Problems Solving on Integral Calculus

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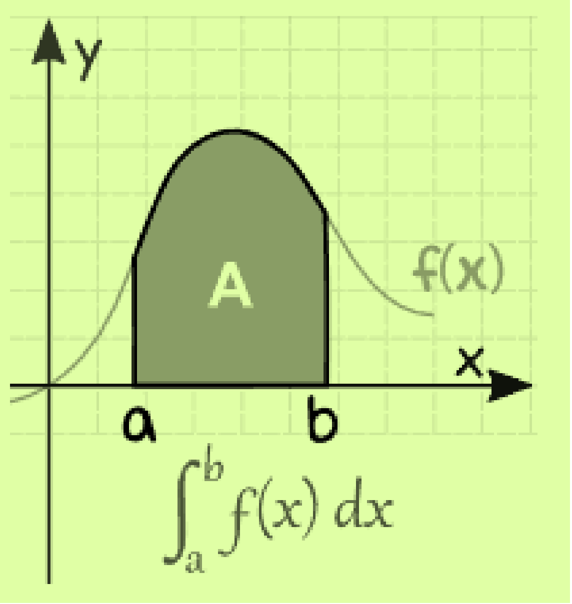

Q. The value of $\displaystyle\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)}\,dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$

(B) $\frac{1}{2}\ln\frac{3}{2}$

(C) $\ln\frac{3}{2}$

(D) $\frac{1}{6}\ln\frac{3}{2}$

$$I = \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2\,dx}{\sin x^2 + \sin(\ln 6 - x^2)}$$

$$\Rightarrow x^2 = t$$

$$2x\,dx = dt$$

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t\,dt}{\sin t + \sin(\ln 6 - t)}$$

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t}\,dt$$

$$2I = \frac{1}{2}\int_{\ln 2}^{\ln 3} dt$$

$$2I = \frac{1}{2}[\ln 3 - \ln 2]$$

$$2I = \frac{1}{2}\ln\left(\frac{3}{2}\right)$$

$$\underline{I = \frac{1}{4}\ln\left(\frac{3}{2}\right)}$$

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Q. $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\ldots..+\frac{1}{n}\right]$ is equal to

(A) $\frac{\pi}{4}+\frac{1}{2}\log 2$

(B) $\frac{\pi}{4}-\frac{1}{2}\log 2$

(C) $\frac{\pi}{4}-2\log\frac{1}{2}$

(D) None of these

$$L=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n+r}{n^2+r^2}$$

$$\frac{n+r}{n^2\left[1+\frac{r^2}{n^2}\right]}$$

$$\frac{1}{n}\frac{\left[1+r/n\right]}{1+\left(\frac{r}{n}\right)^2}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1+(r/n)}{1+(r/n)^2}$$

$$L=\int_0^1\frac{1+x}{1+x^2}\,dx$$

$$L=\int_0^1\frac{1}{1+x^2}dx+\int_0^1\frac{x}{1+x^2}dx$$

$1+x^2=t$

$2x\,dx=dt$

$$=\tan^{-1}x\Big|_0^1+\frac{1}{2}\int_1^2\frac{dt}{t}$$

$$=\left(\frac{\pi}{4}\right)+\frac{1}{2}\log t\Big|_1^2$$

$$\boxed{L=\frac{\pi}{4}+\frac{1}{2}\log 2}$$

Q. Let $f(x) = \int_{2}^{x} \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then the value of $g'(0)$ is

(A) 1

(B) 17

(C) $\sqrt{17}$

(D) None of thes

$f'(x) = 1\cdot\left(\frac{1}{\sqrt{1+x^4}}\right) - 0$

$f(x) = \int_{2}^{x} \frac{dt}{\sqrt{1+t^4}}$

$g(x) = f^{-1}(x)$

$f^{-1}(f(x)) = x$

$\boxed{g(f(x)) = x}$

$g'(f(x))\cdot f'(x) = 1$

at $x=2$, $f(x)=0$

$\boxed{g'(f(x)) = \frac{1}{f'(x)}} \leftarrow x=2$

$f'(x) = \frac{1}{\sqrt{1+x^4}}$

$g'(0) = \frac{1}{f'(2)}$

$f'(2) = \frac{1}{\sqrt{17}}$

$\boxed{g'(0) = \sqrt{17}}$

Q. $\int e^{\tan\theta}(\sec\theta - \sin\theta)d\theta$ equals

(A) $-e^{\tan\theta}\sin\theta + C$ (B) $e^{\tan\theta}\sin\theta + c$

(C) $e^{\tan\theta}\sec\theta + c$ (D) $e^{\tan\theta}\cos\theta + c$

*** Can do by option chaking

$\int e^{\tan\theta}(\sec\theta - \sin\theta)$

Applying by-parts

$\int e^{\tan\theta}\sec\theta\, d\theta - \int e^{\tan\theta}\sin\theta\, d\theta$

$\int e^{\tan\theta}\sec\theta\, d\theta - \left[e^{\tan\theta}(-\cos\theta) - \int e^{\tan\theta}\sec^2\theta\,(\cancel{-\cos\theta})\,d\theta \right]$

$\cancel{\int e^{\tan\theta}\sec\theta\, d\theta} + e^{\tan\theta}\cos\theta \cancel{- \int e^{\tan\theta}\sec\theta\, d\theta}$

$= \underline{e^{\tan\theta}\cos\theta + C}$

The value of $\int \frac{x^2-1}{x^4+1}\,dx$ equals -

(A) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)+C$

(B) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)+C$

(C) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+C$

(D) None of these

$$I = \int \frac{x^2-1}{x^4+1}$$

$$I = \int \frac{x^2-1}{x^2\left(x^2+\frac{1}{x^2}\right)}$$

$$I = \int \frac{\left(1-1/x^2\right)dx}{x^2+\frac{1}{x^2}}$$

$$x+1/x = t \implies$$

$$\left(1-\frac{1}{x^2}\right)dx = dt$$

$$I = \int \frac{dt}{t^2-2}$$

$$= \frac{1}{2\sqrt{2}}\log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|$$

$$= \frac{1}{2\sqrt{2}}\log\left(\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right)$$

$$= \frac{1}{2\sqrt{2}}\log\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)$$

If $\int \frac{dx}{\sqrt{2-3x-x^2}} = fog(x) + C$, then Find g(x) and f(x)

(A) $\frac{2x+3}{\sqrt{17}}$, [redacted] $-\cos^{-1}x$

(B) $\sin^{-1}x, \frac{2x+3}{\sqrt{17}}$

$\frac{\pi}{2} - \cos^{-1}\left(\frac{2x+3}{\sqrt{17}}\right) + C$

(C) $\frac{2x+3}{\sqrt{17}}, \sec^{-1}x$

(D) $\frac{2x+3}{\sqrt{17}}, \operatorname{cosec}^{-1}x$

$$I = \int \frac{dx}{\sqrt{2-3x-x^2}}$$

$$= \int \frac{dx}{\sqrt{2-(x^2+3x)}}$$

$$= \int \frac{dx}{\sqrt{2-\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]}}$$

$$= \int \frac{dx}{\sqrt{2-\left(x+\frac{3}{2}\right)^2+\frac{9}{4}}}$$

$$I = \int \frac{dx}{\sqrt{\frac{17}{4}-\left(x+\frac{3}{2}\right)^2}}$$

$$I = \int \frac{dx}{\sqrt{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x+\frac{3}{2}\right)^2}}$$

$$I = \sin^{-1}\left(\frac{x+3/2}{\frac{\sqrt{17}}{2}}\right) + C$$

$$I = \sin^{-1}\left(\frac{2x+3}{\sqrt{17}}\right) + C$$

Q. The area bounded in the first quadrant by the nornal at $(1,2)$ on the the curve $y^2 = 4x$, x-axis & the curve is given by.

(A) $\frac{10}{3}$

(B) $\frac{7}{3}$

(C) $\frac{4}{3}$

(D) $\frac{9}{2}$

$y^2 = 4x$

$2yy' = 4$

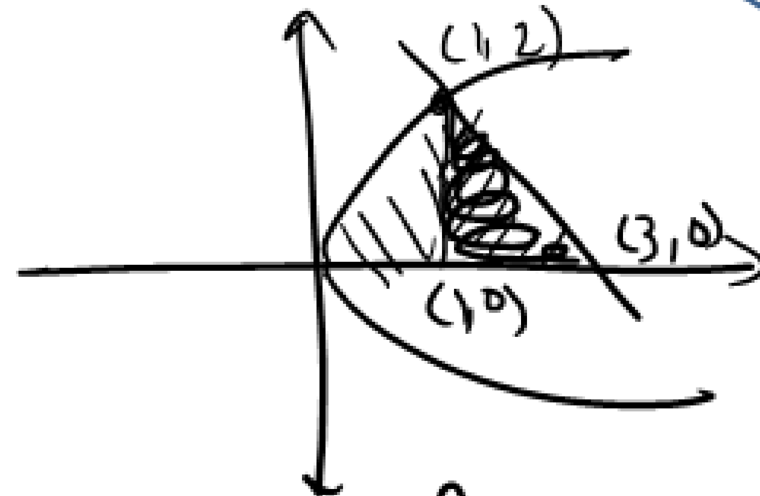
$y' = \frac{4}{2y}\Big|_{(1,2)}$

$y' = 1 \quad (m_T = 1)$

$\boxed{m_N = -1}$

$(y-2) = -1(x-1)$

$\boxed{y+x=3}$



$A_T = \frac{1}{2}\times 2\times 2$

$= 2$ sq. unit

$A_C = \int_0^1 2\sqrt{x}\,dx$

$= \frac{2x^{3/2}2}{3}$

$= 4/3$

$\frac{4}{3}+2$

$\frac{10}{3}$

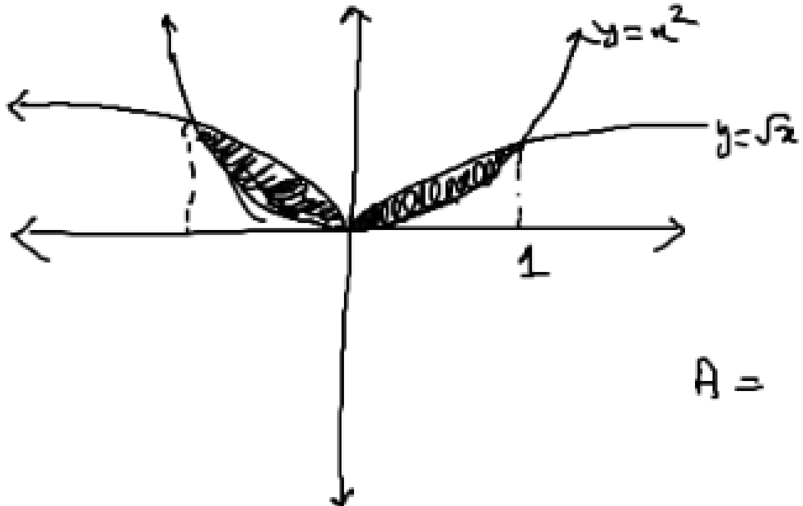
Q. The area of the region(s) enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{6}$

(D) 1



$y = \sqrt{x} \quad x>0$

$y = \sqrt{-x} \quad x<0$

$\int x^{1/2} dx$

$A = 2\int_0^1 (\sqrt{x} - x^2)\, dx$

$A = 2\left[\frac{2x^{3/2}}{3} - \frac{x^3}{3}\right]_0^1$

$= 2\left[\frac{2}{3} - \frac{1}{3}\right]$

$\boxed{A = 2/3}$