PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Radiation (Heat Transfer)

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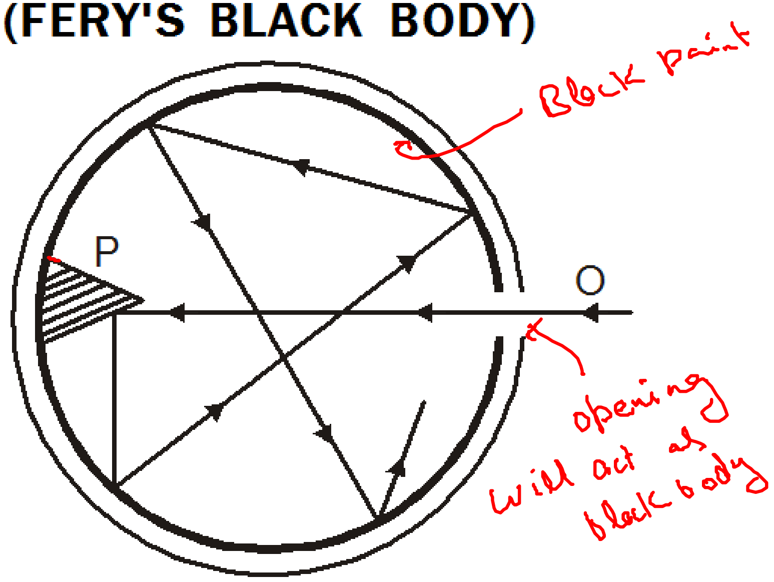
Radiation

The process of the transfer of heat from one place to another place without the requirement of medium is called radiation. The term radiation used here is another word for electromagnetic waves.

Perfectly black body and black body radiation

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.

(FERY'S BLACK BODY)



Absorption, reflection and emission of radiations

$$Q = Q_r + Q_t + Q_a$$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$1 = r + t + a$$

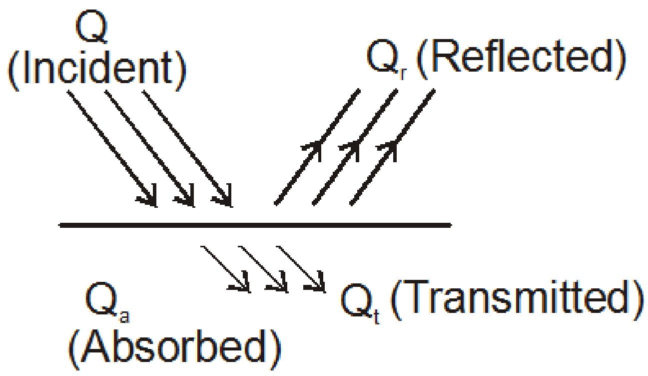
where r = reflecting power , a = absorptive power

and t = transmission power.

(i) $r = 0$, $t = 0$, $a = 1$, perfect black body

(ii) $r = 1$, $t = 0$, $a = 0$, perfect reflector

(iii) $r = 0$, $t = 1$, $a = 0$, perfect transmitter



Absorptive Power, Emissive Power and Emissivity

Absorptive power :

In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As all the radiations incident on a black body are absorbed, a = 1 for a black body.

Emissive power:

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$$E = \frac{Q}{\Delta A \, \Delta t}$$ ← Intensity

(Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

Emissivity:

$$e = \frac{\text{Emissive power of a body at temperature } T}{\text{Emissive power of a black body at same temperature } T} = \frac{E}{E_0}.$$

$$e = \frac{E_{body}}{E_{black\,body}}$$

Kirchoff's Law

For the radiation of a given wavelength at the same temperature

$$\frac{E\,(body)}{E\,(black\ body)} = a\,(body)$$

Hence we can conclude that good emitters are also good absorbers.

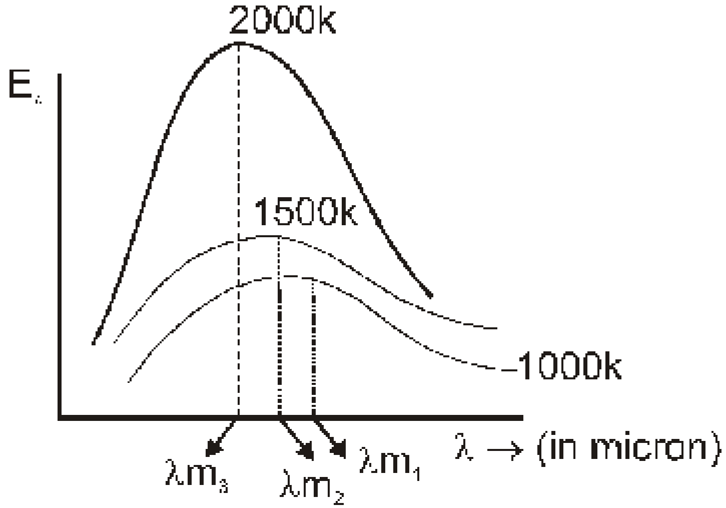
$e = a$

Nature of thermal radiations : (Wien's displacement law)

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

This is called Wien's displacement law.

Here b = 0.282 cm-K, is the Wien’s constant.



Stefan-Boltzmann's law

STEFAN-BOLTZMANN'S LAW

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$e = a \simeq 1$

$$\boxed{P = \sigma A T^4}$$

where σ is Stefan's constant $= 5.67 \times 10^{-8}\ W/m^2K^4$

A body which in not a black body absorbs and hence emits less radiation then

$u_a = a\sigma A T^4$

For such a body, $u = e\sigma A T^4$

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

With the surroundings of temperature T_0, net energy radiated by an area A per unit time..

$$\Delta u = u - u_0 = e\sigma A\,(T^4 - T_0^4)$$

Net power radiated $= \sigma e A T^4 - \sigma e A T_0^4$

$= \sigma e A(T^4 - T_0^4)$

Newton's law of Cooling

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

$\frac{dQ}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

From above expression , $\boxed{\frac{d\theta}{dt} = -k(\theta - \theta_0)}$, where $k = \frac{4e\sigma\theta_0^3}{mc} A$

c ← specific heat

θ ← T body

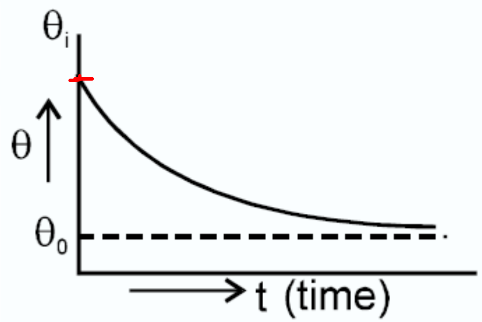
θ_0 ← T surr.

$(\theta_f - \theta_0) = (\theta_i - \theta_0)\, e^{-kt}$

θ_f ← final temp. of body

θ_i ← Initial Temp. of body

$\Rightarrow \quad \theta_f = \theta_0 + (\theta_i - \theta_0)\, e^{-kt}$



$\Delta Q = ms\Delta T$

$\frac{dQ}{dt} = ms\frac{dT}{dt}$

Approximate method for applying Newton's law of cooling

$$\left\langle \frac{d\theta}{dt} \right\rangle = -k(<\theta> - \theta_0)$$

If θ_i & θ_f be initial and final temperature of the body then,

$$<\theta> = \frac{\theta_i + \theta_f}{2}$$

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

$$= -K(\theta_{avg} - \theta_0)$$

$$\frac{\Delta\theta}{\Delta t} = -K\left(\frac{\theta_i + \theta_f}{2} - \theta_0\right)$$

$$\frac{\theta_f - \theta_i}{\Delta t} = -K\left(\frac{\theta_i + \theta_f}{2} - \theta_0\right)$$

Example

A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30°C.

Sol.

for the interval in which temperature falls from 40 to 35°C

$$<\theta> = \frac{40+35}{2} = 37.5^\circ C$$

$$\Rightarrow \quad \frac{(35^\circ C - 40^\circ C)}{10(\text{min})} = -K(37.5^\circ C - 20^\circ C)$$

$$\Rightarrow \quad K = \frac{1}{35}(\text{min}^{-1})$$

for the interval in which temperature falls from 35°C to 30°C

$$<\theta> = \frac{35+30}{2} = 32.5^\circ C$$

$$\Rightarrow \quad \frac{(30^\circ C - 35^\circ C)}{t} = -(32.5^\circ C - 20^\circ C)$$

$$\Rightarrow \quad \text{required time,} \quad t = \frac{5}{12.5} \times 35\,\text{min} = 14\text{ min}$$

$$\frac{\theta_f - \theta_i}{\Delta t} = -K(\theta_{avg} - \theta_0)$$

$40 \to 35$

$$\frac{35-40}{10} = -K(37.5-20) \Rightarrow -\frac{1}{2} = -K\left(\frac{35}{2}\right)$$

$$K = \frac{1}{35}$$

$35 \to 30$

$$\frac{30-35}{t} = -\frac{1}{35}(32.5-20) \Rightarrow t = 14\text{ min}$$

$$-\frac{\cancel{5}}{t} = -\frac{1}{\cancel{35}_{7}}\left(\frac{\cancel{25}}{2}\right)$$