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PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

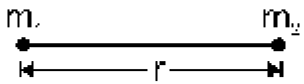
GRAVITATION

By,
Ritesh Agarwal, B. Tech. IIT Bombay

Universal Law of Gravitation

Force of attraction between two point masses:

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{r^2}$$



where $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2} is the universal gravitational constant.

Dimensional formula of G :

$$F = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1} L^3 T^{-2}]$$

Example

The centres of two identical spheres are at a distance 1.0 m apart. If the gravitational force between them is 1.0 N, then find the mass of each sphere. ($G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-1}$)

Sol.

Gravitational force; $F = \dfrac{Gm.m}{r^2}$

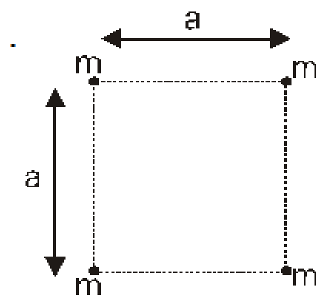
on substituting F = 1.0 N , r = 1.0 m

and $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-1}$

we get $m = 1.225 \times 10^5$ kg

Example

Four point masses each of mass 'm' are placed on the corner of square of side 'a' . Calculate magnitude of gravitational force experienced by each particle.



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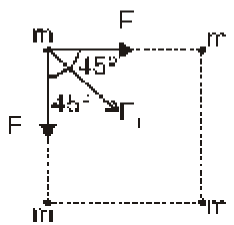


F_r = resultant force on each particle = $2F\cos 45^\circ + F_1$

$$= \frac{2Gm^2}{a^2}\cdot\frac{1}{\sqrt{2}} + \frac{Gm^2}{(\sqrt{2}a)^2} = \frac{Gm^2}{2a^2}\,(2\sqrt{2}+1)$$

Gravitational Field

The intensity of gravitational field at a points is defined as the force experienced by a unit mass placed at that point.

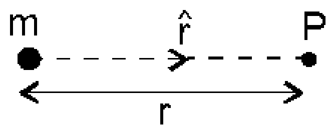
$$\vec{E} = \frac{\vec{F}}{m}$$

SI Unit : N kg^{-1}.

Dimensional formula = $\frac{F}{m} = \frac{[MLT^{-2}]}{[M]} = [M^0 LT^{-2}]$

Intensity of gravitational field due to point mass :

The force due to mass m on test mass m_0 placed at point P is given by :



$$F = \frac{GMm_0}{r^2}$$

Hence $E = \frac{F}{m_0} \Rightarrow E = \frac{GM}{r^2}$

In vector form $\vec{E} = -\frac{GM}{r^2}\hat{r}$

Example

Find the relation between the gravitational field on the surface of two planets A & B of masses m_A, m_B & radius R_A & R_B respectively if

(i) they have equal mass

(ii) they have equal (uniform) density

Sol.

Let E_A & E_B be the gravitational field intensities on the surface of planets A & B.

then, $$E_A = \frac{Gm_A}{R_A^2} = \frac{G\frac{4}{3}\pi R_A^3 \rho_A}{R_A^2} = \frac{4G\pi}{3}\rho_A R_A$$

Similarly, $$E_B \quad \frac{Gm_B}{R_B^2} = \frac{4G}{3}\pi \rho_B R_B$$

(i) for $m_A = m_B$ $$\frac{E_A}{E_B} = \frac{R_B^2}{R_A^2}$$

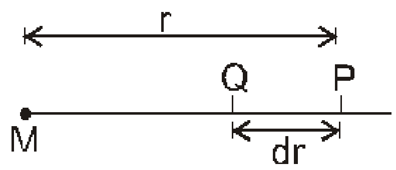
(ii) For & $\rho_A = \rho_B$ $$\frac{E_A}{E_B} \quad \frac{R_A}{R_B}$$

Gravitational Potential

The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done by an external agent in bringing a body of unit mass from infinity to that point, slowly (no change in kinetic energy).

Gravitational potential due to a point mass :

Let the unit mass be displaced through a distance dr towards mass M, then work done is given by



$$dW = F\,dr = \frac{GM}{r^2}dr$$

Total work done in displacing the particle from infinity to point P is

$$W = \int dW = \int_{\infty}^{r} \frac{GM}{r^2}dr = \frac{-GM}{r}.$$

Thus gravitational potential, $\boxed{V = -\frac{GM}{r}}$.

SI Unit : J kg^{-1}.

Dimensional Formula = $\frac{\text{Work}}{\text{mass}} = \frac{[ML^2T^{-2}]}{[M]} = [M^{\circ}L^2T^{-2}]$.

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Relation Between Gravitational Field and Potential

$$dV = -E\,dr \quad \Rightarrow \quad E = -\frac{dV}{dr}$$

Some Important Formulas

Uniform Solid Sphere (Radius = a)

(a) Point P inside the shell. $r \le a$, then

$$V = -\frac{GM}{2a^3}(3a^2 - r^2) \;\&\; E = -\frac{GMr}{a^3}, \text{ and at the centre } V = -\frac{3GM}{2a} \text{ and } E = 0$$

(b) Point P outside the shell. $r \ge a$, then $V = -\dfrac{GM}{r}$ & $E = -\dfrac{GM}{r^2}$

Uniform Thin Spherical Shell

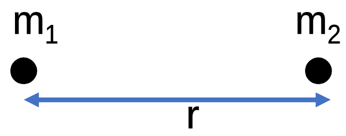
(a) Point P Inside the shell. $r \le a$, then $V = \dfrac{-GM}{a}$ & $E = 0$

(b) Point P outside shell. $r \ge a$, then $V = \dfrac{-GM}{r}$ & $E = -\dfrac{GM}{r^2}$

Gravitational Potential Energy

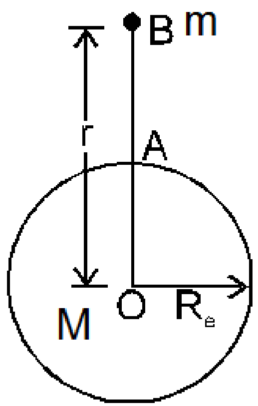
For 2 point masses separated by distance ‘r’.

$$U = -\frac{Gm_2m_2}{r}$$



Potential energy of any particle in gravitational field of earth/planet

Gravitational potential energy, $\boxed{U = -\frac{GMm}{r}}$



Acceleration due to Gravity

It is the acceleration, a freely falling body near the earth s surface acquires due to the earths gravitational pull.

The gravitational field intensity on the surface of earth is therefore numerically equal to the acceleration due to gravity (g), there.

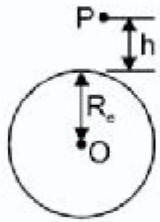
$$g = \frac{GM_e}{R_e^2}$$

where , M_e = Mass of earth
R_e = Radius of earth

Variation of Acceleration due to Gravity

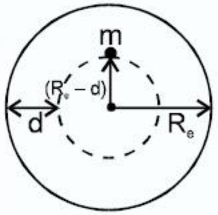
(a) Effect of Altitude

$$g_h = \frac{GM_e}{(R_e + h)^2} = g\left(1 + \frac{h}{R_e}\right)^{-2} \simeq g\left(1 - \frac{2h}{R_e}\right) \text{ when } h << R.$$



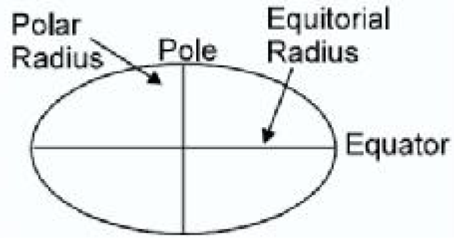
(b) Effect of depth

$$g_d = g\left(1 - \frac{d}{R_e}\right)$$



(c) Effect of the surface of Earth

$$g_{pole} > g_{equator}$$



Escape Speed

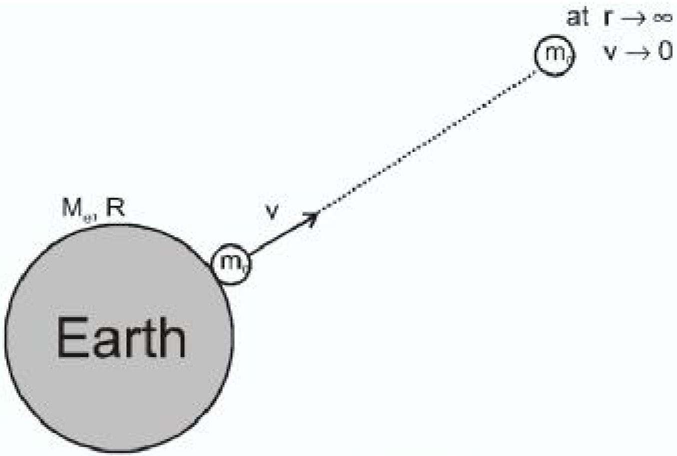
The minimum speed required to send a body out of the gravity field of a planet (send it to $r \to \infty$)

Escape speed at earth's surface :

$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv^2 + m_0\left(-\frac{GM_e}{R}\right) = 0 + m_0\left(-\frac{GM_e}{\infty}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM_e}{R}} = \sqrt{2gR}$$



Escape speed depends on :

(i) Mass (M_e) and size (R) of the planet

(ii) Position from where the particle is projected.

Escape speed does not depend on :

(i) Mass of the body which is projected (m_0)

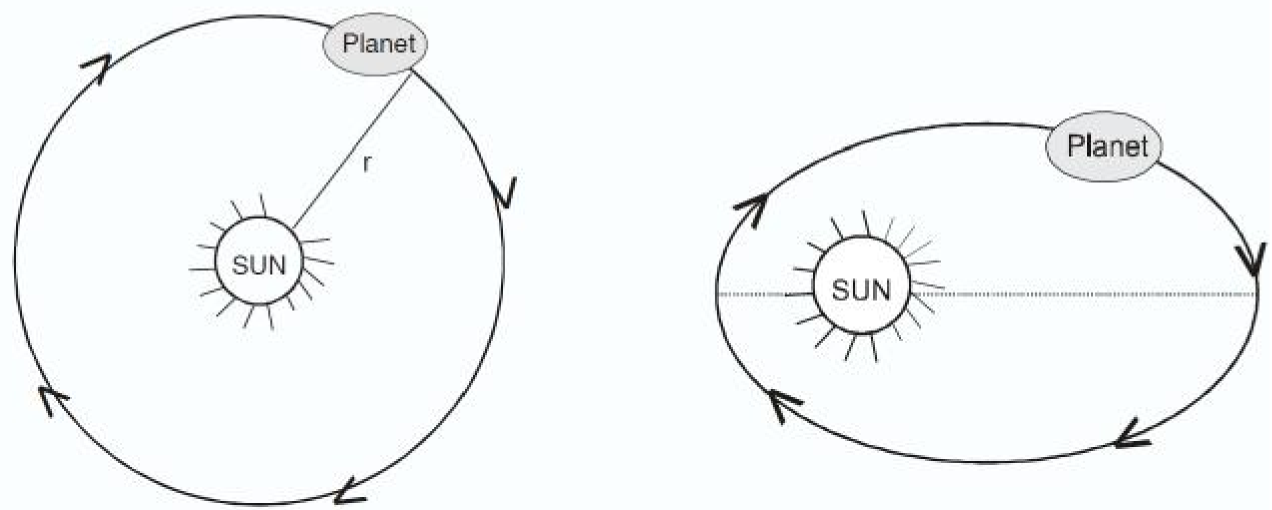
(ii) Angle of projection.

Kelper's Law for Planetary Motion

Suppose a planet is revolving around the sun, or a satellite is revolving around the earth, then the planetary motion can be studied with help of Kepler's three laws.

Kepler's Law of orbit

Each planet moves around the sun in a circular path or elliptical path with the sun at its focus. (Infact circular path is a subset of elliptical path)



Kelper's Laws for Planetary Motion

Kepler's law of time period :

$$\frac{m_0 v^2}{r} = \frac{GM_s m_0}{r^2}$$

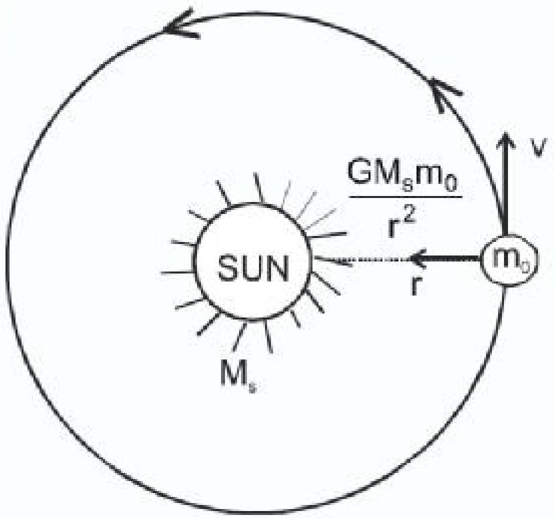
$$v = \sqrt{\frac{GM_s}{r}}$$

Time period of revolution is

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM_s}}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 \qquad \Rightarrow T^2 \alpha\, r^3$$

For all the planet of a sun , $T^2 \propto r^3$



If planets are moving in elliptical orbit, then $T^2 \propto a^3$ where a = semimajor axis of the elliptical path.

Circular Motion of a Satellite around a Planet

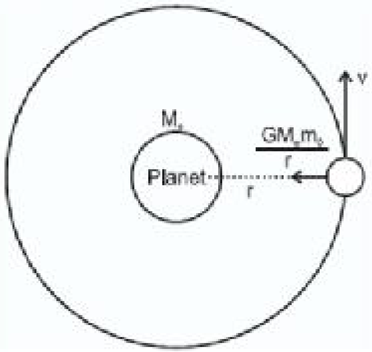
Orbital velocity (v_0)

Suppose a satellite of mass m_0 is at a distance r from a planet. If the satellite does not revolve, then due to the gravitational attraction, it may collide to the planet.
To avoid the collision, the satellite revolve around the planet, for circular motion of satellite.

$\Rightarrow \quad \frac{GM_e m_0}{r^2} = \frac{m_0 v^2}{r}$

$\Rightarrow \quad v = \sqrt{\frac{GM_e}{r}}$ this velocity is called orbital velocity (v_0)

$$v_0 = \sqrt{\frac{GM_e}{r}}$$



Time Period of a Satellite:-

$$T = \frac{2\pi r}{v_0} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi r^{3/2}}{R\sqrt{g}} \Rightarrow \left(T^2 = \frac{4\pi^2}{GM} r^3\right) \Rightarrow T^2 \propto r^3 \quad (\text{here } r = R + h)$$

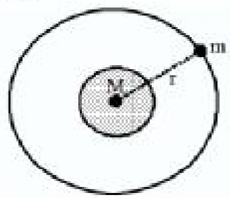
For Geostationary Satellite:-

$T = 24$ hrs. $\quad H = 36{,}000$ km $\simeq 6\,R_e$ $(r \simeq 7R_e)$ $\quad v_0 = 3.1$ km/sec.

Energy of a Satellite

Total M.E. of a Satellite:-

$$K.E. = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$$



$$P.E. = \frac{-GMm}{r}$$

$$T.E. = P.E. + K.E. = \frac{-GMm}{2r}$$

$$T.E. = -K.E. = P.E./2$$

Binding energy of satellite (system):-

B.E. = − T.E.

$$B.E. = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$$

Hence $B.E. = K.E. = -T.E. = \frac{-P.E.}{2}$

Work done in Changing the Orbit of Satellite:-

W = Change in M.E. of system

$$E = \frac{-GMm}{2r}$$

$$W = E_{II} - E_{I}$$

$$W = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$