Ellipse & Hyperbola

Vishal Garg (B.Tech, IIT Bombay)

$\frac{SP}{PM} = e$

Ellipse $(e<1)$

Standard equation of an ellipse referred to its principal axes along the co-ordinate axes is $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$.

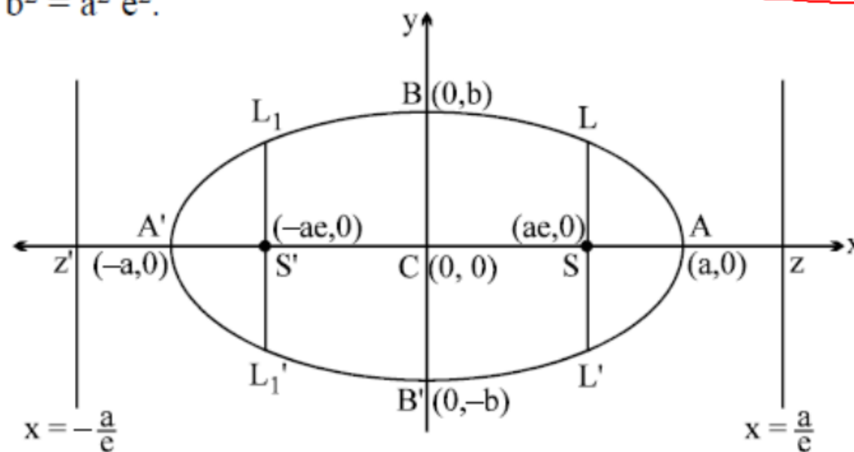
Where a > b & $\boxed{b^2 = a^2(1 - e^2)}$ $\Rightarrow$ $a^2 - b^2 = a^2 e^2$.

Where e = eccentricity ($0 < e < 1$).

FOCI : $S \equiv (a\,e,\ 0)$ & $S' \equiv (-a\,e,\ 0)$.

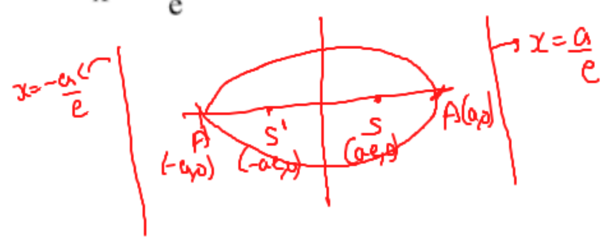
EQUATIONS OF DIRECTRICES :

$x = \frac{a}{e}$ & $x = -\frac{a}{e}$.



VERTICES :

$A' \equiv (-a, 0)$ & $A \equiv (a, 0)$.



Ellipse

~~Standard equation~~ of an ellipse referred to its principal axes along the co-ordinate axes is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Where $a > b$ & $b^2 = a^2(1 - e^2) \Rightarrow a^2 - b^2 = a^2 e^2$.

Where e = eccentricity $(0 < e < 1)$.

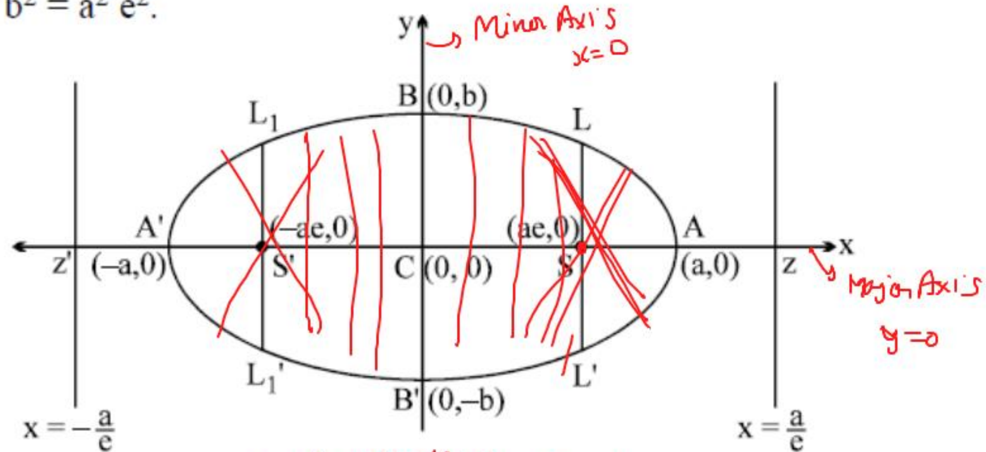
FOCI : $S \equiv (a e, 0)$ & $S' \equiv (-a e, 0)$.

~~EQUATIONS OF~~ DIRECTRICES :

$x = \frac{a}{e}$ & $x = -\frac{a}{e}$.

VERTICES :

$A' \equiv (-a, 0)$ & $A \equiv (a, 0)$.



Double ordinate] + (LR)
Focal Chord
Latus Rectum

FOCAL CHORD : A chord which passes through a focus is called a **focal chord**.

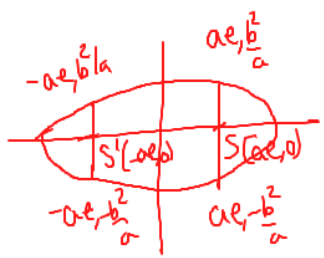
DOUBLE ORDINATE :

A chord perpendicular to the major axis is called a **double ordinate.**

LATUS RECTUM :

The focal chord perpendicular to the major axis is called the **latus rectum**. Length of latus rectum

$$\boxed{(\mathbf{LL'}) = \frac{2b^2}{a}} = \frac{(minor\ \ axis)^2}{major\ \ axis} = 2a(1-e^2) = 2e\ (\text{distance from focus to the corresponding directrix})$$



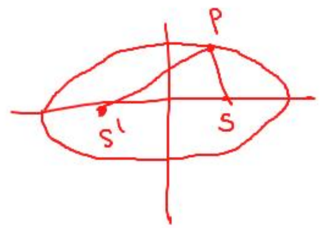
$\pm ae, \pm \frac{b^2}{a}.$

2nd definition of Ellipse

NOTE :

(i) The sum of the focal distances of any point on the ellipse is equal to the major Axis. Hence distance of focus from the extremity of a minor axis is equal to semi major axis. **i.e. BS = CA.**

(ii) If the equation of the ellipse is given as $\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$ & nothing is mentioned then the rule is to assume that $a > b$.



$\boxed{PS + PS' = 2a}$

Horizonal Ellipse $a > b$

$\frac{x^2}{9}+\frac{y^2}{4}=1$

Vertical Ellipse $b > a$

$\frac{x^2}{4}+\frac{y^2}{9}=1$

Problems

The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at (0, 3) is : [JEE (Main)]

(1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$ (3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

$\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ $\quad S(\pm ae, 0) \equiv S(\pm\sqrt{7}, 0)$ $\quad C(0,3)$

$a=4,\ b=3$

$b^2 = a^2(1-e^2)$

$1-e^2 = \frac{b^2}{a^2}$

$e^2 = 1-\frac{b^2}{a^2} \Rightarrow e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\sqrt{7+9} = 4$

$(x-0)^2+(y-3)^2 = 16$

$x^2+y^2-6y = 7$

$x^2+y^2-6y-7=0$

Problems

If LR of an ellipse is half of its minor axis, then its eccentricity is -

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{3}$

$\frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2}$

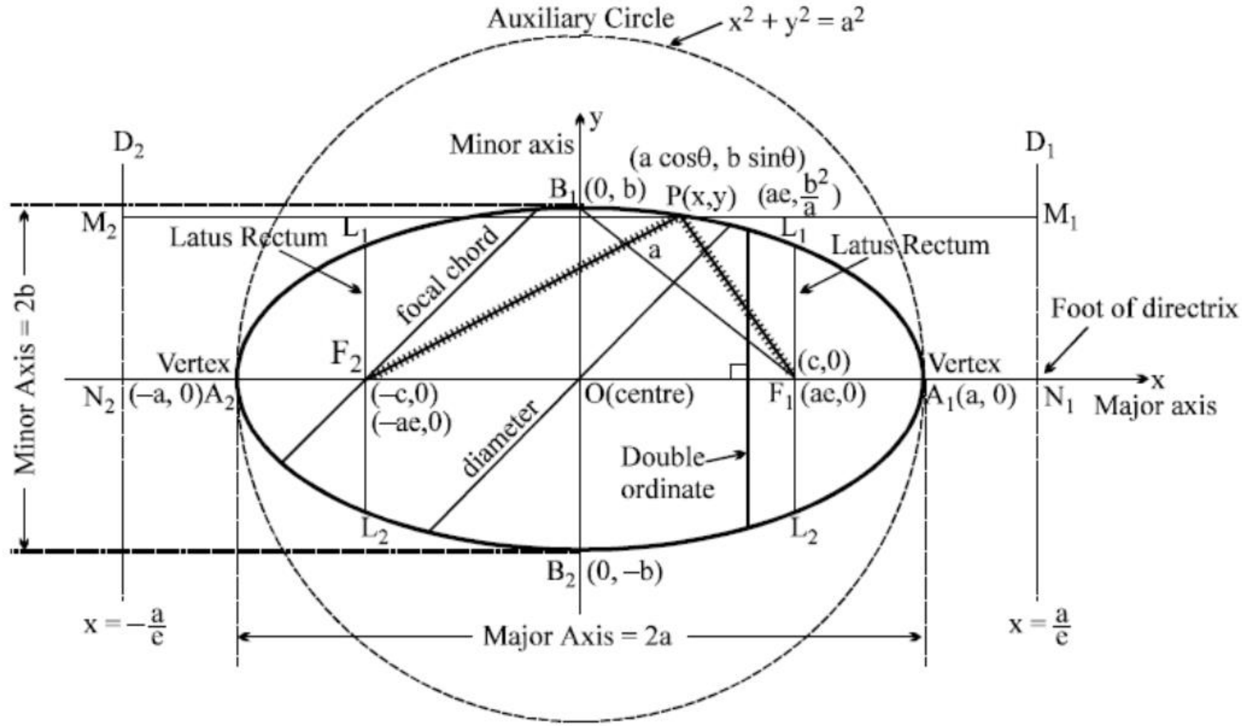
$b^2 = a^2(1-e^2)$

$1-e^2 = \frac{b^2}{a^2}$

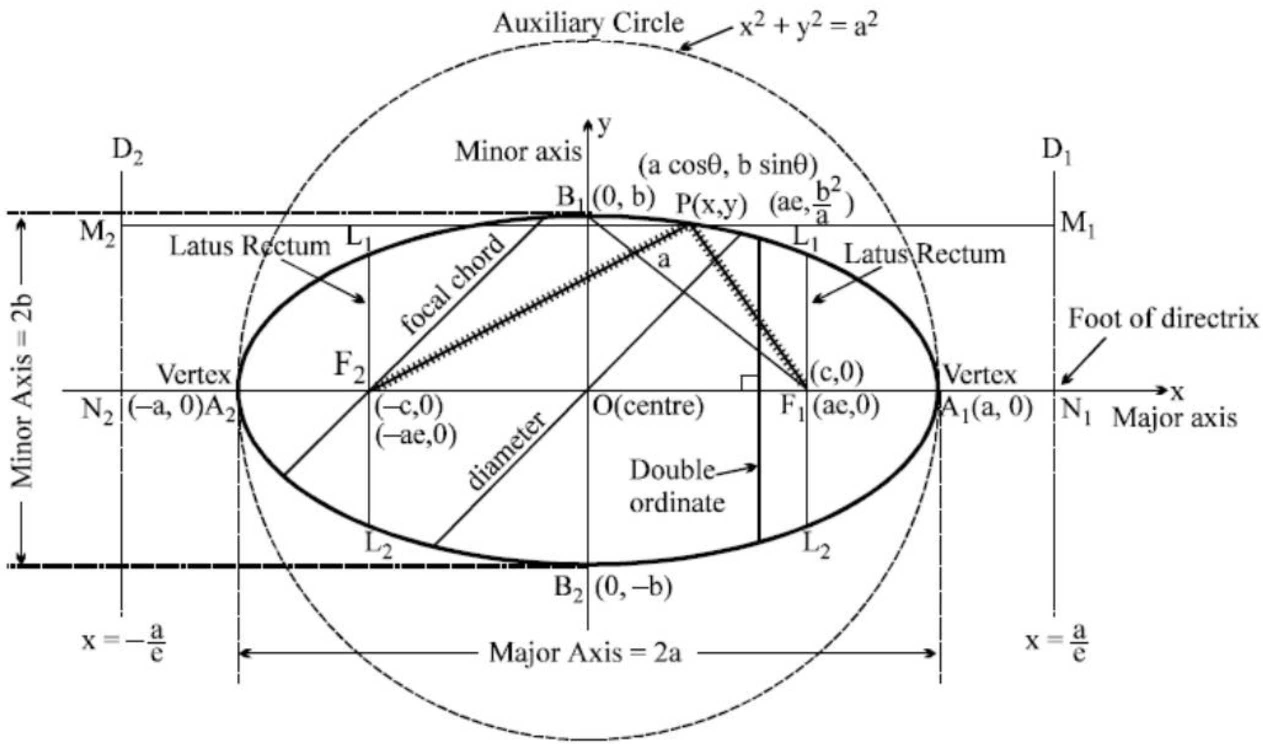
$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{1}{4} = \frac{3}{4}$

$e = \sqrt{3}/2$

ELLIPSE AT A GLANCE

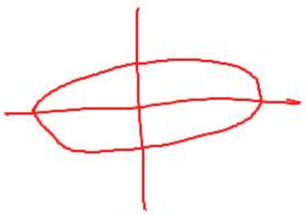


ELLIPSE AT A GLANCE



POSITION OF A POINT w.r.t. AN ELLIPSE :

The point $P(x_1, y_1)$ lies outside, inside or on the ellipse according as ; $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 > < \text{ or } = 0$.



$S_1 > 0$ (outside)

$S_1 = 0$ (on)

$S_1 < 0$ (inside)

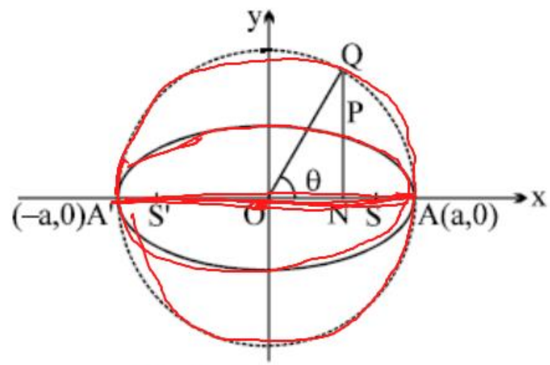
$S : \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

$\boxed{S_1 : \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1}$

AUXILIARY CIRCLE / ECCENTRIC ANGLE :

A circle described on major axis as diameter is called the **auxiliary circle**.

Let Q be a point on the auxiliary circle $x^2 + y^2 = a^2$ such that QP produced is perpendicular to the x-axis then P & Q are called as the **CORRESPONDING POINTS** on the ellipse & the auxiliary circle respectively 'θ' is called the **ECCENTRIC ANGLE** of the point P on the ellipse $(0 \le \theta < 2\pi)$.



$x^2+y^2=a^2$

Parametric Coordinates

$x^2+y^2=r^2$

$x=r\cos\theta,\ y=r\sin\theta$

$y^2=4ax$

$x=at^2,\ y=2at$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$x=a\cos\theta,\ y=b\sin\theta$

Eccentric Angle.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$x=a\sec\theta,\ y=b\tan\theta$

eccentric angle

PARAMETRIC REPRESENTATION :

The equations $x = a\cos\theta$ & $y = b\sin\theta$ together represent the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

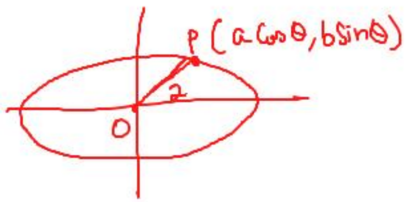
Where θ is a parameter. Note that if $P(\theta) \equiv (a\cos\theta, b\sin\theta)$ is on the ellipse then ; $Q(\theta) \equiv (a\cos\theta, a\sin\theta)$ is on the auxiliary circle.

Problems

If the distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2, then the eccentric angle is-

(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

$P(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$



$OP = 2$

$\sqrt{(\sqrt{6}\cos\theta)^2+(\sqrt{2}\sin\theta)^2} = 2$

$6\cos^2\theta + 2\sin^2\theta = 4$

$2+4\cos^2\theta = 4$

$4\cos^2\theta = 2$

$\cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$a > cm$ $a = cm$ $a < cm$

LINE AND AN ELLIPSE :

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two points real, coincident or imaginary according as c^2 is $<$ = or $>$ $a^2m^2 + b^2$.

Hence $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $\boxed{c^2 = a^2m^2 + b^2.}$

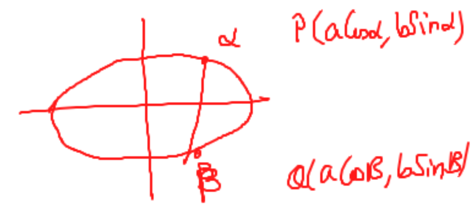
The equation to the chord of the ellipse joining two points with eccentric angles α & β is given by

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}.$$

**

$$\boxed{y = mx \pm \sqrt{a^2m^2+b^2}}$$

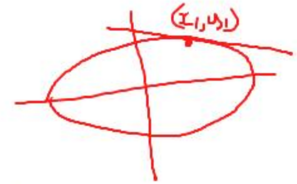
Eqn of tangent in slope form.



Two point form

TANGENTS :

(i) $\dfrac{xx_1}{a^2}+\dfrac{yy_1}{b^2}=1$ is tangent to the ellipse at (x_1, y_1).



$T=0$

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1 \Rightarrow \dfrac{xx_1}{a^2}+\dfrac{yy_1}{b^2}=1$

(ii) $\boxed{y=mx\pm\sqrt{a^2m^2+b^2}}$ is tangent to the ellipse for all values of m.

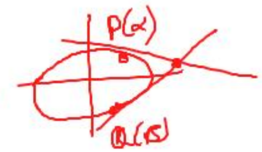
Note that there are two tangents to the ellipse having the same m, i.e. there are two tangents parallel to any given direction.

$\dfrac{x(a\cos\theta)}{a^2}+\dfrac{y(b\sin\theta)}{b^2}=1$

(iii) $\boxed{\dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}=1}$ is tangent to the ellipse at the point $(a\cos\theta, b\sin\theta)$.

(iv) The eccentric angles of point of contact of two parallel tangents differ by π. Conversely if the difference between the eccentric angles of two points is p then the tangents at these points are parallel.

(v) Point of intersection of the tangents at the point α & β is $\boxed{a\dfrac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}},\ b\dfrac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}}$.



NORMALS :

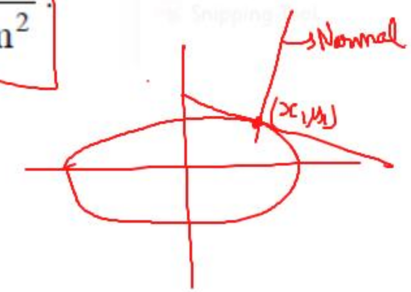
~~(i)~~ Equation of the normal at (x_1, y_1) is $\dfrac{a^2x}{x_1} - \dfrac{b^2y}{y_1} = a^2 - b^2 = a^2e^2$.

~~(ii)~~ Equation of the normal at the point $(a\cos\theta, b\sin\theta)$ is ; $ax\sec\theta - by\,\mathrm{cosec}\,\theta = (a^2 - b^2)$.

(iii) Equation of a normal in terms of its slope 'm' is $\boxed{y = mx - \dfrac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}}$.

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = mx - \dfrac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$



DIRECTOR CIRCLE :

Locus of the point of intersection of the tangents which meet at right angles is called the **Director Circle**. The equation to this locus is $x^2 + y^2 = a^2 + b^2$ i.e. a circle whose centre is the centre of the ellipse & whose radius is the length of the line joining the ends of the major & minor axis.

Circle: $x^2+y^2=a^2$

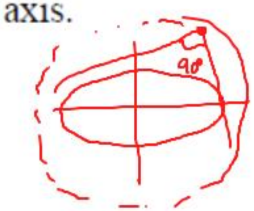
D.C: $x^2+y^2=2a^2$

Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

D.C $\boxed{x^2+y^2=a^2+b^2}$

Hyperbola. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

D.C= $x^2+y^2=a^2-b^2$



Chord of contact, pair of tangents, chord with a given middle point, are to be interpreted as they are in parabola.

C.O.C $\equiv$ $T=0$

Pair of Tangents $\equiv$ $SS_1=T^2$

Chord with a mid Point $\equiv$ $T=S_1$

Hyperbola

The **HYPERBOLA** is a conic whose eccentricity is greater than unity. ($e > 1$).

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

STANDARD EQUATION & DEFINITION(S)

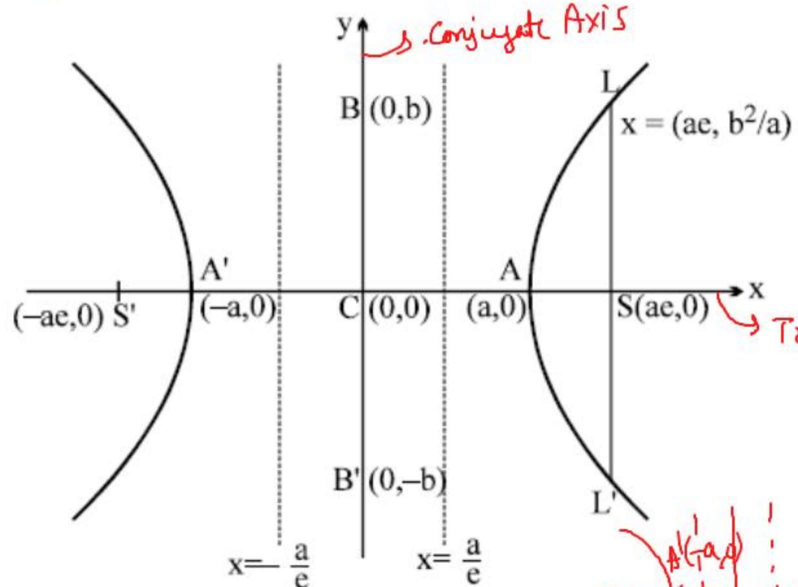
Standard equation of the hyperbola is

~~$b^2 = a^2(1-e^2)$~~

$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$. Where $\boxed{b^2 = a^2(e^2 - 1)}$

or $a^2e^2 = a^2 + b^2$ i.e. $e^2 = 1 + \frac{b^2}{a^2}$

$$= 1 + \left(\frac{C.A}{T.A}\right)^2$$



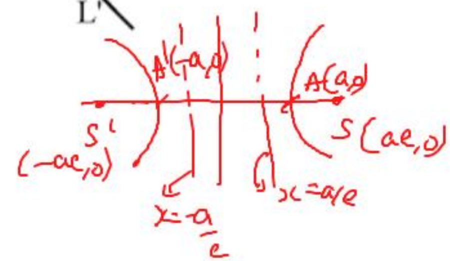
FOCI :

$S \equiv (ae, 0)$ & $S' \equiv (-ae, 0)$.

EQUATIONS OF DIRECTRICES :

$$x = \frac{a}{e} \quad \& \quad x = -\frac{a}{e}.$$

VERTICES : $A \equiv (a, 0)$ & $A' \equiv (-a, 0)$.



$$l\text{ (Latus rectum)} = \frac{2b^2}{a} = \frac{(C.A.)^2}{T.A.} = 2a\,(e^2 - 1).$$

Note : l (L.R.) = 2e (distance from focus to the corresponding directrix)

TRANSVERSE AXIS : The line segment $A'A$ of length 2a in which the foci S' & S both lie is called the **T.A. OF THE HYPERBOLA**.

CONJUGATE AXIS : The line segment $B'B$ between the two points $B' \equiv (0, -b)$ & $B \equiv (0, b)$ is called as the **C.A. OF THE HYPERBOLA**.

The T.A. & the C.A. of the hyperbola are together called the Principal axes of the hyperbola.

FOCAL PROPERTY :

The difference of the focal distances of any point on the hyperbola is constant and equal to transverse axis i.e. $\left|\,|PS| - |PS'|\,\right| = 2a$. The distance SS' = focal length.

$$\boxed{l \text{ (Latus rectum)} = \frac{2b^2}{a}} = \frac{(\text{C.A.})^2}{\text{T.A.}} = 2a\,(e^2 - 1).$$

Note : l (L.R.) = 2e (distance from focus to the corresponding directrix)

TRANSVERSE AXIS : The line segment A′A of length 2a in which the foci S′ & S both lie is called the **T.A. OF THE HYPERBOLA**.

CONJUGATE AXIS : The line segment B′B between the two points $B' \equiv (0, -b)$ & $B \equiv (0, b)$ is called as the **C.A. OF THE HYPERBOLA**.

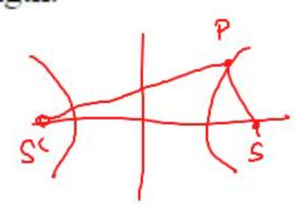
The T.A. & the C.A. of the hyperbola are together called the Principal axes of the hyperbola.

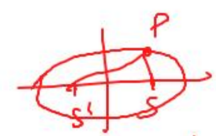
FOCAL PROPERTY :

The difference of the focal distances of any point on the hyperbola is constant and equal to transverse axis i.e. $\big||PS| - |PS'|\big| = 2a$. The distance SS' = focal length.

2nd defn of Hyperbola

$$\boxed{|PS - PS'| = 2a}$$





$|PS + PS'| = 2a$

Problems

The eccentricity of the hyperbola $4x^2 - 9y^2 - 8x = 32$ is -

(A) $\frac{\sqrt{5}}{3}$ ✓(B) $\frac{\sqrt{13}}{3}$ (C) $\frac{\sqrt{13}}{2}$ (D) $\frac{3}{2}$

$4x^2 - 9y^2 - 8x = 32$

$4x^2 - 8x - 9y^2 = 32$

$4(x^2 - 2x + 1 - 1) - 9y^2 = 32$

$4(x-1)^2 - \cancel{4} - 9y^2 = 36$

$4(x-1)^2 - 9y^2 = 36$

$$\boxed{\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1}$$

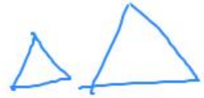
$$\frac{X^2}{3^2} - \frac{y^2}{2^2} = 1$$

$a = 3,\ b = 2$

$b^2 = a^2(e^2 - 1)$

$e^2 - 1 = \frac{b^2}{a^2}$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{4}{9}$



CONJUGATE HYPERBOLA :

Two hyperbolas such that transverse & conjugate axes of one hyperbola are respectively the conjugate & the transverse axes of the other are called **CONJUGATE HYPERBOLAS** of each other.

eg. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are conjugate hyperbolas of each.

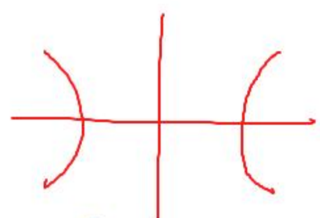
Note That : (a) If e_1 & e_2 are the eccentrcities of the hyperbola & its conjugate then $e_1^{-2}+e_2^{-2}=1$.

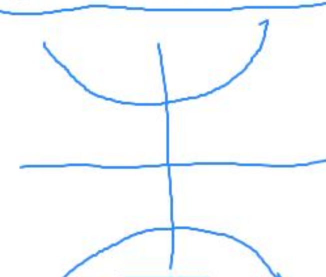
(b) The foci of a hyperbola and its conjugate are concyclic and form the vertices of a square.

(c) Two hyperbolas are said to be similiar if they have the same eccentricity.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=$

$\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$





$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$ ← Conjugate → $\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1}$

RECTANGULAR OR EQUILATERAL HYPERBOLA :

The particular kind of hyperbola in which the lengths of the transverse & conjugate axis are equal is called an **Equilateral Hyperbola**. Note that the eccentricity of the rectangular hyperbola is $\sqrt{2}$ and the length of its latus rectum is equal to its transverse or conjugate axis.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Rectangular Hyp

$2a = 2b$

$a = b$

$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \Rightarrow x^2 - y^2 = a^2$

T.A $= 2a$

C.A $= 2b$

$e^2 = 1 + \frac{b^2}{a^2}$

$e^2 = 1 + \cancel{\frac{a^2}{a^2}} \Rightarrow e^2 = 1 + 1 \Rightarrow e^2 = 2$

$\Rightarrow e = \sqrt{2}$

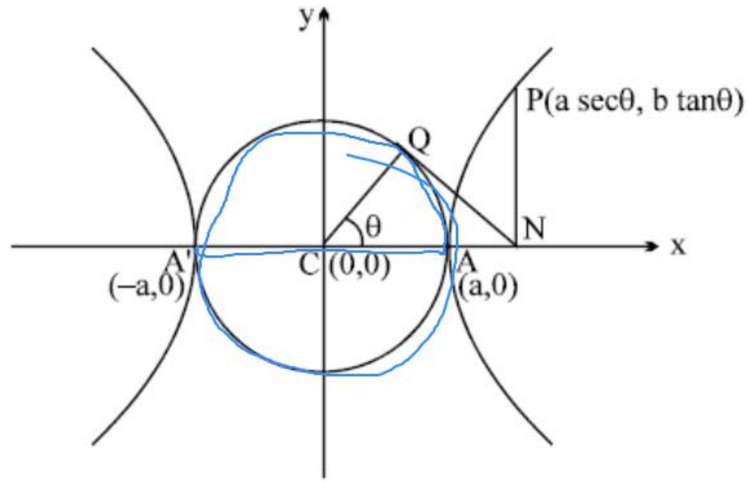
General Note :

Since the fundamental equation to the hyperbola only differs from that to the ellipse in having $-b^2$ instead of b^2 it will be found that many propositions for the hyperbola are derived from those for the ellipse by simply changing the sign of b^2.

AUXILIARY CIRCLE :

A circle drawn with centre C & T.A. as a diameter is called the **AUXILIARY CIRCLE** of the hyperbola. Equation of the auxiliary circle is $x^2 + y^2 = a^2$.

Note from the figure that P & Q are called the **"CORRESPONDING POINTS "** on the hyperbola & the auxiliary circle. 'θ' is called the eccentric angle of the point 'P' on the hyperbola. $(0 \le \theta < 2\pi)$.



The equations $\boxed{x = a \sec\theta \;\&\; y = b \tan\theta}$ together represents the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

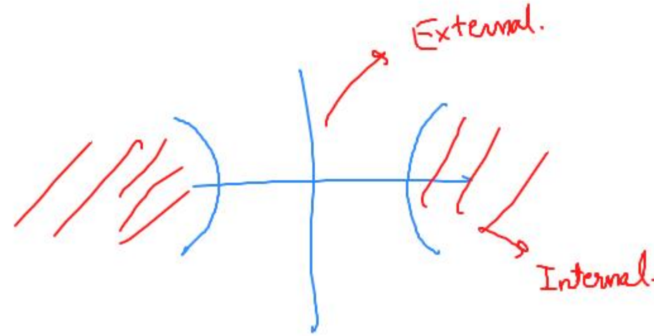
POSITION OF A POINT 'P' w.r.t. A HYPERBOLA : (Exception)

The quantity $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$ is positive, zero or negative according as the point (x_1, y_1) lies within, upon or without the curve.

$S_1 > 0$ inside*

$S_1 = 0$ on.

$S_1 < 0$ outside*



$c^2 = a^2m^2 + b^2$

LINE AND A HYPERBOLA :

The straight line $y = mx + c$ is a secant, a tangent or passes outside the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ according as: $c^2 > = < a^2m^2 - b^2$.

$c^2 = a^2m^2 - b^2$ $\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$

$y = mx + c$

If the line $y = 5x + 1$ touch the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$ $\{b > 4\}$, then -

(A) $b^2 = \frac{1}{5}$ (B) $b^2 = 99$ (C) $b^2 = 4$ (D) $b^2 = 100$

$m = 5$ $a = 2$

$c = 1$ $b = b$

$c^2 = a^2m^2 - b^2$

$1 = 4 \times 5^2 - b^2 \Rightarrow b^2 = 99.$

$(x_1, y_1) \to (a\sec\theta, b\tan\theta)$

TANGENTS :

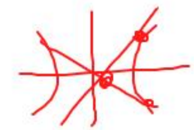
T = 0

(a) Equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

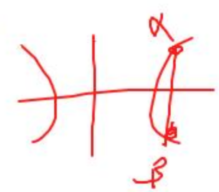
Note: In general two tangents can be drawn from an external point $(x_1 y_1)$ to the hyperbola and they are $y - y_1 = m_1(x - x_1)$ & $y - y_1 = m_2(x - x_2)$, where m_1 & m_2 are roots of the equation $(x_1{}^2 - a^2)m^2 - 2x_1y_1m + y_1{}^2 + b^2 = 0$. If $D < 0$, then no tangent can be drawn from $(x_1 y_1)$ to the hyperbola.

(b) Equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sec\theta, b\tan\theta)$ is $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.

Note : Point of intersection of the tangents at θ_1 & θ_2 is $x = a\frac{\cos\frac{\theta_1 - \theta_2}{2}}{\cos\frac{\theta_1 + \theta_2}{2}}$, $y = b\frac{\sin\frac{\theta_1 + \theta_2}{2}}{\cos\frac{\theta_1 + \theta_2}{2}}$



(c) $\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$ can be taken as the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.



Note that there are two parallel tangents having the same slope m.

(d) Equation of a chord joining α & β is

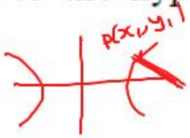
$$\frac{x}{a}\cos\frac{\alpha - \beta}{2} - \frac{y}{b}\sin\frac{\alpha + \beta}{2} = \cos\frac{\alpha + \beta}{2}$$

NORMALS:

(a) The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $P(x_1, y_1)$ on it is

$$\boxed{\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 - b^2 = a^2 e^2.}$$

**.



(b) The equation of the normal at the point P (a secθ, b tanθ) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$\frac{a\,x}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2 = a^2 e^2.$$

Find the equation of normal to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ at (5, 0).

x_1 y_1

$$\frac{25x}{5} + \frac{16y}{0} = 25 - 16$$

$$\frac{16y}{0} = 9 - 5x$$

$16y = 0 \Rightarrow y = 0$

DIRECTOR CIRCLE :

The locus of the intersection of tangents which are at right angles is known as the **DIRECTOR CIRCLE** of the hyperbola. The equation to the director circle is :

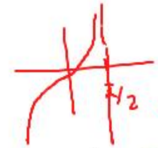
$$x^2 + y^2 = a^2 - b^2.$$

If $b^2 < a^2$ this circle is real; if $b^2 = a^2$ the radius of the circle is zero & it reduces to a point circle at the origin. In this case the centre is the only point from which the tangents at right angles can be drawn to the curve. If $b^2 > a^2$, the radius of the circle is imaginary, so that there is no such circle & so no tangents at right angle can be drawn to the curve.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$x^2 + y^2 = a^2 + b^2 \qquad x^2 + y^2 = a^2 - b^2$$

$xy = c^2$



RECTANGULAR HYPERBOLA :

Rectangular hyperbola referred to its asymptotes as axis of coordinates.

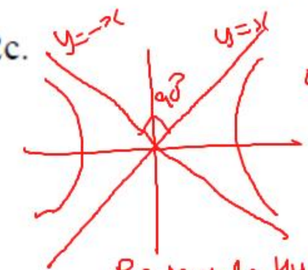
Asymptote (tangent at infinity)

(a) Equation is $\boxed{xy = c^2}$ with parametric representation $x = ct$, $y = c/t$, $t \in R - \{0\}$.

(b) Equation of a chord joining the points $P(t_1)$ & $Q(t_2)$ is $x + t_1 t_2 y = c(t_1 + t_2)$ with slope $m = -\frac{1}{t_1 t_2}$

(c) Equation of the tangent at $P(x_1, y_1)$ is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ & at $P(t)$ is $\frac{x}{t} + ty = 2c$.

$T = 0$



(d) Equation of normal : $y - \frac{c}{t} = t^2(x - ct)$

(e) Chord with a given middle point as (h, k) is $kx + hy = 2hk$.

$T = S_1$

Find the equation of tangent at the point (1, 2) to the rectangular hyperbola xy = 2.

$y = x$, $y = -x$

For Rectangular Hyperbola, asymptotes are $\perp$ to each other.

$y=\frac{1}{x} \Rightarrow xy=1$

RECTANGULAR HYPERBOLA :

$x^2 \to x x_1$ $\quad 2xy \to x y_1 + x_1 y$
$y^2 \to y y_1$
$2x \to x + x_1$
$2y \to y + y_1$

Rectangular hyperbola referred to its asymptotes as axis of coordinates.

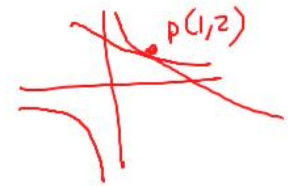
(a) Equation is $xy = c^2$ with parametric representation $x = ct$, $y = c/t$, $t \in R - \{0\}$.

(b) Equation of a chord joining the points $P(t_1)$ & $Q(t_2)$ is $x + t_1t_2y = c(t_1 + t_2)$ with slope $m = -\frac{1}{t_1t_2}$

(c) Equation of the tangent at $P(x_1 \cdot y_1)$ is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ & at $P(t)$ is $\frac{x}{t} + ty = 2c$.

(d) Equation of normal : $y - \frac{c}{t} = t^2(x - ct)$

(e) Chord with a given middle point as (h, k) is $kx + hy = 2hk$.



Find the equation of tangent at the point (1, 2) to the rectangular hyperbola xy = 2.

$T=0$

$2xy = 4$

$xy_1 + x_1y = 4$ (eqn of tangent)

$\boxed{2x+y=4} \to$ Ans.