Problem Solving on DE, BT & P&C

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The order of the differential equation whose general solution is given by $y = (c_1 + c_2) \cos(x + c_3) - c_4 e^{x+c5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constant is –

(A) 5 (B) 4

~~(C)~~ 3 (D) 2

$y = K_1 \cos(x + c_3) - c_4 e^x e^{c_5}$

$y = K_1 \cos(x + c_3) - K_2 e^x$

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Let $(1+t)\frac{dy}{dt} - ty = 1, y(0) = -1$, find $y(t)$ at $t = 1$?

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) $e - \frac{1}{2}$ (D) $e + \frac{1}{2}$

Sol^n

$$\frac{dy}{dt} - \frac{t}{(1+t)} y = \frac{1}{1+t}$$

$$I.F = e^{\int -\frac{t}{1+t} dt}$$

$$= e^{\int \frac{-(t+1)+1}{(t+1)} dt}$$

$$= e^{-\int dt + \int \frac{1}{1+t} dt}$$

$$= e^{-t + \ln(1+t)}$$

$$I.F = e^{-t} \cdot (1+t)$$

$$y \times e^{-t}(1+t) = \int \frac{1}{\cancel{1+t}} \times e^{-t} \cancel{(1+t)} dt$$

$$y \cdot e^{-t}(1+t) = -e^{-t} + C$$

$$(-1) \times 1 \times (1) = -1 + C$$

$$-\cancel{1} = -\cancel{1} + C$$

$$\boxed{C = 0}$$

$$y \cdot e^{-t}(1+t) = -e^{-t}$$

$$y \cdot \cancel{e^{-1}}(2) = -\cancel{e^{-1}} \Rightarrow \boxed{y = -\frac{1}{2}}$$

The solution of the differential equation $(2x - 10y^3)\dfrac{dy}{dx} + y = 0$ is

(A) $x + y = ce^{2x}$ (B) $y^2 = 2x^3 + c$

(C) $xy^2 = 2y^5 + c$ (D) $x(y^2 + xy) = 0$

$$(2x - 10y^3) + y\frac{dx}{dy} = 0$$

$$y\frac{dx}{dy} + 2x = 10y^3$$

$$\frac{dx}{dy} + \left(\frac{2}{y}\right)x = 10y^2$$

$$\boxed{\frac{dx}{dy} + P(y)\cdot x = Q(y)}$$

$$I.F = e^{\int P(y)dy}$$

$$= e^{\int \frac{2}{y}dy}$$

$$= e^{2\ln y}$$

$$= e^{\ln y^2} = y^2$$

$$x\cdot y^2 = \int 10y^2 \times y^2\,dy$$

$$xy^2 = \frac{\overset{2}{\cancel{10}}y^5}{\cancel{5}} + c$$

$$\boxed{xy^2 = 2y^5 + c}$$

$(x^2 + y^2)dy = xy\,dx$ (initial value problem), y [illegible] $) = e$

then find $x_0 = ?$

(A) $\sqrt{\dfrac{e^2-1}{2}}$ (B) $\sqrt{2e^2-1}$

(C) $\sqrt{e^2-2}$ (D) $\sqrt{3}e$

Sol^n $(x^2+y^2)dy = xy\,dx$

$\dfrac{dy}{dx} = \left(\dfrac{xy}{x^2+y^2}\right)$

$\dfrac{dy}{dx} = \dfrac{(y/x)}{1+(y/x)^2}$

$y = vx$

$\dfrac{dy}{dx} = x\dfrac{dv}{dx} + v$

$\rightarrow v + x\dfrac{dv}{dx} = \dfrac{v}{1+v^2}$

$x\dfrac{dv}{dx} = \dfrac{v}{1+v^2} - v$

$x\dfrac{dv}{dx} = \dfrac{-v^3}{1+v^2}$

$-\int \dfrac{(1+v^2)}{v^3}dv = \int \dfrac{dx}{x}$

$-\int\dfrac{dv}{v^3} - \int\dfrac{dv}{v} = \int\dfrac{dx}{x}$

$\left(\dfrac{1}{2v^2}\right) - \ln v = \ln x + c$

$\dfrac{1}{2} = c$

r and n are positive integers $r>1$, $n>2$ and coefficient of $(r+2)^{th}$ term and $(3r)^{th}$ term in expansion of $(1+x)^{2n}$ are equal, then n equals.

(A) 3r (B) 3r+1

(C) 2r (D) 2r+1

$(1+x)^n$, $T_{r+1} = {}^nC_r x^r$

$(1+x)^{2n}$ $T_{r+2} = {}^{2n}C_{r+1} x^{(r+1)}$

$T_{3r} = {}^{2n}C_{3r-1} (x)^{3r-1}$

${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$

${}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1}$

$2n-(r+1) = 3r-1$

$2n = 3r-1+r+1$

$\boxed{2n = 4r}$

$n = 2r$

The coefficient of the middle term in the binomial expansion in powers of x of $(1-\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals.

$$6C_3 = \frac{6\times5\times4}{3!}$$

(A) $-\frac{5}{3}$

(B) $\frac{3}{5}$

(C) $-\frac{3}{10}$

(D) $\frac{10}{3}$

$(1-\alpha x)^4 = 5$ term Middle term $= T_3$

$(1-\alpha x)^6 = 7$ term " $= T_4$

$T_3 = {}^4C_2(-\alpha x)^2 = [4C_2(-\alpha)^2]$

$T_4 = 6C_3(-\alpha x)^3 = 6C_3(-\alpha)^3$

$4C_2(-\alpha)^2 = 6C_3(-\alpha)^3$

$\alpha = -\frac{4C_2}{6C_3}$

$= -\frac{6}{20} = -\frac{3}{10}$

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The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(A) $(n-1)$

~~(B)~~ $(-1)^n(1-n)$

(C) $(-1)^{n+1}(n-1)^2$

(D) $(-1)^{n-1}n$

Solⁿ

$(1+x)(1-x)^n$

$(1+x)\left[a x^n + b x^{n-1}\right]$

$a x^n + b \cdot x^n = (a+b)$

$a+b = (-1)^n + n(-1)^{n-1}$

$= (-1)^n[1-n]$

$=$

$T_{r+1} = {}^nC_r(-x)^r$

$T_{n+1} = {}^nC_n(-x)^n$

$= {}^nC_n(-1)^n x^n$

$a = (-1)^n$

$T_n = {}^nC_{n-1}(-x)^{n-1}$

$= {}^nC_{n-1}(-1)^{n-1}x^{n-1}$

b

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If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ Equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$.

Then a and b satisfy the relation

(A) $a - b = 1$ (B) $a + b = 1$

(C) $\frac{a}{b} = 1$ ~~(D)~~ $ab = 1$

$$\cancel{{}^{11}C_5}\,\frac{a^6}{\cancel{b^5}} = \cancel{{}^{11}C_6}\,\frac{\cancel{a^5}}{b^6}$$

$$\boxed{ab = 1}$$

$$\left(ax^2 + \frac{1}{bx}\right)^{11}, \quad T_{r+1} = {}^nC_r\,(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r = {}^{11}C_r\,\frac{a^{11-r}}{b^r}\left(\frac{x^{22-2r}}{x^r}\right) = x^{22-3r}$$

$$22 - 3r = 7$$
$$15 = 3r$$
$$\boxed{r = 5}$$

$${}^{11}C_5\,\frac{a^6}{b^5}$$

$$\left(ax - \frac{1}{bx^2}\right)^{11} = T_{r+1} = {}^{11}C_r\,(ax)^{11-r}\left(\frac{-1}{bx^2}\right)^r$$

$$= {}^{11}C_r\,\frac{a^{11-r}}{(-b)^r}\cdot\frac{x^{11-r}}{x^{2r}} \Rightarrow (\quad)\,x^{11-3r}$$

$$11 - 3r = -7$$
$$18 = 3r$$
$$\boxed{r = 6}$$

$$= {}^{11}C_6\,\frac{a^5}{(-b)^6}$$

... 5^{th} and 6^{th} terms is zero, then $\frac{a}{b}$ equals.

(A) $\frac{5}{n-4}$ (B) $\frac{6}{n-5}$

(C) $\frac{n-5}{6}$ (D) $\frac{n-4}{5}$

$(a-b)^n$

$T_{r+1} = {}^nC_r\, a^{n-r}(-b)^r$

$T_5 = {}^nC_4\, a^{n-4}(-b)^4$

$T_6 = {}^nC_5\, a^{n-5}(-b)^5$

$T_5 + T_6 = 0$

${}^nC_4\, a^{n-4}(-b)^4 + {}^nC_5 (a)^{n-5}(-b)^5 = 0$

$a^{n-5}(-b)^4\left[{}^nC_4\, a + {}^nC_5(-b)\right] = 0$

${}^nC_4\, a = b\cdot {}^nC_5$

$\frac{\cancel{n!}}{4!(n-4)!}\, a = b\cdot \frac{\cancel{n!}}{5!(n-5)!}$

$5a = b(n-4)$

$\boxed{\frac{a}{b} = \frac{n-4}{5}}$

Total number of four digit odd numbers that can be formed using 0, 1,2,3,5,7 (using repetition allowed) are

(A) 216 (B) 375

(C) 400 ~~(D)~~ 720

5 6 6 4

$5 \times 6 \times 6 \times 4$

30×24

720

repetition not allowed

4 4 3 4

$4 \times 4 \times 3 \times 4$

16×12

192

Number greater than 1000 but less than 4000 is formed using the digits 0,1,2,3,4(repetition allowed) is

374

(A) 125

(B) 105

(C) 375

(D) 625

0, 1, 2, 3, 4

1 0 0 0

$3 \times 5 \times 5 \times 5$

$125 \times 3 = (375 - 1)$

$= 374$

A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is.

(A) 140 (B) 196

(C) 280 (D) 346

Sol^n

13 → 10 answer

$$5 \to 4 \quad 8 \to 6 \quad \text{and} \quad 5 \to 5 \quad 8 \to 5$$

$$^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5 = 5 \times \frac{\cancel{8}^{4} \times 7}{\cancel{2}} + 1 \times \frac{8 \times 7 \times \cancel{6}}{\cancel{3!}}$$

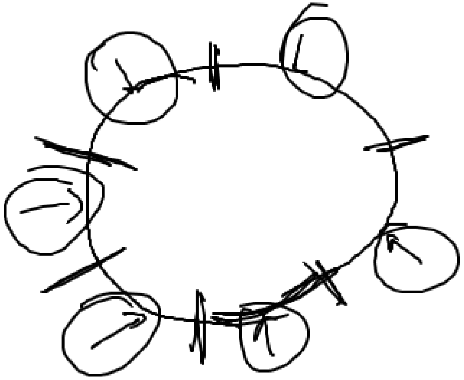
$$= 140 + 56$$

$$= \boxed{196}$$

The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(A) $6! \times 5!$ (B) 30

(C) $5! \times 4!$ (D) $7!' 5!$



$(6-1)! = 5!$

$6 \times 5 \times 4 \times 3 \times 2 = 6!$

$6! \times 5!$