PHYSICS

NEET and JEE Main 2020 : 45 Days Crash Course

Problem Solving Class

(Rotation, Fluid, Centre of Mass)

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PQ8Q18

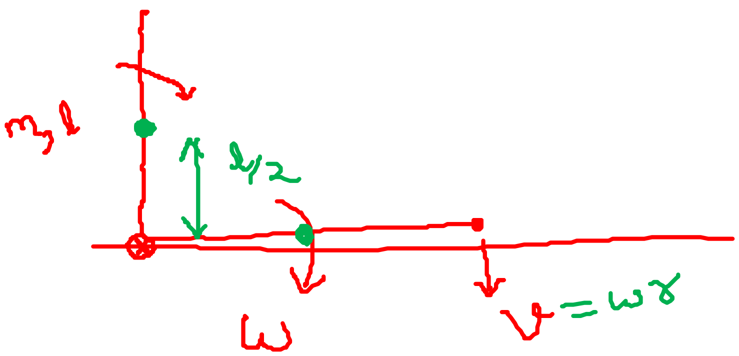
A thin uniform rod of mass m and length l is hinged at the lower end to a level floor and stands vertically. It is now allowed to fall, then its upper end will strike the floor with a velocity given by

(A) $\sqrt{mgl}$

(B) $\sqrt{3gl}$

(C) $\sqrt{5\,mgl}$

(D) $\sqrt{2\,mgl}$



E/C

Loss in P.E. = Gain in K.E

$$mg\frac{l}{2} = \frac{1}{2}\cdot\frac{ml^2}{3}\cdot\omega^2$$

$$\omega = \sqrt{\frac{3g}{l}}$$

$$v = \omega l = \sqrt{3gl}$$

PQ8S18

Ans [B]

$$mg\frac{l}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\omega^2\right)$$

Loss in potential energy = Gain in kinetic energy

$$\Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

$$\Rightarrow v = l\omega = \sqrt{3gl}$$

PQ8Q52

A stone of mass **m**, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, **r** is the instantaneous radius of the circle. Find n.

(A) -3

(B) 3

(C) 4

(D) -4

$$T = \frac{mv^2}{r}$$

$$L = mvr = \text{const}$$

$$v \propto \frac{1}{r}$$

$$T \propto \frac{1}{r^3}$$

$$T \propto r^{-3}$$

PQ8S52

Ans [A]

Tension in the string provides centripetal force.

$$\Rightarrow \quad T = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{L}{mr}\right)^2 = \left(\frac{L^2}{m}\right) r^{-3}$$

Angular momentum, $L = mvr \quad \Rightarrow \quad v = \frac{L}{mr}$

Comparing with $T = Ar^n$, we get $n = -3$.

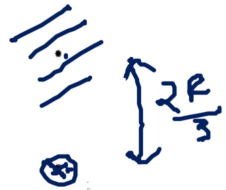
From a circular disc of radius **R** and mass **9M**, a small disc of radius $\frac{R}{3}$ is removed. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is,

(A) $4MR^2$ ✓

(B) $\frac{40}{9}MR^2$

(C) $10MR^2$

(D) $\frac{37}{9}MR^2$



$I = I_1 - I_2$

$M' = \sigma \cdot \pi \frac{R^2}{9} = \frac{9M}{\pi R^2} \cdot \frac{\pi R^2}{9} = M$

$\sigma = \frac{9M}{\pi R^2}$

$I = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right]$

Ans [A]

$I_{remaining} = I_{total} - I_{removed}$

The moment of inertia of disc of mass 9M and radius R about an axis perpendicular to its plane and passing through its centre O is,

$$I_{total} = \frac{1}{2}(9M)R^2 = \frac{9}{2}MR^2.$$

The mass of removed disc is $\frac{9M}{\pi R^2}\frac{\pi R^2}{9} = M$. The moment of inertia of the removed disc about axis passing through O is,

$$I_{removed} = \frac{1}{2}M\left(\frac{R}{3}\right)^2 + Md^2 = \frac{1}{18}MR^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2}MR^2.$$

$$I_{remaining} = 4MR^2$$

$I_{removed}$ can be found out by parallel axis theorem.

PQ8Q60

A particle of mass **m** is projected with a velocity **v** making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height **h** is,

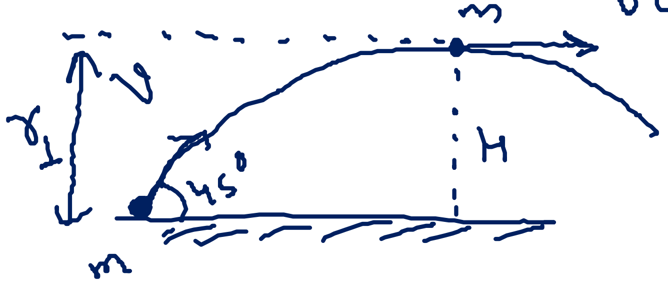
(A) Zero

(B) $\frac{mv^3}{4\sqrt{2}g}$

(C) $\frac{mv^3}{\sqrt{2}g}$

(D) $m\sqrt{gh^3}$

$v\cos 45^\circ = \frac{v}{\sqrt{2}}$



$L = r_\perp p$

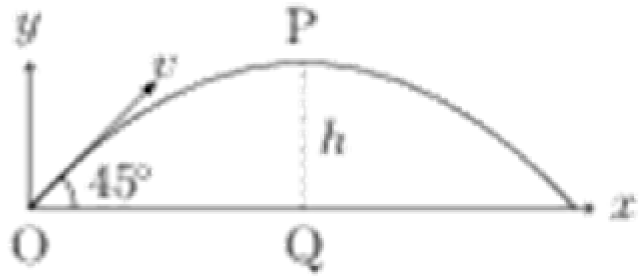
$r_\perp = H = \frac{u^2 \sin^2\theta}{2g} = \frac{v^2 \cdot \frac{1}{2}}{2g} = \frac{v^2}{4g}$

$L = r_\perp p = \frac{v^2}{4g} \cdot m\frac{v}{\sqrt{2}}$

Ans [B]

At the highest point ‘P’, vertical component of the velocity becomes zero.

The maximum height is given by,



$$h = \frac{v^2 \sin^2 45°}{2g} = \frac{v^2}{4g}.$$

The position vector of ‘P’ and velocity at ‘P’ are,

$$\vec{r}_P = |OQ|\,\hat{i} + h\,\hat{j}$$

$$\vec{r}_P = \frac{R}{2}\,\hat{i} + h_{max}\,\hat{j}$$

$$\vec{v}_P = v\cos 45°\,\hat{i} = \frac{v}{\sqrt{2}}\,\hat{i}.$$

The angular momentum about ‘O’ is given by,

$$\vec{L} = m\vec{r}_P \times \vec{v}_P = -\frac{mvh}{\sqrt{2}}\,\hat{k}$$

Angular momentum, $\vec{L} = m\vec{r}_P \times \vec{v}_P$

$$\Rightarrow \left|\vec{L}\right| = \frac{mvh}{\sqrt{2}} = \frac{mv^3}{4\sqrt{2}g} = m\sqrt{2gh^3}.$$

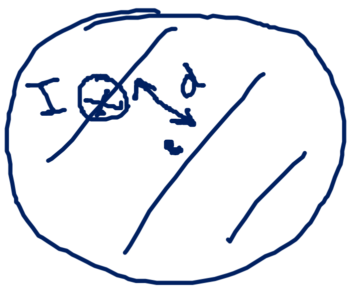
The radius of the gyration of a uniform disc about a line perpendicular to the disc equals its radius R. find the distance of the line from the center.

(A) $\frac{R}{\sqrt{2}}$

(B) $\frac{R}{2}$

(C) $\sqrt{2}R$

(D) R



$$\frac{mR^2}{2} + md^2 = mK^2 = mR^2$$

$$d^2 = R^2 - \frac{R^2}{2} = \frac{R^2}{2}$$

$$d = \frac{R}{\sqrt{2}}$$

Ans [A]

$$R = \sqrt{\frac{I}{m}} = \sqrt{\frac{\frac{1}{2}mR^2 + md^2}{m}}$$

Using the parallel axis theorem

$$I_{CM\,disc} = \frac{1}{2}mR^2$$

$$I = MK^2 \Rightarrow K = \sqrt{\frac{I}{M}}$$

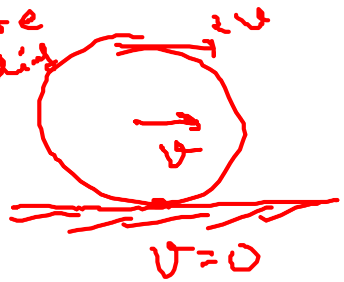
$$\therefore \; d = \frac{R}{\sqrt{2}}$$

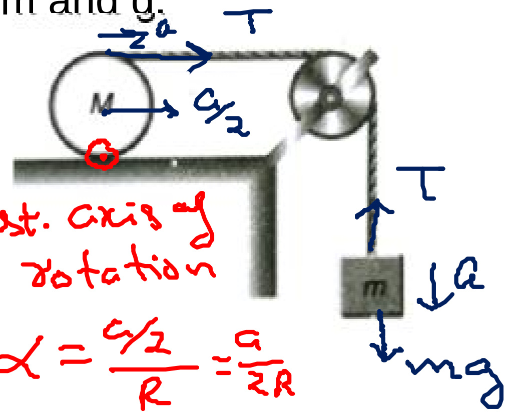
Consider the arrangement shown in figure. The string is wrapped around a uniform cylinder which rolls without slipping. The other end of the string is passed over a massless, frictionless pulley to a falling weight. Determine the acceleration of the falling mass m in terms of only the mass of the cylinder M, the mass m and g.

(A) $\dfrac{8mg}{3M+8m}$ (B) $\dfrac{3mg}{3M+8m}$

(C) $\dfrac{8mg}{8M+3m}$ (D) $\dfrac{3mg}{8M+3m}$

Pure rolling





$\alpha = \dfrac{a/2}{R} = \dfrac{a}{2R}$

For block, $mg - T = ma$ —(1)

For cylinder $\tau_P = I_P\alpha$

$T \cdot 2\cancel{R} = \frac{3}{2}MR^{\cancel{2}} \cdot \frac{a}{2\cancel{R}}$

$T = \frac{3M}{8}a$ —(2)

(1) + (2)

$mg = \left(m + \frac{3M}{8}\right)a$

$\boxed{a = \dfrac{8mg}{8m+3M}}$

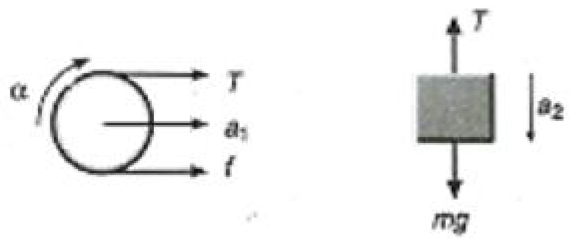
PQ8S66

Ans [A] Let T be the tension in the string and f the force of (static) friction, between the cylinder and the surface

a_1 = acceleration of center of mass of cylinder towards right

a_2 = downward acceleration of block m.

α = angular acceleration of cylinder (clockwise)



Equations of motion are

For block, $mg - T = ma_2$ ... (i)

For cylinder, $T + f = Ma_1$ ... (ii)

Using second law of motion

$$\alpha = \frac{(T - f)R}{1/2MR^2}$$

$\tau_{CM} = I_{CM}\alpha$

The string attaches the mass m to the highest point of the cylinder, hence

$$\mathbf{v_m} = \mathbf{v_{COM}} + \mathbf{R\omega}$$

Differentiating, we get $a_2 = a_1 + R\alpha$

We have (for rolling without slipping) $a_1 = R\alpha$

Solving these equations , we get $a_2 = \dfrac{8mg}{3M + 8m}$

P-Q1302

Density of ice is $\mathbf{900}\boldsymbol{kg}/\boldsymbol{m^3}$. A piece of ice is floating in water of density $\mathbf{1000}\boldsymbol{kg}/\boldsymbol{m^3}$.

Find the fraction of the piece of ice outside the water

(A) 0.9 (B) 0.5 (C) 0 (D) 0.1

$R.D = \frac{900}{1000} = 0.9$

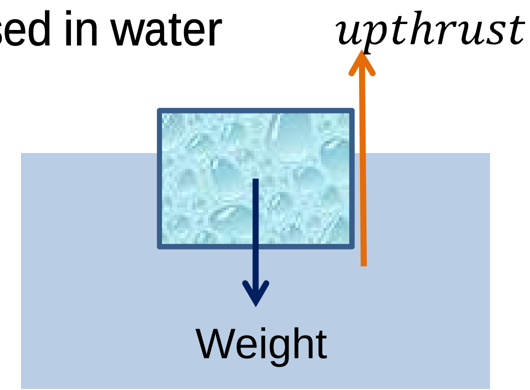
$V \times 900 \times g = (n-1) V \times 1000 \cdot g$

P-Q1302-Solution

Ans [D]

let V be the total volume and V_i the volume of ice piece immersed in water

for equilibrium of ice piece



$$weight = upthrust$$

Force balance at equilibrium

$$V\rho_i g = V_i \rho_w g$$

Upthurst due to water

$$\rho_i = 900 kg/m^3$$

$$\rho_w = 1000 kg/m^3$$

$$\frac{V_i}{V} = \frac{900}{1000} = 0.9$$

$$f = 1 - 0.9 = 0.1$$

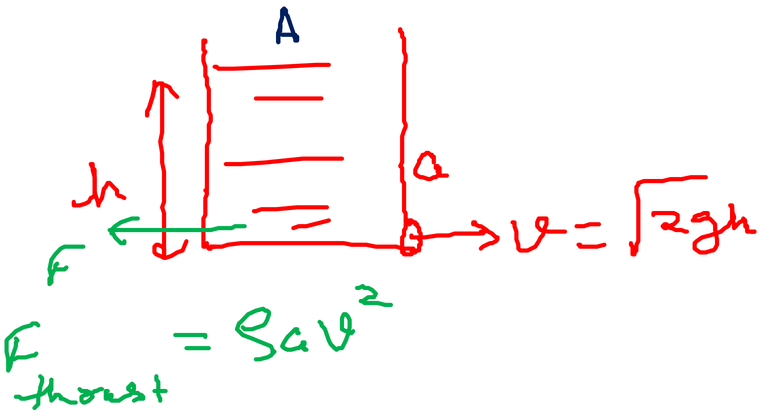
Fraction of piece outside

P-Q1329

A light cylindrical vessel is kept on a horizontal surface. Its base area is A. A hole of cross sectional area a is made just at its bottom side. The minimum coefficient of friction necessary for sliding of the vessel due to the impact force of the emerging liquid is (a<< A)

(A) varying (B) a/A

✓(C) 2a/A (D) None of these



$$\rho a v^2 = \mu m g$$

$$\rho a \cdot 2gh = \mu \cdot A h \cdot \rho g$$

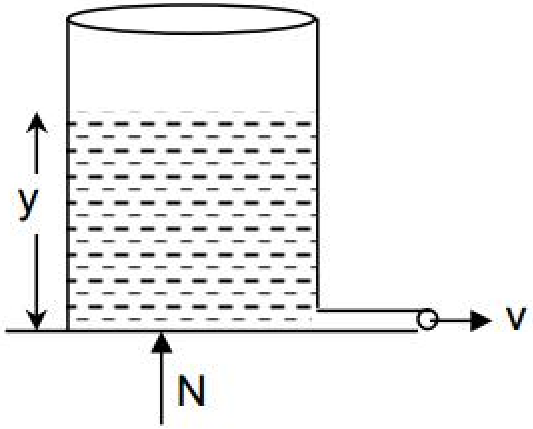
$$\boxed{\mu = \frac{2a}{A}}$$

P-Q1329-Solution

Ans [C]

The velocity of efflux of the liquid is given by v

$$v = \sqrt{2gy}$$



The impact force on the emerging liquid on the vessel +liquid content is equal to

$$F = v\frac{dm}{dt} = va\rho v = a\rho v^2$$

$$\Rightarrow F = a\rho\,(\sqrt{2gy})^2 = 2a\rho gy$$

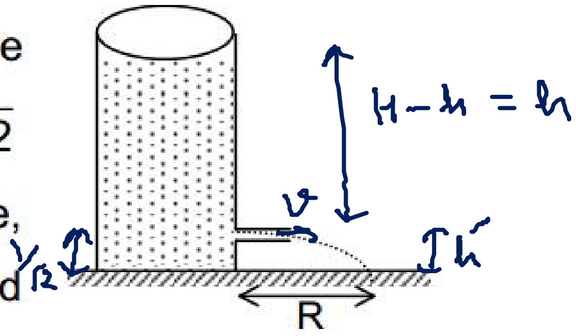
The force of friction $= f = F = 2a\rho gy$

$$\Rightarrow \mu N = 2a\rho gy \quad \Rightarrow \mu\,(A\rho gy) = 2a\,\rho gy$$

$$\Rightarrow \mu = \frac{2a}{A}.$$

P-Q1333

A small hole is made at a height of h′ = (1/$\sqrt{2}$) m from the bottom of a cylindrical water tank and at a depth of h = $\sqrt{2}$ m from the upper level of water in the tank. The distance, where the water emerging from the hole strikes the ground is:



(A) 2√2m

(B) 1 m

(C) 2 m

(D) None of these.

$$t = \sqrt{\frac{2h'}{g}} = \sqrt{\frac{2}{\sqrt{2}g}}$$

$$v = \sqrt{2gh} = \sqrt{2g\sqrt{2}}$$

$$R = vt = \sqrt{2\sqrt{2}g}\ \sqrt{\frac{2}{\sqrt{2}g}}$$

$$= 2m$$

P-Q1333-Solution

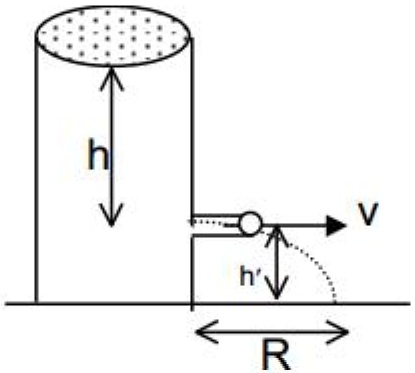
Ans [C]

$v = \sqrt{2gh}$. The range $R = v_2 \times t$,

where t = time of fall can be given by

$h' = 1/2\, gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h'}{g}}$

Using 2nd eq. of motion and putting $u = 0$



$\Rightarrow \; R = v\sqrt{\frac{2h'}{g}}$; putting $v = \sqrt{2gh}$, we obtain $R = 2\sqrt{hh'}$

Putting $h' = \frac{1}{\sqrt{2}}$ m and $h = \sqrt{2}$ m, we obtain R = 2 m.

P-Q13105

The velocity of kerosene oil in a horizontal pipe is 5 m/s. If $g = 10 m / s^2$ then the velocity head of oil will be

(A) 1.25 m

(B) 12.5 m

(C) 0.125 m

(D) 125 m

$$\frac{v^2}{2g} = \frac{25}{2 \times 10} = 1.25 m$$

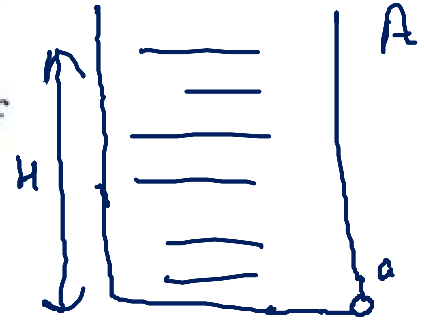
P-Q13105-Solution

Ans [A]

Velocity head $h = \frac{v^2}{2g} = \frac{(5)^2}{2 \times 10} = 1.25\ m$

$\because v = \sqrt{2gh}$

A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to $\frac{H}{\eta}(\eta > 1)$; and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is

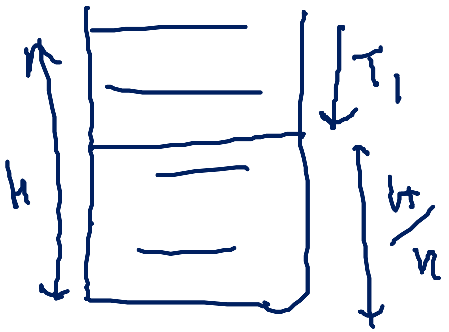


(A) 2 (B) 3

(C) 4 (D) $2\sqrt{2}$

For vessel to become empty

$$t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$



$$T_1 = \frac{A}{a}\left(\sqrt{\frac{2H}{g}} - \sqrt{\frac{2H}{g\eta}}\right)$$

$$T_2 = \frac{A}{a}\sqrt{\frac{2}{g}\frac{H}{\eta}} = \frac{A}{a}\sqrt{\frac{2H}{g}}\cdot\frac{1}{\sqrt{\eta}}$$

$$T_1 = T_2 \Rightarrow \frac{A}{a}\sqrt{\frac{2H}{g}}\left(1 - \sqrt{\frac{1}{\eta}}\right)$$

$$1 = \frac{2}{\sqrt{\eta}} \Rightarrow \eta = 4$$

Ans [C]

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$$

Using efflux speed relation

The velocity gained after fall from *h* height in air will be , $v = \sqrt{2gh}$

$$\text{Now, } T_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right] \quad (1)$$

$$\text{and } T_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - \sqrt{0}\right] \quad (2)$$

According to problem $T_1 = T_2$

Use equation 1 and 2

$$\therefore \sqrt{H} - \sqrt{\frac{H}{\eta}} = \sqrt{\frac{H}{\eta}} - 0 \Rightarrow \sqrt{H} = 2\sqrt{\frac{H}{\eta}} \Rightarrow \eta = 4$$

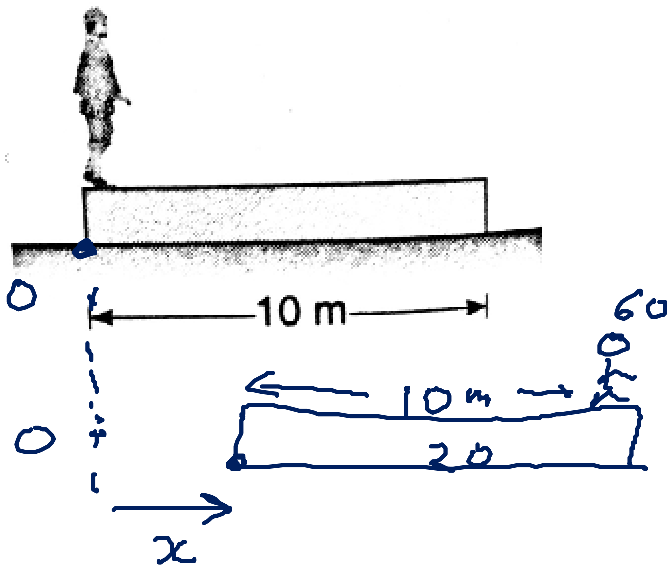
A wooden plank of mass **20kg** is resting on a smooth horizontal floor . A man of mass **60kg** starts moving from one end of the plank to the order end . The length of plank is **10m** . Find the displacement of the plank over the floor when the man reaches the other end of the plank

A) 1.25m

B) 4.75m

C) 10m

D) 7.5m ✓



$$\Delta x_{cm} = 0$$

$$m_1 \Delta x_1 + m_2 \Delta x_2 = 0$$

$$60(x + 10 - 0) + 20(x - 0) = 0$$

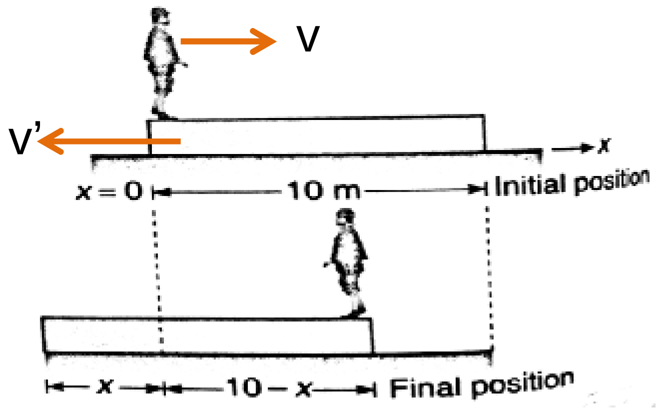
$$3x + 30 + x = 0$$

$$4x = -30$$

$$x = -7.5\,m$$

P-Q708-Solution

Ans [D]



$$x_i = x_f$$

$$\frac{(60)(0)+20\left(\frac{10}{2}\right)}{60+20} = \frac{(60)(10-x)+20\left(\frac{10}{2}-x\right)}{60+20}$$

$$\frac{5}{4} = \frac{6(10-x)+2\left(\frac{10}{2}-x\right)}{8} = \frac{60-6x+10-2x}{8}$$

COM is not moving so COM also same as initial and final position of man

$$5 = 30 - 3x + 5 - x$$

$$4x = 30$$

$$x = \frac{30}{4}\ \text{m}$$

$$x = 7.5\ \text{m}$$

Centre of mass of plank lies at its centre

We can also solve by conservation of linear momentum

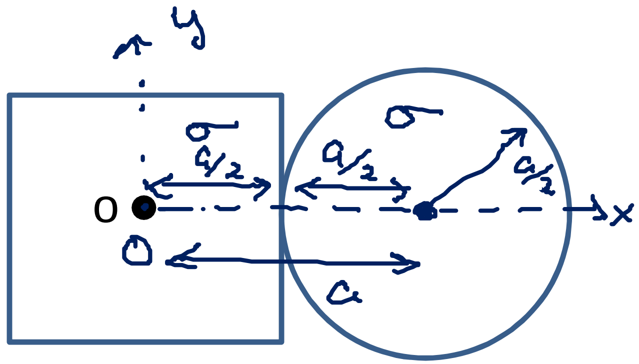
$$m(v - v') - Mv' = 0 \quad (1)$$

$$v = \frac{10}{t}$$

$$x_{plank} = t \times v'_{plank}$$

P-Q735

A square lamina of side a and a circular lamina of diameter a are placed touching each other as shown in figure. Find distance of their Centre of mass from point O, the Centre of square.



(A) $\frac{\pi}{\pi+4}a$ ✓

(B) $\frac{2\pi}{\pi+2}a$

(C) $\frac{2\pi}{\pi+4}a$

(D) $\frac{\pi}{\pi+2}a$

$$x_{cm} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$x_{cm} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{0 + \frac{\pi a^2}{4}\cdot a}{a^2 + \frac{\pi a^2}{4}} = \frac{\pi a^3}{4}\cdot\frac{4}{(4+\pi)a^2} = \frac{\pi a}{\pi + 4}$$

P-Q735-Solution

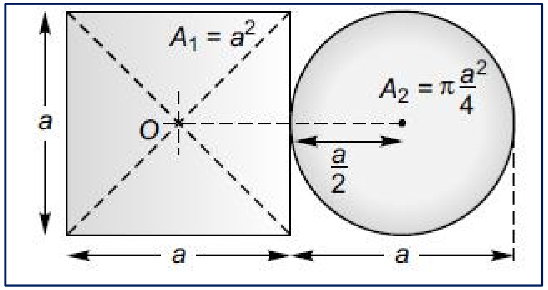
Ans [A]

$$x_{CM} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

This formula will be used for calculating the centre of mass

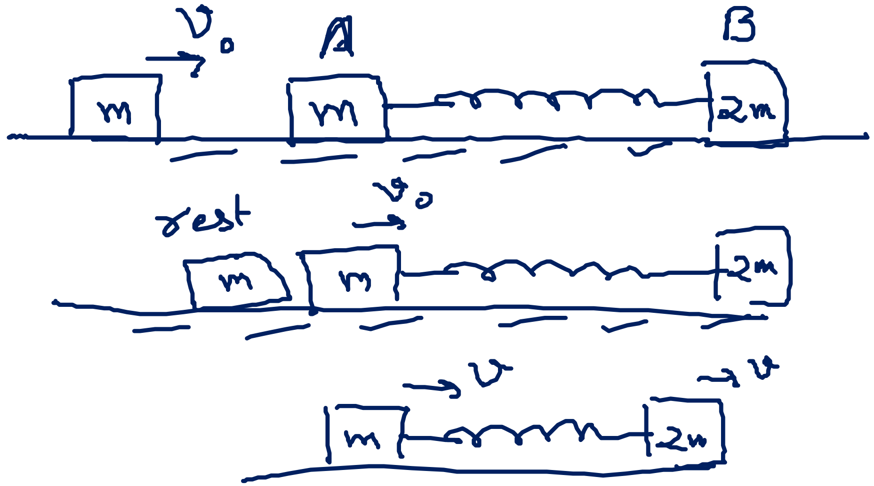
$$= \frac{a^2 \cdot 0 + \frac{\pi a^2}{4} \cdot (a)}{a^2 + \frac{\pi a^2}{4}}$$

$$= \frac{\pi}{4 + \pi} a$$



Q 32

Two blocks A and B of masses m and 2m respectively are placed on a smooth floor. They are connected by a spring. A third block C of mass m moves with a velocity v_0 along the line joining A and B and collides elastically with A, as shown in figure. At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further, at this instant the compression of the spring is found to be x_0. Find the common velocity of A and B at time t_0.



$\vec{P}$/c

$$3mv = mv_0$$

$$v = v_0/3$$

Sol 32

Ans [A]

An external force on the system is zero the law of conservation of linear momentum gives

$$mv_0 = mv + (2m)v$$

$$v = \frac{v_0}{3}$$

The law of conservation of energy gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 + \frac{1}{2}kx_0^2$$

$$\frac{1}{2}mv_0^2 = \frac{3}{2}mv^2 + \frac{1}{2}kx_0^2$$

$$\frac{1}{2}mv_0^2 = \frac{3}{2}m\left(\frac{v_0}{3}\right)^2 + \frac{1}{2}kx_0^2$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2 - \frac{1}{6}mv_0^2$$

$$\frac{1}{2}kx_0^2 = \frac{1}{3}mv_0^2$$

$$k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$$

P-Q728

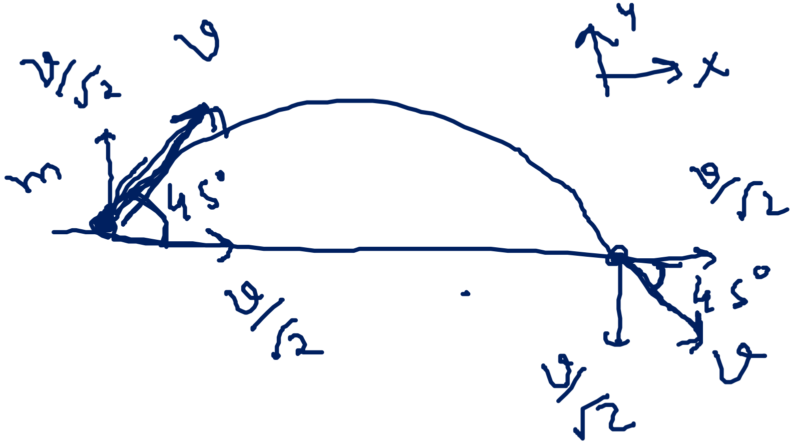
A projectile of mass m is fired with a velocity v from point P at an angle 45°. Neglecting air resistance, the magnitude of the change in momentum leaving the point P and arriving at Q is

(A) $mv\sqrt{2}$

(B) $2mv$

(C) $\frac{mv}{2}$

(D) $\frac{mv}{\sqrt{2}}$



$$\vec{p}_i = m\left(\frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}\right)$$

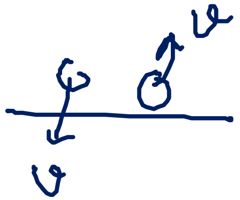
$$\vec{p}_f = m\left(\frac{v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}\right)$$

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$= 2m \cdot \frac{v}{\sqrt{2}}(-\hat{j})$$

$$= \sqrt{2}mv(-\hat{j})$$

$$|\Delta\vec{p}| = \sqrt{2}mv$$

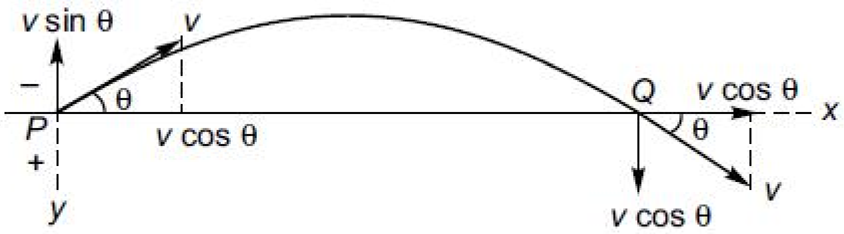


$|\Delta p| = 2mv$

P-Q728-Solution

Ans [A]

Change in momentum along *x*-axis



$$= m\,(v\cos\theta - v\cos\theta) = 0$$

So the momentum along x-axis will be constant

∴ Net change in momentum= Change in momentum along *y*-axis

$$= m[(+v\sin\theta) - (-v\sin\theta)]$$
$$= 2\,mv\sin\theta$$
$$= mv\sqrt{2}$$