PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Electrostatics

(Part 01)

By,
Ritesh Agarwal, B. Tech. IIT Bombay

Electric Charge

Positive charges develop on a body due to removal of electrons. Negative charges develop due to addition of electrons.

Basic Properties of Charges

(i) Charge is a scalar. It adds algebraically.

(ii) Like charges repel each other while unlike charges attract each other.

(iii) Net charge of isolated system remains conserved.

(iv) Change is Quantized. $Q = ne$

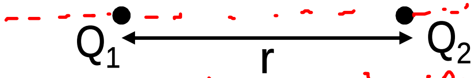
(v) Charge is always associated with mass.

(vi) Charge is invariant i.e. does not depends on speed or reference frame.

(vii) Methods of charging : Friction, Induction & Conduction.

Coulomb's Law

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2}$$ ← permittivity of free space



Point charges

$\varepsilon = \varepsilon_0 \varepsilon_r$

↑ Absolute permittivity of medium.

$\varepsilon_r \geq 1$

In vector form formula can be given as below.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{|\vec{r}|^3}\vec{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

(q_1 & q_2 are to be substituted with sign.)

← relative permittivity

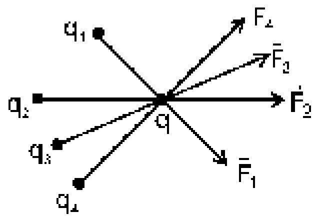
here $\vec{r}$ is position vector of the test charge (on which force is to be calculated) with respect to the source charge (due to which force is to be calculated).

In medium

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2}$$

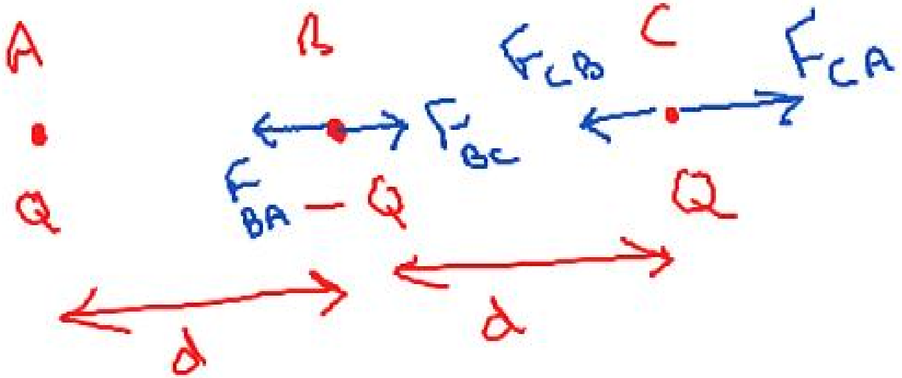
$$F_{medium} = \frac{F_{vacuum}}{\varepsilon_r}$$

Principle of Superposition



$$\vec{F} = \vec{F}_1 - \vec{F}_2 + \vec{F}_3 + \dots$$

Example



Find force on charge at B and at C

At B, $F_{net} = 0$

$$F_{CA} = \frac{KQ^2}{(2d)^2} = \frac{KQ^2}{4d^2}$$

$$F_{CB} = \frac{KQ^2}{d^2}$$

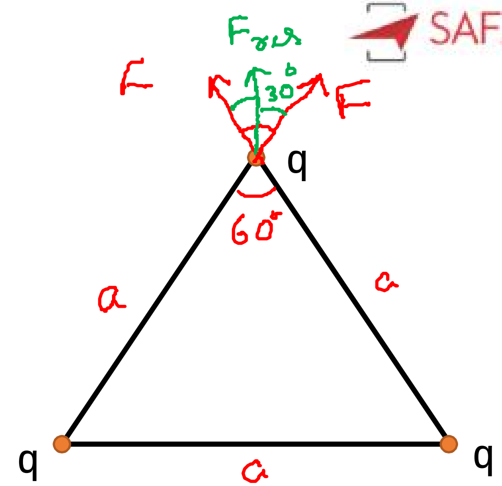
$$F_{net} = F_{CB} - F_{CA}$$

$$= \frac{KQ^2}{d^2}\left(1 - \frac{1}{4}\right)$$

$$= \frac{3}{4}\frac{KQ^2}{d^2}$$

Example

Three identical charges are kept at three vertices of an equilateral triangle of side ‘a’. Find net electrostatic force on any one of the charge.



Sol.

$$F = \frac{Kq^2}{a^2}$$

$$F_{res.} = 2F\cos 30^\circ = 2F\cdot\frac{\sqrt{3}}{\cancel{2}}$$

$$= \sqrt{3}\,\frac{Kq^2}{a^2}$$

Electric Field

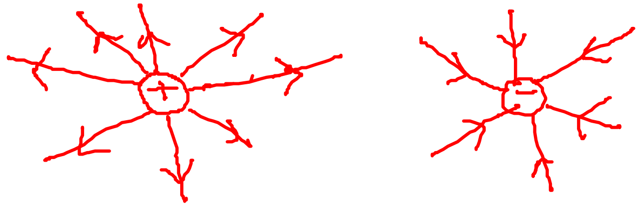
Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Properties of electric field intensity $\vec{E}$:

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Direction of electric field due to positive charge is always away from it while due to negative charge, always towards it.



(iii) Its S.I. unit is Newton/Coulomb.

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) It obeys the superposition principle

i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

Example

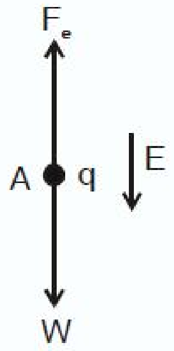
Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge –10 μc and mass 10 mg. (take g = 10 ms²)

Sol.

As force on a charge q in an electric field $\vec{E}$ is

$$\vec{F}_q = q\vec{E}$$

So according to given problem



$$|\vec{F}_q| = |\vec{W}| \qquad \text{i.e.,} \qquad |q|E = mg$$

i.e., $E = \dfrac{mg}{|q|}$ = 10 N/C., in downward direction.

$F_E = q\vec{E}$

E

$W = mg$

$q = -10\mu C = -10\times10^{-6} C$

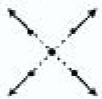
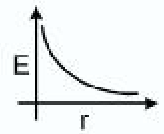
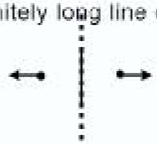
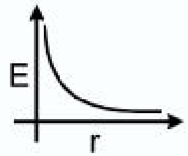
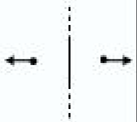
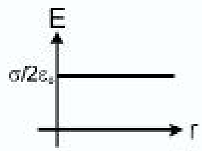
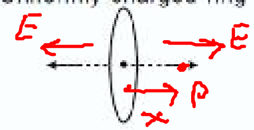
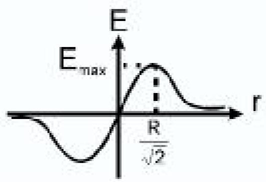
$m = 10 mg = 10\times10^{-6} kg$

$qE = mg$

$\cancel{10\times10^{-6}} E = \cancel{10\times10^{-6}} \times 10$

$E = 10 N/C$

Electric Field Intensities due to various Charge Distributions

Name/Type	Formula	Note	Graph
Point charge 	$\frac{Kq}{\lvert\vec{r}\rvert^2}\cdot\hat{r} = \frac{Kq}{r^3}\vec{r}$ Kq/r^2	* q is source charge. * $\vec{r}$ is vector drawn from source charge to the test point. * Electric field is nonuniform, radially outwards due to +charges & inwards due to –charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda\hat{r}}{r}$	* λ is linear charge density (assumed uniform) * r is perpendicular distance of point from line charge. * $\hat{r}$ is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	* σ is surface charge density. (assumed uniform) * $\hat{n}$ is unit normal vector. * Electric field intensity is independent of distance.	
Uniformly charged ring 	$E = \frac{KQx}{(R^2+x^2)^{3/2}}$ $E_{centre} = 0$	* Q is total charge of the ring. * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	

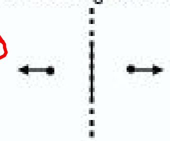
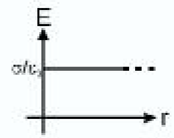
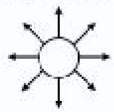
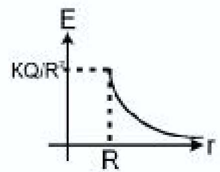
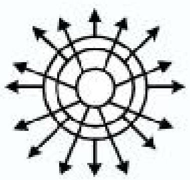
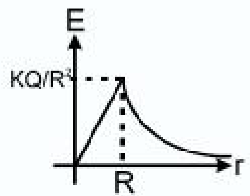
$\frac{Q}{2A\varepsilon_0}$

where Q = Total charge

Electric Field Intensities due to various Charge Distributions

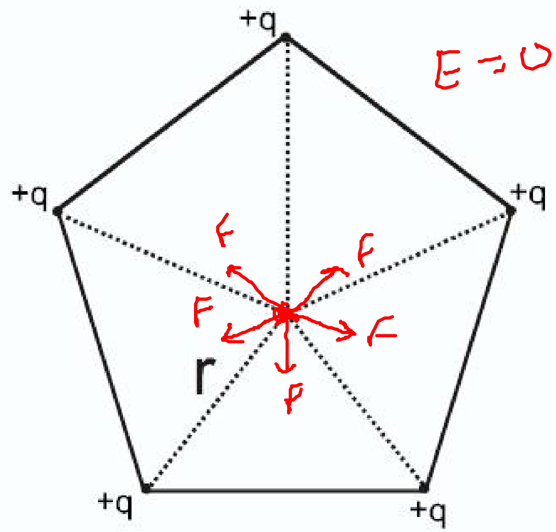
$E = \frac{Q}{2A\varepsilon_0}$

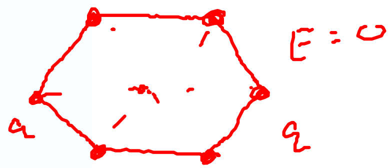
Area

Name/Type	Formula	Note	Graph
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	* σ is the surface charge density (assumed uniform) * $\hat{n}$ is the unit vector perpendicular is the surface. * Electric field intensity is independent of distance	
Uniformly charged hollow conducting/ nonconducting /solid conducting sphere 	(i) for $r \geq R$ (ii) for $r < R$ $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ $\vec{E} = 0$	* R is radius of the sphere. * $\vec{r}$ is vector drawn from centre of sphere to the point. * Sphere acts like a point charge, placed at centre for points outside the sphere. * $\vec{E}$ is always along radial direction. * Q is total charge ($= \sigma 4\pi R^2$). (σ = surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \frac{KQ\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$	* $\vec{r}$ is vector drawn from centre of sphere to the point * Sphere acts like a point charge placed at the centre for points outside the sphere * $\vec{E}$ is always along radial dir^n * Q is total charge $(\rho.\frac{4}{3}\pi R^3)$. ($\rho$ = volume charge density) * Inside the sphere $E \propto r$. * Outside the sphere $E \propto 1/r^2$.	

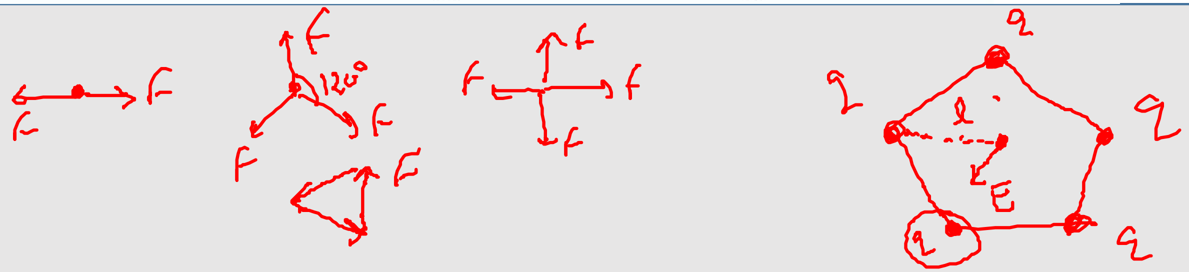
Example

Four charges are placed at the corners of a regular pentagon. Find E at centre?





Sol.

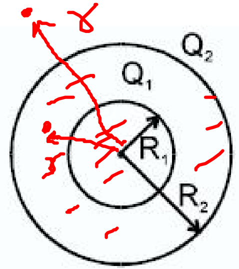


$$E = \frac{KQ}{l^2}$$

Example

Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.

(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$



Sol.

(i) for $r < R_1$,

therefore point lies inside both the spheres

$$E_{net} = E_{Inner} + E_{outer} \qquad = 0 + 0$$

(ii) for $R_1 \le r < R_2$,

therefore point lies outside inner sphere but inside outer sphere:

$$E_{net} = E_{inner} + E_{outer}$$

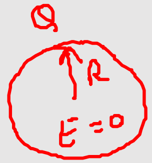
$$= \frac{KQ_1}{r^2}\hat{r} + 0 \qquad = \frac{KQ_1}{r^2}\hat{r}$$

(iii) for $r \ge R_2$

point lies outside inner as well as outer sphere therefore.

$$E_{Net} = E_{inner} + E_{outer}$$

$$= \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} \qquad = \frac{K(Q_1+Q_2)}{r^2}\hat{r}$$



$E = \frac{KQ}{r^2}$

$E(r < R_1) = 0$

$E(R_1 \le r < R_2) = \frac{KQ_1}{r^2} + 0$

$E(r \ge R_2) = \frac{KQ_1}{r^2} + \frac{KQ_2}{r^2}$

Electric Dipole

If two point charges equal in magnitude q and opposite in sign separated by a distance a such that the distance of field point r>>a, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude p = (q × a) and direction from negative charge to positive charge.

Net Charge of dipole = zero

SI Unit : C-m   Dimensional Formula : [LAT]

Example.

A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the net charge and electric dipole moment of the system ?

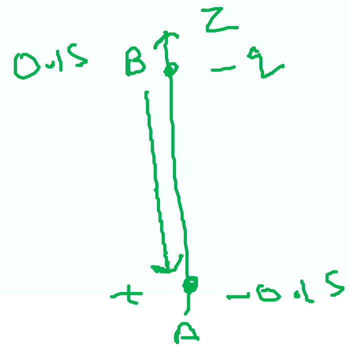
Sol. Net charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric dipole moment,

$$P = (\text{Magnitude of charge}) \times (\text{Separation between charges})$$

$$= 2.5 \times 10^{-7} [0.15 + 0.15] \text{ C m} \quad = 7.5 \times 10^{-8} \text{ C m}$$

The direction of dipole moment is from B to A.



Electric Field Intensity Due to Dipole

At the axial point :-

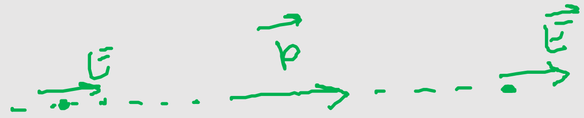
$$E = \frac{Kq}{\left(x-\frac{a}{2}\right)^2} - \frac{Kq}{\left(x+\frac{a}{2}\right)^2}(\rightarrow) = \frac{Kq(2xa)}{\left(r2-\frac{a}{2}\right)^2}(\rightarrow)$$

If $r >> a$ then

$$E = \frac{Kq2ra}{r^4}(\rightarrow) = \frac{2KP}{r^3}(\rightarrow),$$

As the direction of electric field at axial position is along the dipole moment ($\vec{P}$)

so $$\boxed{\vec{E}_{axial} = \frac{2K\vec{P}}{r^3}}$$

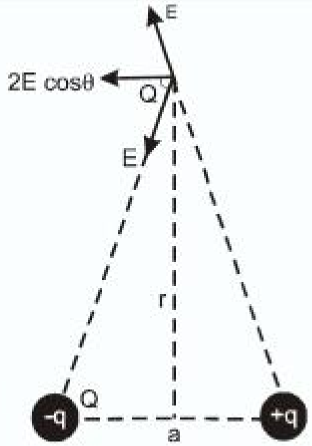


Electric Field Intensity Due to Dipole

Electric field at perpendicular Bisector (Equatorial Position)

$E_{net} = 2\,E\cos\theta\ (\rightarrow)$

$$E_{net} = 2\left(\frac{Kq}{\left(\sqrt{r^2+\left(\frac{a}{2}\right)^2}\right)^2}\right)\frac{\frac{a}{2}}{\sqrt{r^2+\left(\frac{a}{2}\right)^2}}(\leftarrow) = 2\ \frac{Kqa}{\left(r^2+\left(\frac{a}{2}\right)^2\right)^{3/2}}(\leftarrow)$$



If r >> a then

$$E_{net} = \frac{KP}{r^3}(\leftarrow)$$

As the direction of $\vec{E}$ at equatorial position is opposite of $\vec{P}$ so we can write in vector form:

$$\boxed{\vec{E}_{eqtorial} = -\frac{K\vec{P}}{r^3}}$$

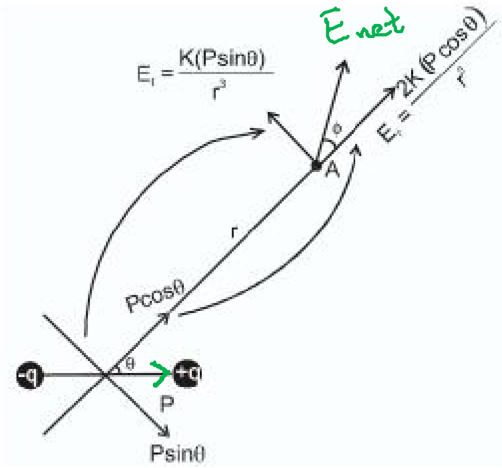
Electric Field Intensity Due to Dipole

Electric field at general point (r, θ) :

$$E_{net} = \sqrt{E_r^2 + E_t^2} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\phi = \frac{E_t}{E_r} = \frac{\frac{KP\sin\theta}{r^3}}{\frac{2KP\cos\theta}{r^3}}$$

$$\boxed{E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta} \quad ; \quad \tan\phi = \frac{\tan\theta}{2}}$$



Angle with dipole moment $= \theta + \phi$

Example

The electric field due to a short dipole at a distance r, on the axial line, from its mid point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

Sol.

$$\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$$

or $$\frac{2}{r^3} = \frac{1}{r'^3}$$

or $$\frac{r^3}{r'^3} = 2$$ or $$\frac{r}{r'} = 2^{1/3}$$

$E_{axial} = \frac{2Kp}{r^3}$

$E_{eq.} = \frac{Kp}{r'^3}$

$\frac{2\cancel{Kp}}{r^3} = \frac{\cancel{Kp}}{r'^3}$

$\frac{r^3}{r'^3} = 2$

$\frac{r}{r'} = 2^{1/3}$

Electric Lines of Force (ELOF)

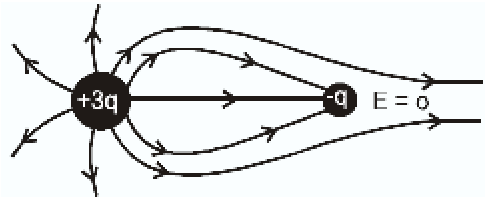
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

net

Properties

E_1 not possible E_2

(i) They are imaginary
(ii) Can never cross each other
(iii) Can never be closed loops
(iv) The number of lines originating or terminating
on a charge is proportional to the magnitude of charge.



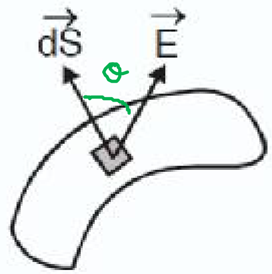
(v) Lines of force ends or starts normally at the surface of a conductor.
(vi) If there is no electric field there will be no lines of force.
(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded
lines represent strong field while distant weak field.

$E_A > E_B$ A B

(viii) Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

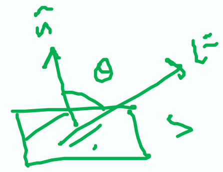
Electric Flux

Consider some surface in an electric field $\vec{E}$. Let us select a small area element $\overrightarrow{dS}$ on this surface. The electric flux of the field over the area element is given by $\boxed{d\phi_E = \vec{E}.\overrightarrow{dS} = EdS\cos\theta}$



Direction of $\overrightarrow{dS}$ is normal to the surface. It is along $\hat{n}$

The electric flux over the whole area is given by $\phi_E = \int_S \vec{E}.\overrightarrow{dS} = \int_S E_n dS$



If the electric field is uniform over that area then $\phi_E = \vec{E}\cdot\vec{S}$

$\phi = ES\cos\theta$

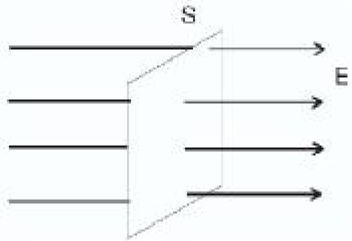
Special Cases

Case I : If the electric field in normal to the surface,

then angle of electric field $\vec{E}$ with normal will be zero

So $\phi = ES \cos 0$

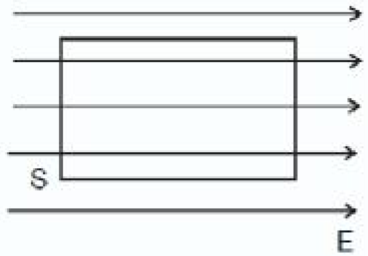
$\phi = ES$



Case II : If electric field is parallel of the surface (glazing),

then angle made by $\vec{E}$ with normal = 90°

So $\phi = ES \cos 90^\circ = 0$



Physical Meaning

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface.

Unit

(i) The SI unit of electric flux is $Nm^2 C^{-1}$ (gauss) or $J\ m\ C^{-1}$.

(ii) Electric flux is a scalar quantity. (It can be positive, negative or zero)

Gauss's Law

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface. Here, ε_0 is the permittivity of free space.

$$\phi_E = \oint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$

Important Points

(i) Flux through gaussian surface is independent of its shape.

(ii) Flux through gaussian surface depends only on total charge present inside gaussian surface.

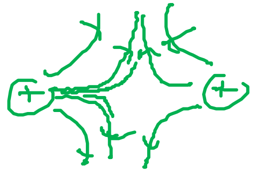
(iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.

(iv) Electric field intensity at the gaussian surface is due to all the charges present inside as well as outside the gaussian surface.

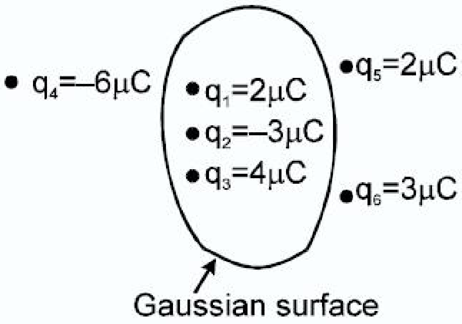
(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.

(vi) In a gaussian surface $\phi = 0$ does not imply $E = 0$ at every point of the surface but $E = 0$ at every point implies $\phi = 0$.

Example



Find out flux through the given gaussian surface.

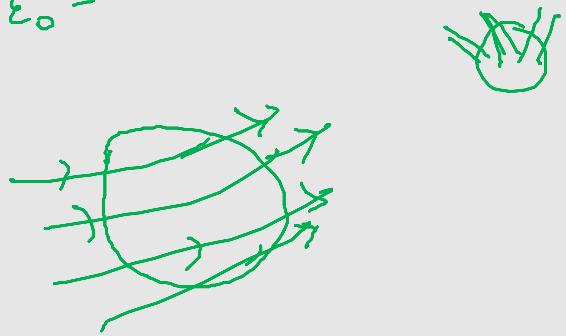


Sol.

$$\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3\times 10^{-6}}{\varepsilon_0} \text{ Nm}^2/\text{C}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times 10^{9} \text{ N-m}^2/\text{C}^2$$

$$\varepsilon_0 = 8.85\times 10^{-12} \text{ C}^2/\text{N-m}^2$$



Example

If a point charge q is placed at the centre of a cube then find out flux through any one surface of cube.

Sol.

Flux through 6 surfaces = $\frac{q}{\varepsilon_0}$.

Since all the surfaces are symmetrical

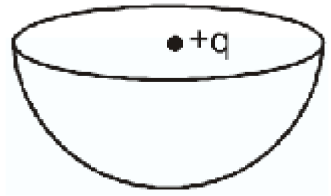
so, flux through one surfaces = $\frac{1}{6}\frac{q}{\varepsilon_0}$

For an imaginary cube

Position of charge	Main centre	Face centre	Side center	Corner
flux from cube	$\frac{q}{\varepsilon_0}$	$\frac{q}{2\varepsilon_0}$	$\frac{q}{4\varepsilon_0}$	$\frac{q}{8\varepsilon_0}$

Example

A point charge +q is placed at the centre of curvature of a hemisphere. Find flux through the hemispherical surface.



Sol.

Lets put an upper half hemisphere.

Now flux passing through the entire sphere = $\frac{q}{\varepsilon_0}$

As the charge q is symmetrical to the upper half and lower half hemispheres , so half-half flux will emit from both the surfaces.

Flux emmittring from lower half surface = $\frac{q}{2\varepsilon_0}$

Flux emmitting from upper half surface = $\frac{q}{2\varepsilon_0}$