

# Problems Solving on Differential Calculus

By  
Ankush Garg(B. Tech, IIT Jodhpur)

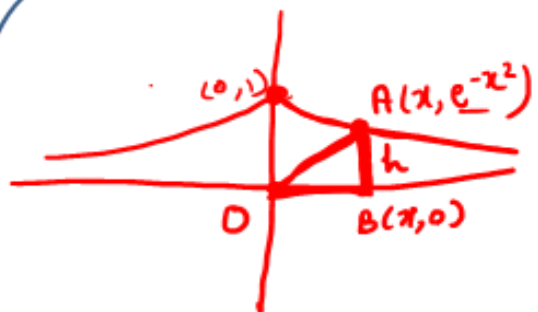
Q. Point 'A' lies on the curve  $y = e^{-x^2}$  and has the coordinate  $(x, e^{-x^2})$  where  $x > 0$ . Point B has the coordinates  $(x, 0)$ . If 'O' is the origin then the maximum area of the triangle AOB is.

(A)  $\frac{1}{\sqrt{2}e}$

(B)  $\frac{1}{\sqrt{4}e}$

(C)  $\frac{1}{\sqrt{e}}$

(D)  $\frac{1}{\sqrt{8}e}$



$$A = \frac{1}{2} \times B \times H$$

$$A = \frac{1}{2} \times x \times e^{-x^2}$$

$$\frac{dA}{dx} = \frac{1}{2} [e^{-x^2} + x e^{-x^2} (-2x)]$$

$$0 = \frac{1}{2} e^{-x^2} [1 - 2x^2]$$

$$1 - 2x^2 = 0$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

$x = \frac{1}{\sqrt{2}}$  ✓

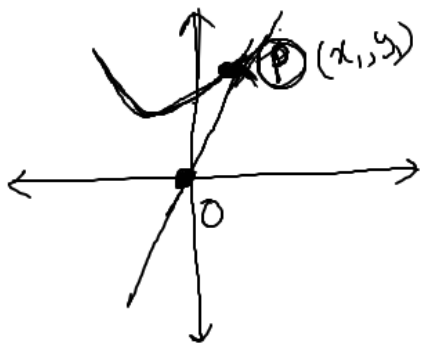
$$A = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times e^{-1/2}$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{1}{\sqrt{e}}$$

$A = \frac{1}{\sqrt{8}e}$

Q. The coordinates of the point on the curve  $y = x^2 + 3x + 4$  the tangent at which passes through the origin is equal to

- ~~(A) (2, 14), (-2, 2)~~  
 (B) (2, 14), (-2, -2)  
 (C) (2, 14), (2, 2)  
 (D) None of these



Mouse
Select
T
Draw
Stamp
Spotlight
Eraser
Format
Undo
Redo
Clear
Save

$$y = x^2 + 3x + 4$$

$$\left. \frac{dy}{dx} \right|_{(x_1, y_1)} = 2x_1 + 3$$

Eq<sup>n</sup> of Tangent

$$y - y_1 = (2x_1 + 3)(x - x_1)$$

(0,0)

$$0 - y_1 = (2x_1 + 3)(-x_1)$$

$$- [x_1^2 + 3x_1 + 4] = -2x_1^2 - 3x_1$$

$$x_1^2 - 4 = 0$$

$$x_1 = \pm 2$$


$$P(2, 2^2 + 3 \times 2 + 4)$$

$$P(2, 14), Q(-2, (-2)^2 + 3(-2) + 4)$$

$$Q(-2, 2)$$

Q. If  $f$  and  $g$  are two increasing functions such that  $g \circ f$  is defined, then

- ~~(A)  $g \circ f$  is an increasing function~~  
 (B)  $g \circ f$  is a decreasing function  
 (C)  $g \circ f$  is neither increasing nor decreasing  
 (D) None of these



$f \uparrow$   
 $x_2 > x_1$   
 $f(x_2) > f(x_1)$  [ + is  $\uparrow$  ]  
 $g(f(x_2)) > g(f(x_1))$  [  $g$  is  $\uparrow$  ]  
 $g \circ f(x_2) > g \circ f(x_1)$   $\uparrow$

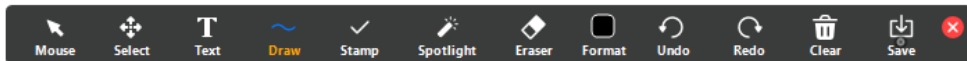
Q. The equation  $\sin x + x \cos x = 0$  has at least one root in Rolle's

(A)  $\left(-\frac{\pi}{2}, 0\right)$

~~(B)  $(0, \pi)$~~

(C)  $\left(\pi, \frac{3\pi}{2}\right)$

(D)  $\left(0, \frac{\pi}{2}\right)$



Handwritten notes illustrating the application of Rolle's Theorem to the equation  $\sin x + x \cos x = 0$ .

The equation is written as  $\sin x + x \cos x = 0$ .

The derivative is calculated as  $\frac{d}{dx} (x \sin x)$ .

The function is defined as  $f(x) = x \sin x$ .

The condition for Rolle's Theorem is stated as  $0 = f(a) = f(b)$ .

The interval  $[a, b]$  is indicated.

The equation  $\sin x + x \cos x = 0$  is written again.

The function  $x \sin x$  is written in a circle.

Two graphs are shown: one with a curve above the x-axis and one with a curve below the x-axis, both labeled  $f(x) = 0$  at the endpoints  $a$  and  $b$ .

Two conditions are listed: (1) Continuous ✓ and (2) diff ✓.

The condition  $f(a) = f(b)$  is written and underlined.

Q. Let  $l = \lim_{x \rightarrow 2} \frac{\{x\}}{[x]}$  and  $m = \lim_{x \rightarrow 2} \frac{[x]}{\{x\}}$

Where  $[y]$  and  $\{x\}$  denotes largest integer less than or equal to  $y$  and frac  $y$  respectively.

- (A)  $l$  exist but  $m$  does not
- (B)  $m$  exist but  $l$  does not
- (C) Both  $l$  and  $m$  exist
- (D) Neither  $l$  nor  $m$  exist

$\{x\}, [x]$

$R.H.L = L.H.L = \text{finite}$

$\{x\} = x - [x]$

$l = \lim_{x \rightarrow 2} \frac{\{x\}}{[x]}$

$L.H.L \lim_{x \rightarrow 2^-} \frac{1}{x - [x]}$

$\lim_{x \rightarrow 2^-} \frac{1}{x - 1}$

$L.H.L = 1$

$R.H.L = \lim_{x \rightarrow 2^+} \frac{[x]}{x - [x]}$

$= \lim_{x \rightarrow 2^+} \frac{2}{x - 2}$

$R.H.L = \infty$

$l$  not exist

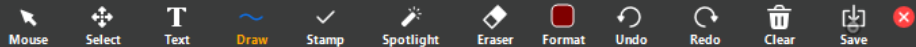


For  $m$   
 $L.H.L = 1$   
 $R.H.L = 0$

$m$  does not exist



Q. Let  $l = \lim_{x \rightarrow 2} \frac{[x]}{\{x\}}$  and  $m = \lim_{x \rightarrow 2} \frac{\{x\}}{[x]}$



Where  $[y]$  and  $\{x\}$  denotes largest integer less than or equal to  $y$  and fractional part of  $y$  respectively.

- (A)  $l$  exist but  $m$  does not  
 (B)  $m$  exist but  $l$  does not  
 (C) Both  $l$  and  $m$  exist  
 (D) Neither  $l$  nor  $m$  exist

$\{x\}, [x]$

$R.H.L = L.H.L = \text{finite}$

$\{x\} = x - [x]$

$l = \lim_{x \rightarrow 2} \frac{[x]}{\{x\}}$

L.H.L  $\lim_{x \rightarrow 2^-} \frac{1}{x - [x]}$

$\lim_{x \rightarrow 2^-} \frac{1}{x - 1}$

$L.H.L = 1$   $R.H.L = \infty$

$l$  not exist

R.H.L  $\lim_{x \rightarrow 2^+} \frac{[x]}{x - [x]}$   
 $= \lim_{x \rightarrow 2^+} \frac{2}{x - 2}$



For  $m$

$L.H.L = 1$   
 $R.H.L = 0$

$m$  does not exist

Q. Let  $f(x) = \frac{x - [x]}{1 - [x] + x}$ , then range of  $f(x)$  is ( $[.] = \text{G.I.F.}$ )

(A)  $[0, 1]$

(B)  $[0, 1/2]$

(C)  $[1/2, 1]$

(D)  $[0, 1/2]$

$$f(x) = \frac{x - [x]}{1 - [x] + x}$$

$$\underline{[x] = x - \{x\}} \quad f(x) = \frac{\{x\}}{1 + \{x\}}$$

$$f(x) = \frac{\{x\} + 1 - 1}{1 + \{x\}}$$

$$f(x) = 1 - \frac{1}{1 + \{x\}}$$

$$0 \leq \{x\} < 1$$

$$1 \leq 1 + \{x\} < 2$$

$$\frac{1}{2} < \frac{1}{1 + \{x\}} \leq 1$$

$$-1 \leq \frac{-1}{1 + \{x\}} < \frac{-1}{2}$$

$$0 \leq 1 - \frac{1}{1 + \{x\}} < \frac{1}{2}$$



Q. Function  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^3 - x$  is.

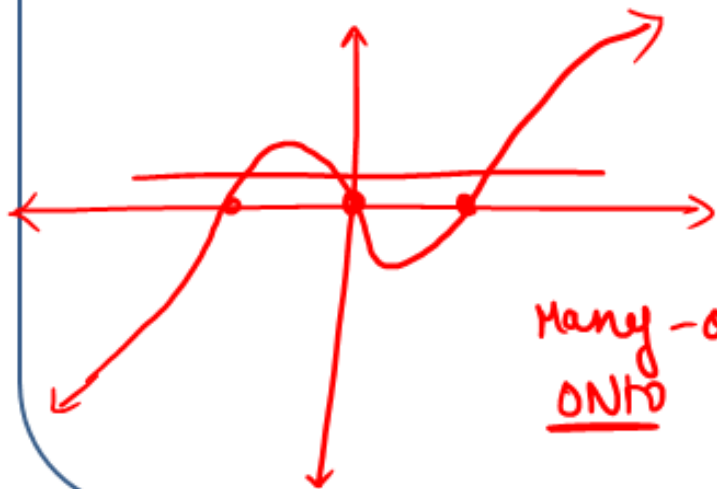
- (A) One-one onto (B) One-one into ~~(C) Many-one onto~~ (D) Many-one into

$$f(x) = x^3 - x$$

$$f(x) = x(x^2 - 1)$$

$$f(x) = x(x-1)(x+1)$$

↳ Odd degree Polynomial  $f^n$   
 will have Range =  $\mathbb{R}$ .



Many-one  
ONTO

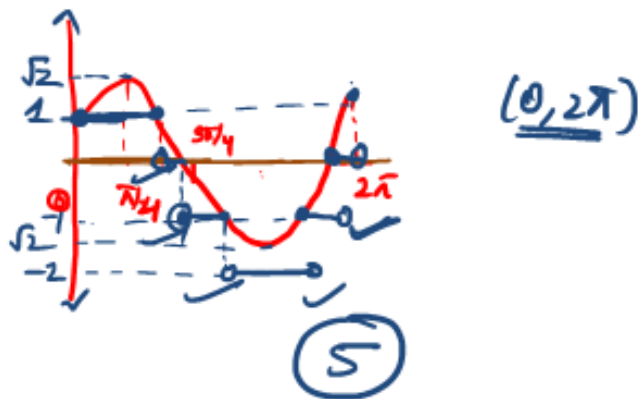
The number of points where  $f(x) = [\sin x + \cos x]$  (where  $[ ]$  denotes the greatest integer function),  $x \in (0, 2\pi)$  is not continuous is :

- (A) 3                      (B) 4                      (C) 5                      (D) 6

$$f(x) = [\sin x + \cos x]$$

$$f(x) = \left[ \sqrt{2} \left( \sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}} \right) \right]$$

$$= \left[ \sqrt{2} \sin \left( x + \frac{\pi}{4} \right) \right]$$



Q. Evaluate  $\lim_{x \rightarrow -\infty} x(x + \sqrt{1+x^2})$

(A)  $\infty$

(B) ~~...~~

(C) From Ayushi Varshney to Me: (Privately)  
yes  
yes

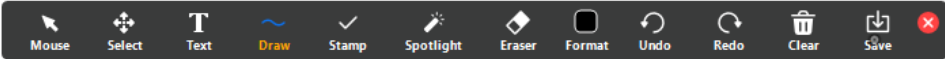
(D) From Murari Prajapati to Me: (Privately)  
SIR MURARI

From Aarish Usmani to Me: (Privately)  
yes

From Gaurav to Me: (Privately)  
sir last step firse bata do  
sir last step firse bata do

To: Roshan Yadav (Privately)

Type message here...



$\lim_{x \rightarrow -\infty} x(x + \sqrt{1+x^2})$

$\lim_{x \rightarrow -\infty} x(x + \sqrt{1+x^2}) \times \frac{(x - \sqrt{1+x^2})}{(x - \sqrt{1+x^2})}$

$\lim_{x \rightarrow -\infty} \frac{x[x^2 - (1+x^2)]}{x - \sqrt{1+x^2}}$

$\lim_{x \rightarrow -\infty} \frac{-x}{x - \sqrt{1+x^2}}$

$= \lim_{x \rightarrow -\infty} \frac{-x}{x - \sqrt{1+x^2}}$

$= \lim_{x \rightarrow -\infty} \frac{-x}{x - \sqrt{\frac{1}{x^2} + 1}}$   
 $= \lim_{x \rightarrow -\infty} \frac{-x}{x + x\sqrt{\frac{1}{x^2} + 1}}$

$\lim_{x \rightarrow -\infty} \frac{-1}{1 + \sqrt{\frac{1}{x^2} + 1}}$   
 $= \frac{-1}{2}$