MATHEMATICS (ASSIGNMENT-3)
TOPIC- INDEFINITE INTEGRATION SOLUTION

1. **(c)**

We have,

$$I = \int \frac{e^{2x} - 2e^x}{e^{2x}+1} dx$$

$$\Rightarrow I = \int \frac{e^{2x}}{e^{2x}+1} dx - \int \frac{2e^x}{e^{2x}+1} dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{1}{e^{2x}+1} d(e^{2x}+1) - 2\int \frac{1}{(e^x)^2 + 1^2} d(e^x)$$

$$\Rightarrow I = \frac{1}{2}\log(e^{2x}+1) - 2\tan^{-1}(e^x) + C$$

2. **(b)**

$$\int \sqrt{1+\sin\left(\frac{x}{4}\right)}\ dx = \int \sqrt{1 + 2\ \sin\frac{x}{8}\cos\frac{x}{8}} dx$$

$$= \int \sqrt{\sin^2\frac{x}{8} + \cos^2\frac{x}{8} + 2\sin\frac{x}{8}\cos\frac{x}{8}}\ dx$$

$$= \int \sqrt{\left(\sin\frac{x}{8} + \cos\frac{x}{8}\right)^2}\ dx$$

$$= \int \left(\sin\frac{x}{8} + \cos\frac{x}{8}\right)\ dx$$

$$= \frac{-\cos\frac{x}{8}}{1/8} + \frac{\sin\frac{x}{8}}{1/8} + c$$

$$= 8\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$$

3. **(a)**

We have,

$$I = \int \frac{3^x}{\sqrt{1-9^x}} dx = \frac{1}{\log_e 3}\int \frac{1}{\sqrt{1^2 - (3^x)^2}} d(3^x)$$

$$= (\log_3 e)\sin^{-1}(3^x) + C$$

4. **(c)**

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore \int \frac{dx}{x\log x} = \int \frac{dt}{t} = \log t + \text{constant}$

$= \log(\log x) + \text{constant} = f(x) + \text{constant}$

$\therefore f(x) = \log(\log x)$

5. **(c)**

Let $I = \int \frac{x^2+1}{x^2-1}\, dx$

$$\Rightarrow I = \int \frac{x^2+1-1+1}{x^2-1}\, dx$$

$$\Rightarrow I = \int \frac{x^2-1}{x^2-1}\, dx + \int \frac{2}{x^2-1}\, dx$$

$$\Rightarrow I = \int 1\, dx + 2\int \frac{1}{x^2-1}\, dx$$

$$\Rightarrow I = x + 2\,\frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + c$$

$$\Rightarrow I = x + \log\left(\frac{x-1}{x+1}\right) + c$$

6. **(a)**

$$I = \int \frac{x+\sin x}{1+\cos x}dx = \int \frac{x}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}dx$$

$$\Rightarrow I = \int x\frac{1}{2}\sec^2\frac{x}{2}dx + \int \tan\frac{x}{2}dx$$

$$\Rightarrow I = x\tan\frac{x}{2} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx + C$$

$$= x\tan\frac{x}{2} + C$$

7. **(c)**

Let $I = \int \frac{dx}{1-\cos x - \sin x}$

Put $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore I = \int \frac{dx}{1 - \frac{\left(1-\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)} - \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$= \int \frac{\sec^2\frac{x}{2}dx}{\left[1+\tan^2\frac{x}{2} - 1 + \tan^2\frac{x}{2} - 2\tan\frac{x}{2}\right]}$$

$$= \int \frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2} - 2\tan\frac{x}{2}} = \int \frac{\frac{1}{2}\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2} - \tan\frac{x}{2}}$$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$\therefore I = \int \frac{dt}{t^2 - t} = \int \frac{dt}{(t-1)}$$

$$= \int \left[\frac{1}{t-1} - \frac{1}{t}\right] dt = \int \frac{dt}{t-1} - \int \frac{dt}{t}$$

$$= \log(t-1) - \log t + c = \log\left|\frac{t-1}{t}\right| + c$$

$$= \log\left|\frac{\tan\frac{x}{2} - 1}{\tan\frac{x}{2}}\right| + c = \log\left|1 - \cot\frac{x}{2}\right| + c$$

8. **(a)**

$$\int f(x)g(x)dx = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\, e^{\sin^{-1}x}\, dx$$

Put $\sin^{-1}x = t \quad \Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$$\therefore \int f(x)\, g(x)dx = \int te^t\, dt = te^t - e^t + c$$

$$= e^{\sin^{-1}x}(\sin^{-1}x - 1) + c$$

9. **(d)**

Given, $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$

Let $1 + x^3 = t \Rightarrow 3x^2dx = dt$

$$\therefore I = \int \frac{(t-1)}{\sqrt{t}} . \frac{dt}{3} = \frac{1}{3}\int \left(\sqrt{t} - t^{-1/2}\right) dt$$

$$= \frac{1}{3}\left[\frac{2t^{3/2}}{3} - 2t^{1/2}\right] + c$$

$$= \frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + c$$

10. **(c)**

Let $I = \int \frac{\sin\theta+\cos\theta}{\sqrt{1+\sin 2\theta-1}} d\theta$

$$= \int \frac{\sin\theta + \cos\theta}{\sqrt{1-(\sin\theta-\cos\theta)^2}} d\theta$$

Put $\sin\theta - \cos\theta = t$

$\Rightarrow (\cos\theta + \sin\theta)d\theta = dt$

$$\therefore\ I = \int \frac{1}{\sqrt{1-t^2}} dt$$

$= \sin^{-1} t + c$

$= \sin^{-1}(\sin\theta - \cos\theta) + c$

11. **(c)**

$$\int \operatorname{cosec}^4 x\, dx = \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x\, dx$$

$$= \int \operatorname{cosec}^2 x(1+\cot^2 x)dx$$

$$= \int \operatorname{cosec}^2 x\, dx + \int \cot^2 x \cdot \operatorname{cosec}^2 x\, dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

12. **(d)**

Let $I = \int e^{\tan^{-1} x} dx + \int e^{\tan^{-1} x} \cdot \frac{x}{(1+x^2)}\, dx$

$$= \int \frac{d}{dx}(x\, e^{\tan^{-1} x})dx + c$$

$= xe^{\tan^{-1} x} + c$

13. **(b)**

Let $I = \int(\sin x - \cos x)^4(\sin x + \cos x)dx$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$

$$\therefore\ I = \int t^4\ dt = \frac{t^5}{5} + c = \frac{(\sin x - \cos x)^5}{5} + c$$

14. **(a)**

Let $\sin x = z \Rightarrow d\,(\sin x) = dz$

$$\therefore \int \frac{dz}{\sqrt{1-z^2}} = \sin^{-1} z + c$$

$$= \sin^{-1}(\sin x) + c = x + c$$

15. **(b)**

Let $I = \int \frac{dx}{2\sqrt{x}(1+x)}$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\therefore I = \int \frac{dt}{1+t^2} = \tan^{-1} t + c = \tan^{-1}\sqrt{x} + c$$

16. **(a)**

$$\int \frac{\cos x - 1}{\sin x + 1} e^x\, dx$$

$$= \int \frac{\cos x}{1+\sin x} e^x\, dx - \int \frac{1}{1+\sin x} e^x\, dx$$

$$= \frac{(\cos x)e^x}{1+\sin x} + \int \frac{-(1+\sin x)\sin x - \cos^2 x}{(1+\sin x)^2} e^x dx$$

$$- \int \frac{e^x}{\sin x + 1} dx + c$$

$$= \frac{e^x \cos x}{1+\sin x} + \int \frac{1}{1+\sin x} e^x dx$$

$$- \int \frac{e^x}{1+\sin x} dx + c$$

$$= \frac{e^x \cos x}{1+\sin x} + c$$

17. **(b)**

LHS $= \int (\log x)^2\, dx$

$= x\,(\log x)^2 - \int x\,.\,2\log x \frac{1}{x}\, dx$

$= x\,(\log x)^2 - 2\left[x\log x - \int x\,.\frac{1}{x}dx\right] + c$

$= x\,(\log x)^2 - 2[x\log x - x]$

$= x\,(\log x)^2 - 2x[\log x - 1] + c$

But RHS is given by

$x[f(x)]^2 + Ax[f(x) - 1] + c$

$\therefore f(x) = \log x$ and $A = -2$

18. **(b)**

We have,

$$I = \int \frac{1+x^2}{1+x^4}dx = \int \frac{(1+1/x^2)}{x^2+(1/x^2)}dx$$

$$\Rightarrow I = \int \frac{1+(1/x^2)}{\left(x-\frac{1}{x^2}\right)^2 + C}dx$$

$$= \int \frac{1}{\left(x-\frac{1}{x}\right)^2 + (\sqrt{2})^2} d\left(x - \frac{1}{x}\right)$$

$$\Rightarrow I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$$

19. **(b)**

$\int \cos^4 x\, dx = \frac{1}{4}(2\cos^2 x)^2\, dx$

$= \frac{1}{4}\int (1+\cos 2x)^2\, dx$

$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)dx$

$= \frac{1}{8}\int \{2 + 4\cos 2x + (1+\cos 4x)\}dx$

$= \frac{1}{8}\left\{3x + 2\sin 2x + \frac{1}{4}\sin 4x\right\} + D$

$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + D$

On comparing, we get

$A = \frac{3}{8}, B = \frac{1}{4}, C = \frac{1}{32}$

20. **(c)**

Let $I = \int (1 - \cos^2 x) \cos^2 x . \sin x \ dx$

Put $\cos x = t$

$\Rightarrow -\sin x \ dx = dt$

$\therefore \ I = -\int (1 - t^2) \, t^2 dt = \int (t^4 - t^2) \, dt$

$= \frac{t^5}{5} - \frac{t^3}{3} + c$

$= \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c$