{\cancel{30}^{15} \times \cancel{30}^{10} \times \mu^{\cancel{2}}}{\cancel{2} \times \cancel{3}\ \cancel{\mu}} = 150\mu J$$

$$U_{Total} = \frac{1}{2} C_{eq}.V^2 = \frac{1}{2} \times 1\mu \times 900 = 450\mu J$$

Heat $= 450\mu J$

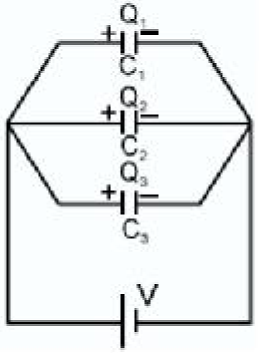
Heat $= U_i + W_{battery} - U_f$

$W_{battery} = Q.\mathcal{E}$

Combination of Capacitors

Parallel Combination :

(i) When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.



(ii) All capacitors have same potential difference but different charges.

(iii) $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

(iv) Equivalent capacitance of parallel combination

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

Example

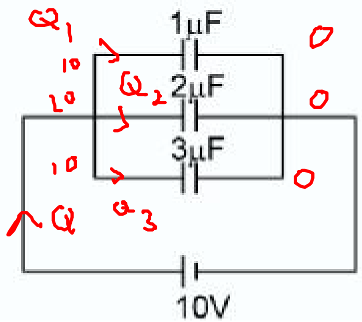
Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following

(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.



Sol.

(i) $Q = (30 + 20 + 10)\mu C$

$= 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced $= 60 \times 10 - 300 = 300\ \mu J$

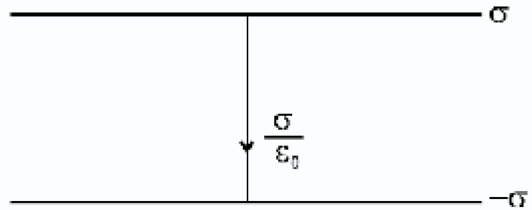
(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

$C_{eq} = 6\mu C$

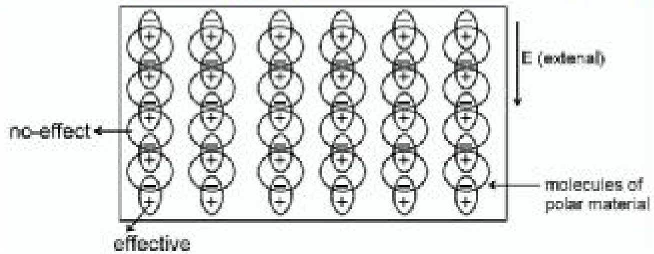
Capacitors with Dielectric

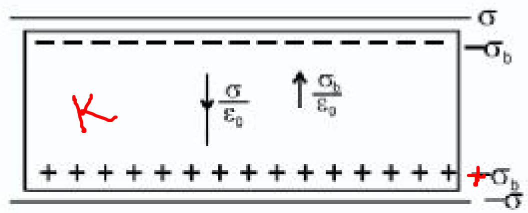
(i) In absence of dielectric

$E = \frac{\sigma}{\varepsilon_0}$



(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.





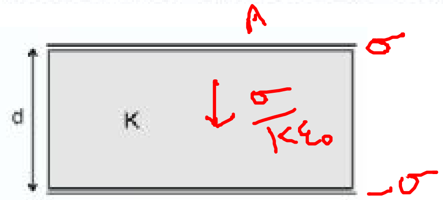
$\sigma_b = \sigma\left(1 - \frac{1}{K}\right)$

$E_{net} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0} = \frac{1}{K}\frac{\sigma}{\varepsilon_0}$

σ_b = induced charge density (called bound charge because it is not due to free electrons).

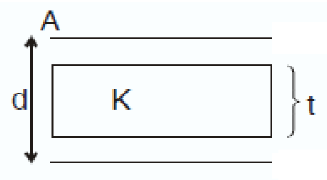
(iii) Capacitance in the presence of dielectric

$$\boxed{C = \frac{AK\varepsilon_0}{d}}$$

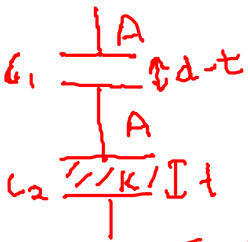


$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{K\varepsilon_0}\cdot d} = \frac{K\varepsilon_0 A}{d}$

Capacitance of common arrangements with Dielectric



$$C = \frac{\varepsilon_0 A}{d - t + t/K}$$



$$C_1 = \frac{\varepsilon_0 A}{d-t}$$

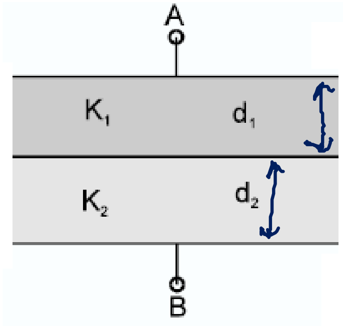
$$C_2 = \frac{K\varepsilon_0 A}{t}$$

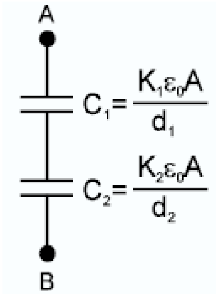
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

If dielectric is metal or conductor ($K = \infty$)

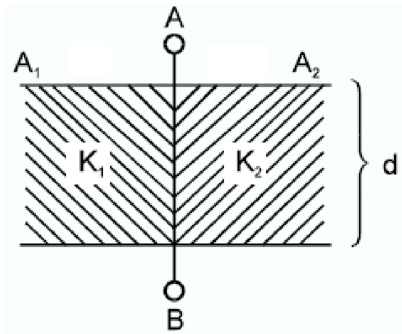
$$C = \frac{\varepsilon_0 A}{d-t}$$

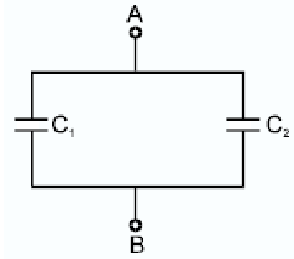
Series





$$\frac{1}{C} = \frac{d_1}{AK_1\varepsilon_0} + \frac{d_2}{AK_2\varepsilon_0} \Rightarrow C = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$



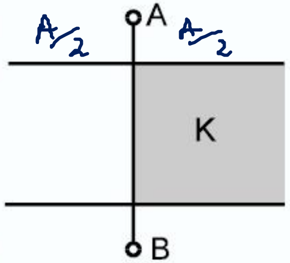


$$C = \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_2\varepsilon_0 A_2}{d}$$

Example

A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-

(A) $\frac{C}{2}$ (B) $\frac{C}{2K}$ (C) $\frac{C}{2}[1 + K]$ (D) $\frac{2[1+K]}{C}$



Sol.

This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

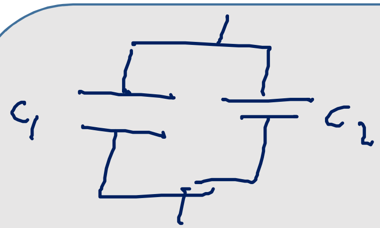
$$C' = C_1 + C_2$$

$$= \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 AK}{2d}$$

$$= \frac{\varepsilon_0 A}{2d}[1 + K]$$

$$= \frac{C}{2}[1 + K]$$

Hence the correct answer will be (C).

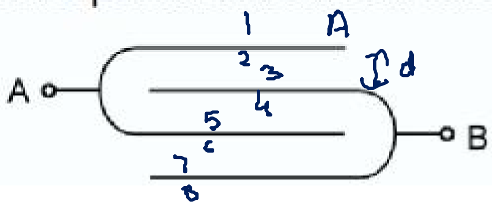


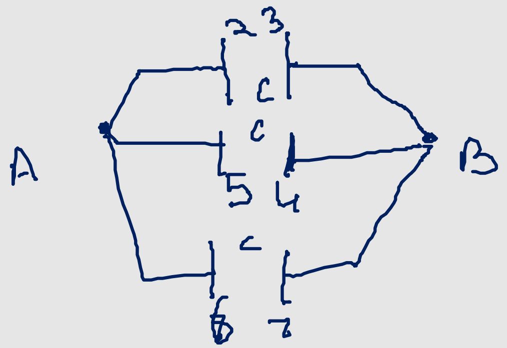
$C_1 = \frac{\varepsilon_0 A/2}{d}$; $C_2 = \frac{K\varepsilon_0 A/2}{d}$

$C = \frac{\varepsilon_0 A}{d}$

Combination of Parallel Plates

Ex. Find out equivalent capacitance between A and B.



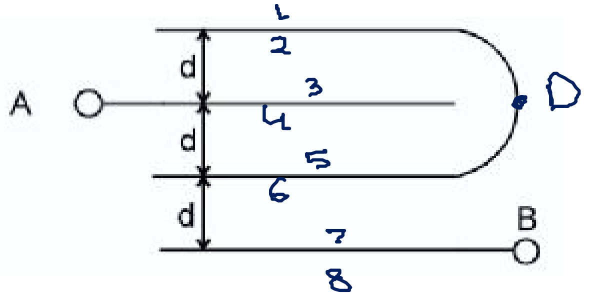


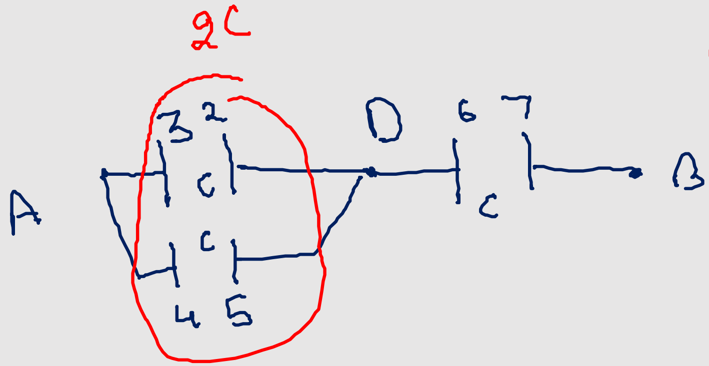
$$C = \frac{\varepsilon_0 A}{d}$$

$$C_{eq} = 3C = \frac{3\varepsilon_0 A}{d}$$

Combination of Parallel Plates

Ex.33 Find out equivalent capacitance between A and B.





$$C_{AB} = \frac{2C \cdot C}{2C+C}$$

$$= \frac{2C}{3}$$

$$= \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

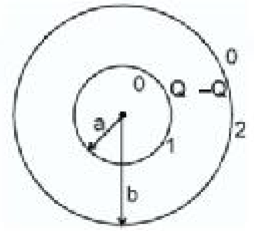
2 in series

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Other Types of Capacitors

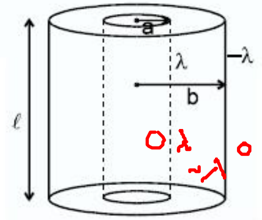
Spherical capacitor :

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$



Cylindrical capacitor

$$\text{Capacitance per unit length} = \frac{2\pi\varepsilon_0}{\ell n \frac{b}{a}}$$



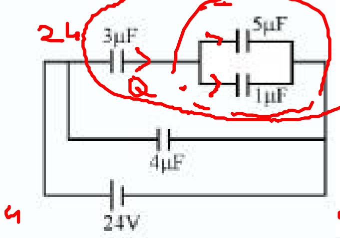
$\boxed{\text{Heat} = U_i + W_{battery} - U_f}$

Example

In the circuit shown, the energy stored in 1μF capacitor is

(A) 40 μJ (B) 64 μJ

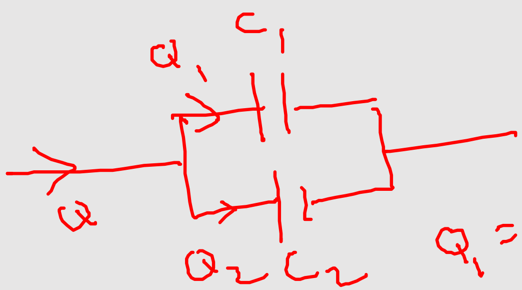
(C) 32 μJ (D) none



Sol.

6 & 3

$\frac{6 \times 3}{6+3} = 2\mu F$



$Q_1 = \frac{C_1}{C_1+C_2} Q$

$Q_2 = \frac{C_2}{C_1+C_2} \cdot Q$

$Q = C_{eq} \cdot V = 2\mu \times 24$

$= 48\mu C$

$q_{1\mu F} = \frac{1\cancel{\mu}}{(1+5)\cancel{\mu}} \cdot 48\mu C$

$= \frac{48}{6}\mu C = 8\mu C$

$U = \frac{q^2}{2C} = \frac{64\mu^{\cancel{2}}}{2 \times 1\cancel{\mu}}$

$= 32\mu J$