Problem Solving on Straight Lines & Circles

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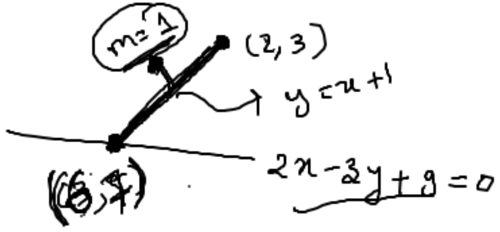
The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$, is -

(A) $\sqrt{2}$ $\quad\quad$ $\sqrt{16+16}$ $\quad\quad$ (B) $4\sqrt{2}$

(C) $\sqrt{8}$ $\quad\quad$ (D) $3\sqrt{2}$

Sol^n



$y - 3 = 1(x - 2)$

$y - 3 = x - 2$

$\boxed{y = x + 1}$

$2x - 3(x + 1) + 9 = 0$

$-x + 6 = 0$

$\boxed{x = 6}$

$\sqrt{1+4}$

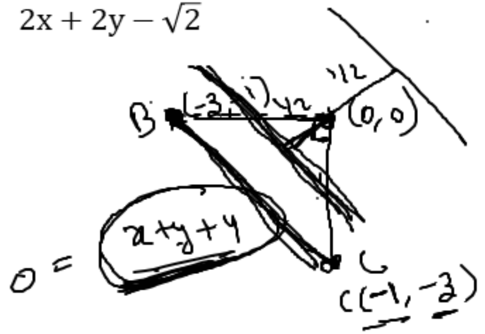
The vertices of ΔOBC are respectively $(0, 0)$, $(-3, -1)$ and $(-1, -3)$. The equation of line parallel to BC and at a distance 1/2 from O which intersects OB and OC is -

(A) $2x + 2y + \sqrt{2} = 0$ (B) $2x - 2y + \sqrt{2} = 0$

(C) $2x + 2y - \sqrt{2}$ (D) None of these

$x + y - 1/\sqrt{2} = 0$ — (b)

Solⁿ



$$\text{slope of } BC = \frac{(-1)-(-3)}{(-3)-(-1)} = \frac{2}{-2} = -1$$

$$\boxed{y = -x + K}$$

$$\boxed{x + y = K}$$

$$\left|\frac{K}{\sqrt{2}}\right| = \frac{1}{2}$$

$$x + y = \frac{1}{\sqrt{2}} \quad \text{— (1)}$$

$$x + y = -\frac{1}{\sqrt{2}} \quad \text{— (2)}$$

$$x + y + \frac{1}{\sqrt{2}} = 0 \quad \text{— (2)}$$

$$K = \frac{1}{\sqrt{2}}, \ -\frac{1}{\sqrt{2}}$$

$$2x + 2y + \sqrt{2} = 0$$

The equation of two straight lines through (7, 9) and making an angle of 60° with the line $x - \sqrt{3}y - 2\sqrt{3} = 0$ is -

(A) $x = 7, x + \sqrt{3}y = 7 + 9\sqrt{3}$ (B) $x = \sqrt{3}, x + \sqrt{3}y = 7 + 9\sqrt{3}$

(C) $x = 7, x - \sqrt{3}y = 7 + 9\sqrt{3}$ (D) $x = \sqrt{3}, x - \sqrt{3}y = 7 + 9\sqrt{3}$

$y - 9 = \frac{-1}{\sqrt{3}}(x - 7)$

$\sqrt{3}y - 9\sqrt{3} = -x + 7$

$\sqrt{3}y + x = 7 + 9\sqrt{3}$

$\tan 60^\circ = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\sqrt{3} = \left|\frac{\frac{1}{\sqrt{3}} - m}{1 + \frac{1}{\sqrt{3}}m}\right|$

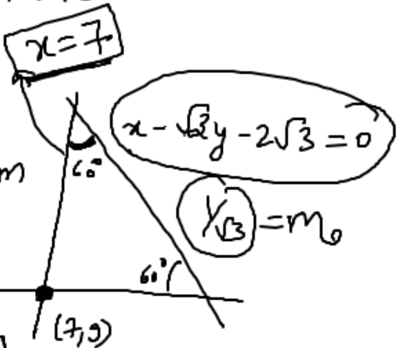
$\sqrt{3} = \left|\frac{1 - \sqrt{3}m}{\sqrt{3} + m}\right| \Longrightarrow \sqrt{3} = -\left(\frac{1 - \sqrt{3}m}{\sqrt{3} + m}\right) \Rightarrow 3 + \sqrt{3}m = \sqrt{3}m - 1$

$m \to \infty$

$\sqrt{3} = \frac{1 - \sqrt{3}m}{\sqrt{3} + m} \Rightarrow 3 + \sqrt{3}m = 1 - \sqrt{3}m$

$\Rightarrow 2 = -2\sqrt{3}m$

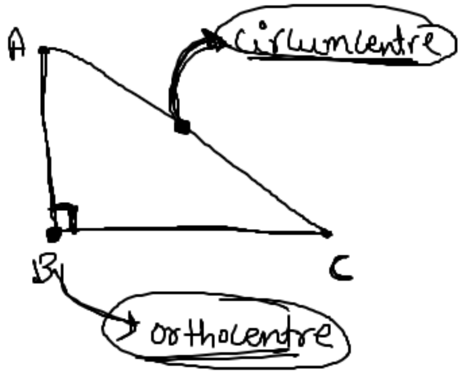
$\boxed{m = -1/\sqrt{3}}$



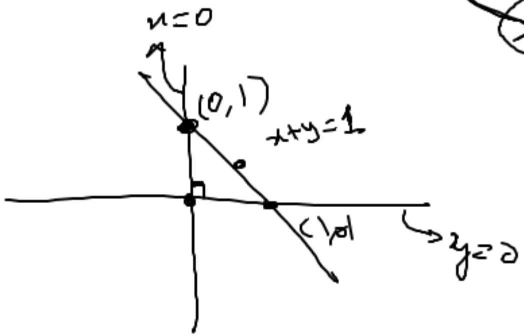
The orthocentre of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(C) $(0,0)$ (D) $\left(\frac{1}{4}, \frac{1}{4}\right)$



Circumcentre = $\left(\frac{1}{2}, \frac{1}{2}\right)$



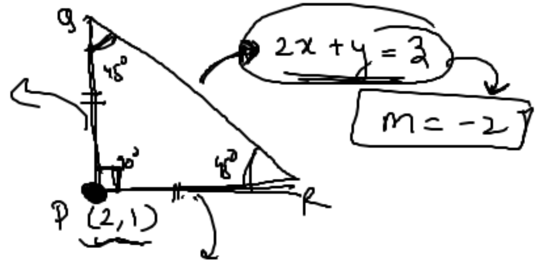
Let PQR be a right angled isosceles triangle, right angled at P (2, 1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is -

(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$

(B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$

(D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$



$$\tan 45^\circ = \left|\frac{m+2}{1-2m}\right|$$

$$1 = \left|\frac{m+2}{1-2m}\right|$$

$2m-1 = m+2$ $\quad$ $1-2m = m+2$

$\boxed{m = 3}$ $\quad$ $\boxed{-\frac{1}{3} = m}$

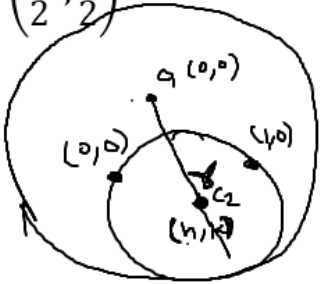
The centre of circle passing through (0 , 0) and (1 , 0) and touching the circle $x^2 + y^2 = 9$ is

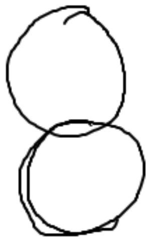
(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(B) $\left(\frac{1}{2}, -\sqrt{2}\right)$

(C) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{1}{2}, \frac{3}{2}\right)$





$$\boxed{C_1C_2 = |r_1 - r_2|}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-h)^2 + (y-k)^2 = h^2 + k^2$$

$$(1-h)^2 + \cancel{k^2} = h^2 + \cancel{k^2}$$

$$\cancel{h^2} + 1 - 2h = \cancel{h^2} \Rightarrow \boxed{h = 1/2}$$

$$\sqrt{h^2+k^2} = |3 - \sqrt{h^2+k^2}|$$

$$2\sqrt{h^2+k^2} = 3$$

$$h^2+k^2 = \frac{9}{4}$$

$$\frac{1}{4} + k^2 = \frac{9}{4}$$

$$\boxed{k^2 = 2}$$

$$k = \pm\sqrt{2}$$

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$. A possible equation of L is

(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$

(C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

$y = \frac{x}{\sqrt{3}} - \frac{5}{\sqrt{3}}$

$\boxed{\sqrt{3}y = x - 5}$

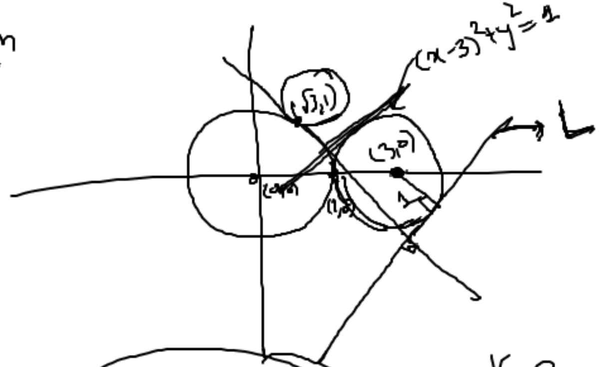
Sol^n

$\boxed{T=0}$

$\boxed{\sqrt{3}x + y = 4} \longrightarrow PT$

$-\sqrt{3}$

$\frac{1}{\sqrt{3}}$



$\boxed{y = \frac{1}{\sqrt{3}}x + c} \Rightarrow \sqrt{3}y = x - 1$

$\boxed{x - \sqrt{3}y = 1}$

$\left|\frac{\sqrt{3}+c}{\sqrt{1+\frac{1}{3}}}\right| = 1$

$\left|\frac{(\sqrt{3}+c)\sqrt{3}}{2}\right| = 1$

$3 + \sqrt{3}c = 2$

$\sqrt{3}c = -1$

$\boxed{c = -1/\sqrt{3}}$

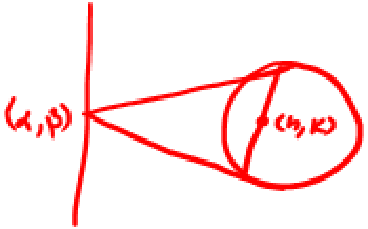
$3 + \sqrt{3}c = -2$

$\boxed{c = -5/\sqrt{3}}$

The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$



$4x-5y=20$

$4\alpha = 5\beta+20$

$\alpha = \frac{5}{4}\beta+5$

comparing coeffi

$\frac{h}{\frac{5\beta}{4}+5} = \frac{k}{\beta} = \frac{h^2+k^2}{9}$

$\frac{9h}{h^2+k^2} = 5+\frac{5\beta}{4}$ | $\beta = \frac{9k}{h^2+k^2}$

$\boxed{T = S_1}$ Chord whose mid-point is given

$hx + ky - \cancel{9} = h^2+k^2 - \cancel{9}$

$\boxed{hx+ky = h^2+k^2}$ —(1)

chord of contact $T=0$

$\alpha x + \beta y = 9$

$\boxed{\left(\frac{5}{4}\beta+5\right)x+\beta y = 9}$ —(2)

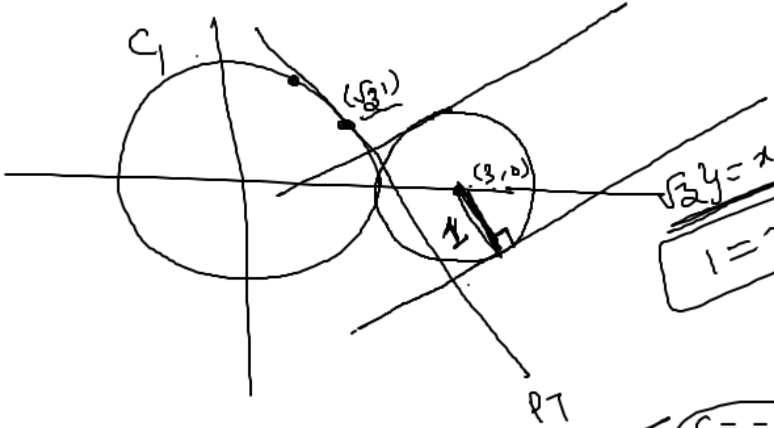
$\frac{9h}{h^2+k^2} = 5+\frac{45k}{4(h^2+k^2)} \Rightarrow \frac{9h}{\cancel{(h^2+k^2)}} = \frac{20(h^2+k^2)+45k}{4\cancel{(h^2+k^2)}}$

$20(h^2+k^2)+45k-36h=0$

A tangent PT is d straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$. A possible equation of L is

(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$

(C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$



$xx_1 + yy_1 = c^2$ $(r=0)$

PT

$\boxed{\sqrt{3}x + y = 4}$

$m = -\sqrt{3}$

L ⊥ $m_1 = \frac{1}{\sqrt{3}}$

$\sqrt{3}y = x - 1$

$\boxed{1 = x - \sqrt{3}y}$

L=0 $\boxed{y = \frac{1}{\sqrt{3}}x + c}$

$$\left|\frac{\sqrt{3}+c}{\sqrt{\frac{1}{3}+1}}\right| = 1$$

$$\left|\frac{3+\sqrt{3}c}{2}\right| = 1$$

$c = -1/\sqrt{3}$

$c = -5/\sqrt{3}$