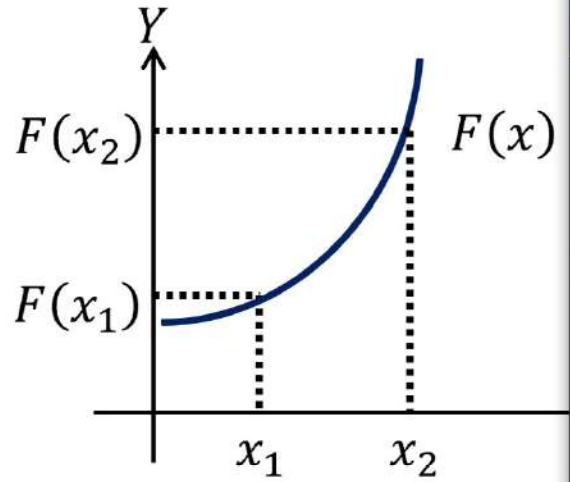
Monotonocity: Strictly Incre

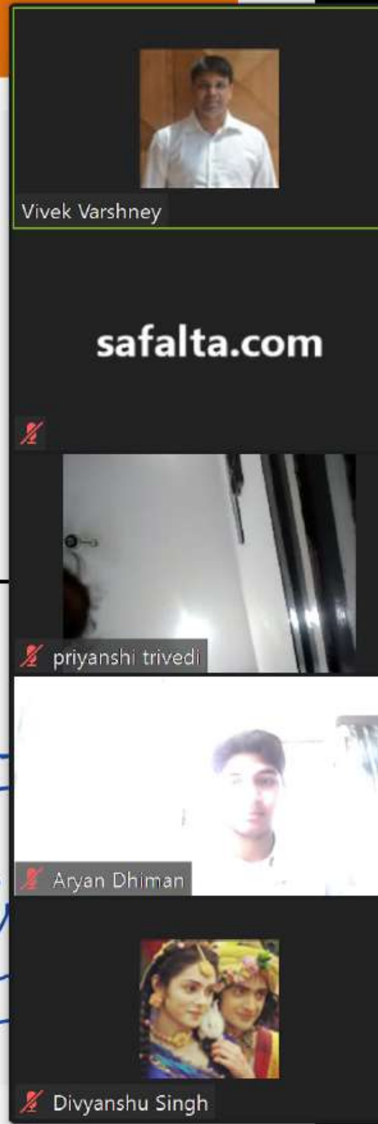
Monotonocity of function means Increasing / Decreasing behaviour of a

Strictly Increasing Function

$x_1 < x_2 \Rightarrow F(x_1) < F(x_2)$

$F'(x) > 0$





Ex $F(x) = x^2 + 3x$

$(-1 \text{ to } +3)$

$F'(x) = 2x + 3$

$F'(x) > 0$ $\quad x \in [-1, 3] \longrightarrow \boxed{x > -1.5}$

$F'(x) > 0$

$2x + 3 > 0$

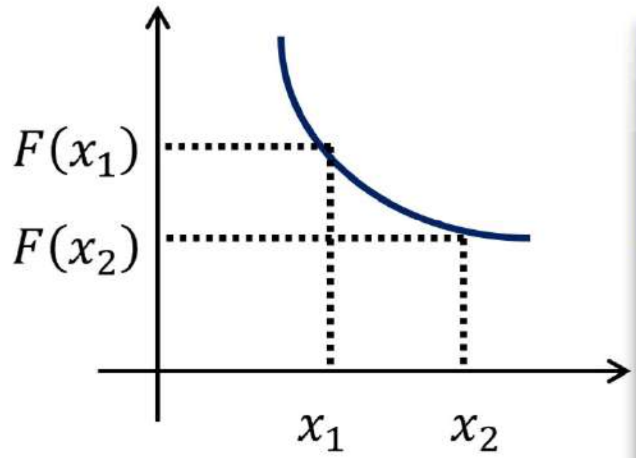
$\Rightarrow 2x > -3$

$x > -1.5$

Decreasing Function

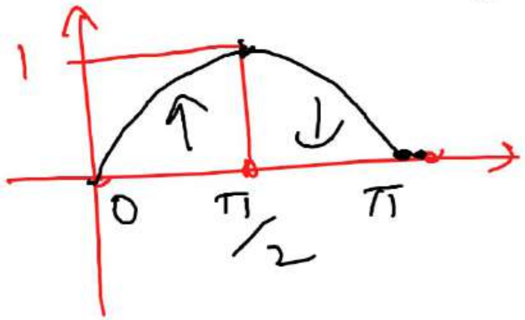
Strictly decreasing Function :-

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$



$f'(x) < 0$

Ex $f(x) = \sin x$



$f'(x) = \cos x$

I $\quad 0 \leftrightarrow \frac{\pi}{2}$

$f'(x) = \cos x$

$+$

$\sin x \uparrow$

$\frac{\pi}{2} \leftrightarrow \pi \quad$ II

$f'(x) = \cos x$

$-$

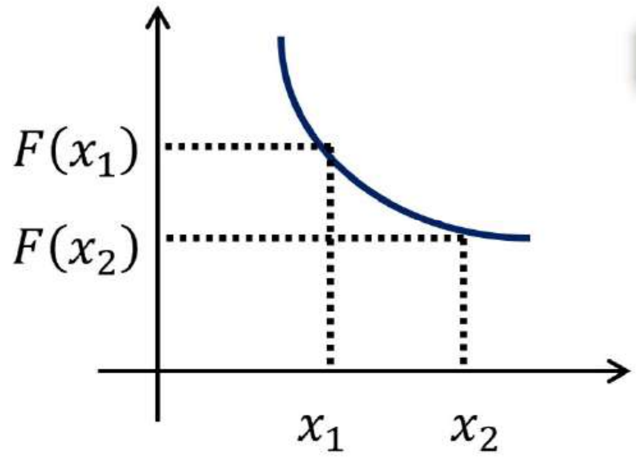
$\sin x \downarrow$

Decreasing Function

Strictly decreasing Function :-

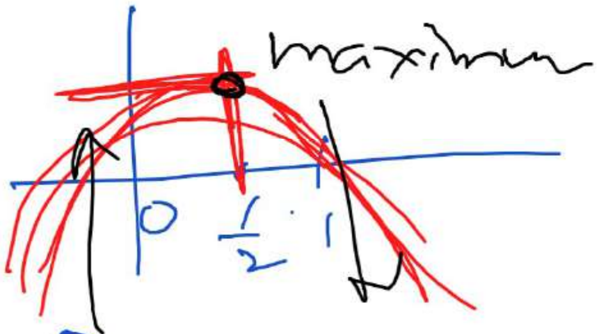
$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$$

$$f'(x) < 0$$



Ex $f(x) = x - x^2$

$f'(x) = 1 - 2x$



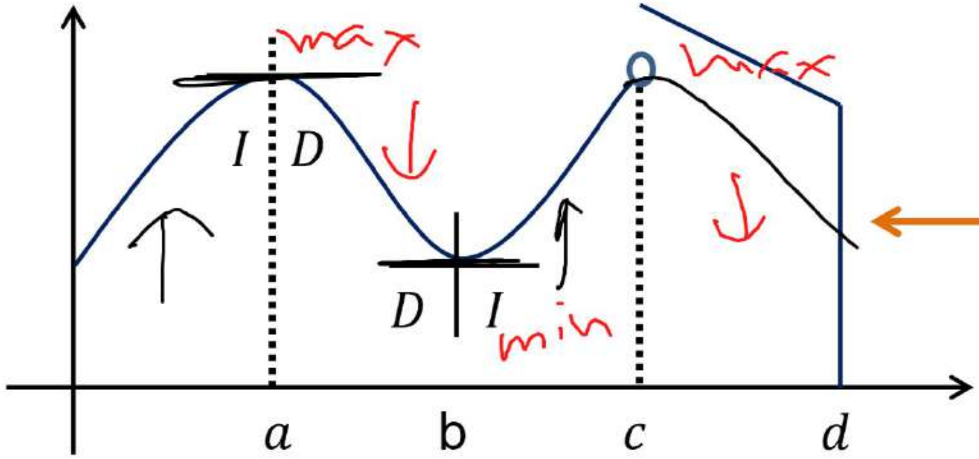
(1) $f'(x) > 0 \Rightarrow 1 - 2x > 0$

$1 > 2x \Rightarrow \frac{1}{2} > x \Rightarrow x < \frac{1}{2}$

(2) $f'(x) < 0 \Rightarrow 1 - 2x < 0$

$1 < 2x \Rightarrow \frac{1}{2} < x \Rightarrow x > \frac{1}{2}$

Interval of Monotonocity



In this graph,

(0,a) → function is increasing

(a,b) → function is decreasing

(b,c) → function is increasing

(c,d) → function is decreasing

$$f'(x) = 0 \text{ , } at\ point\ (a, b)$$

$$f'(x) does\ not\ exists\ at\ point(c)$$

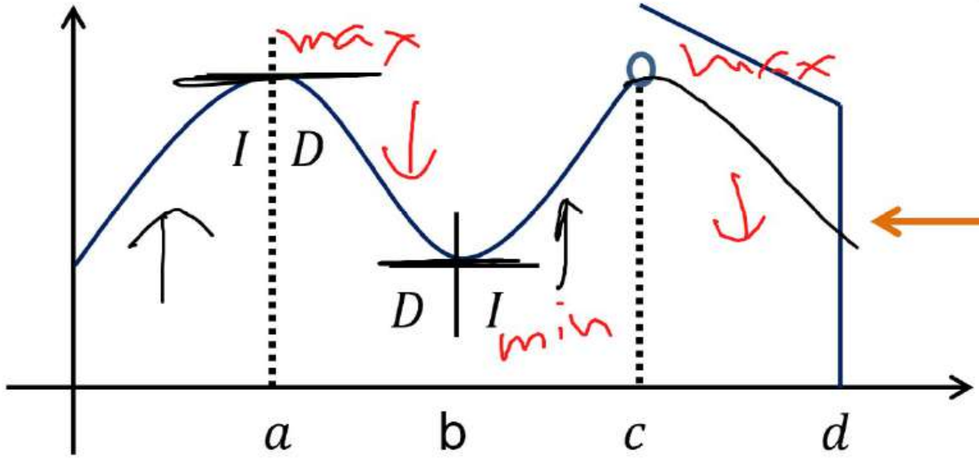
→ **Critical Point**

Critical Point :-

Point in the domain where $\mathbf{f'(x) = 0}$ *or*, $f'(x)$ does not exists

$\boxed{f'(x) \Rightarrow \text{Slope of Tangent}}$

Interval of Monotonocity



In this graph,

(0,a) → function is increasing

(a,b) → function is decreasing

(b,c) → function is increasing

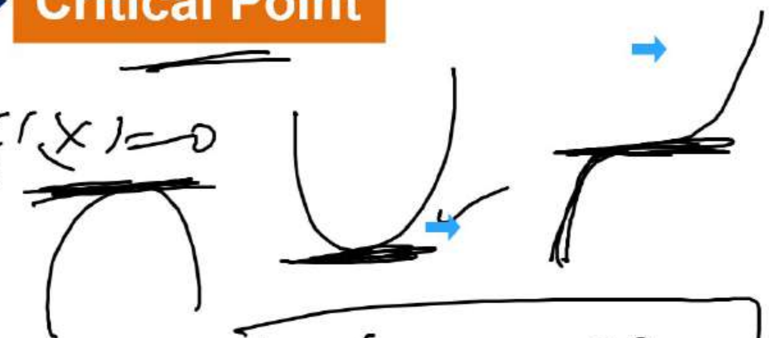
(c,d) → function is decreasing

$$f'(x) = 0 \text{ , at point } (a, b)$$

$$f'(x) \text{ does not exists at point } (c)$$

→ **Critical Point**

$f'(x)=0$



$\boxed{f'(x) \Rightarrow \text{Slope of Tangent}}$

Critical Point :-

Point in the domain where $\mathbf{f}'(x) = \mathbf{0}$ *or*, $f'(x)$ does not exists

M-Q3445

If $y = a\log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$,

$(a)\ a = 2, b = -1$ $\qquad$ $(b)\ a = 2, b = -1/2$

$(c)\ a = -2, b = 1/2$ $\qquad$ (d) *none of these.*

$$\frac{dy}{dx} = a\left(\frac{1}{x}\right) + b(2x) + 1$$

Given $x = -1$ $\quad$ $x = 2$ are extreme values

$$\left.\frac{dy}{dx}\right|_{x=-1} = 0$$

$$a\cdot\left(-\frac{1}{1}\right) + b(2(-1)) + 1 = 0$$

$$-a - 2b = -1$$

$$a + 2b = 1 \quad (1)$$

$$\left.\frac{dy}{dx}\right|_{x=2} = 0$$

$$\frac{a}{2} + 4b = -1$$

$$\boxed{a + 8b = -2} \quad (2)$$

M-Q3445

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$-a + 2b = -1$ — (1)

$\boxed{a + 2b = 1}$ (1)

$$\left.\frac{dy}{dx}\right|_{x=2} = 0$$

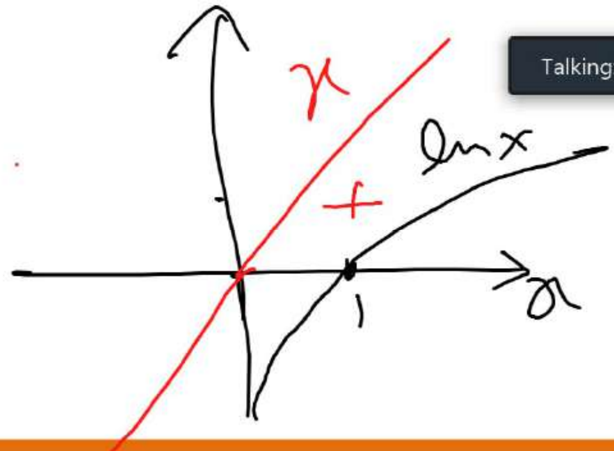
$$\frac{a}{2} + 4b = -1$$

$\boxed{a + 8b = -2}$ (2)

Monotonocity : Operation of

$If\ f(x) and\ g(x)\ are\ strictly\ increasing, then$

$f(x) + g(x)\ is\ also\ strictly\ increasing$



$ex:\ \ y = lnx + x$

$\uparrow \quad \uparrow \quad = \quad \uparrow$ increasing

See here in this example "lnx" and "x" both are increasing function so y is increasing function

$If\ f(x) and\ g(x)\ are\ strictly\ increasing, then$

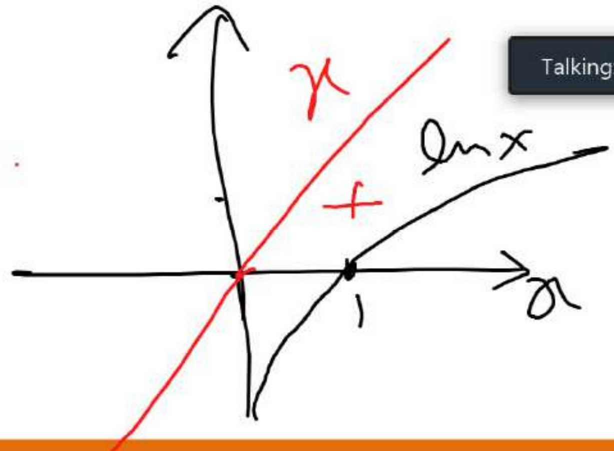
$f(x) - g(x) \rightarrow No\ Comment$

Monotonocity : Operation of $f(x) \pm g(x)$

If $f(x)$ and $g(x)$ are strictly increasing, then

$\Downarrow$

$f(x) + g(x)$ *is also strictly increasing*



ex: $y = lnx + x$

$\uparrow \quad \uparrow \quad = \quad \uparrow$ **increasing**

See here in this example "lnx" and "x" both are increasing function so y is increasing function

---

If $f(x)$ and $g(x)$ are strictly increasing, then

$\Downarrow$

$f(x) - g(x) \rightarrow$ *No Comment*

$\uparrow$ or $\downarrow$

Monotonocity : Operation of $f(x).g(x)$

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Talking:

If $f(x)$ and $g(x)$ are strictly increasing, then

$$\Downarrow$$

$f(x).g(x) \to$ *No Comment*

$\uparrow$ or $\downarrow$

---

*

But if $f(x).g(x)$ are $+ve$, then

Special Case

$$\Downarrow$$

$f(x).g(x)$ is also strictly increasing ✓

$$ex:\ \ y = e^x(\tan^{-1}(x) + \pi)$$

$$\begin{array}{ccccc} + & & + & & \\ \uparrow & & \uparrow & \Rightarrow & \uparrow \end{array}$$

See here in this example e^x and $(\tan^{-1} x + \pi)$ both are increasing function so y is increasing function

Monotonocity : Operation of $f(x).g(x)$

If $f(x)$ and $g(x)$ are strictly increasing, then

$f(x).g(x) \to$ *No Comment*

$\ln x \quad \sin x \ (+)$

$x \in [0, \pi/2)$

$(0, 1) \Rightarrow \ln x \ (-)$

$\pi/2 = 3.14/2 = 1.57$

$\boxed{x \in (1, \pi/2) \ \ln (+)}$

But if $f(x).g(x)$ are $+ve$, then

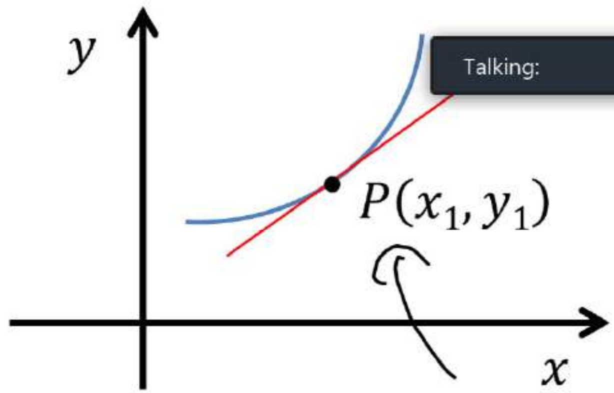
$f(x).g(x)$ *is also strictly increasing*

$ex: \ y = e^x(\tan^{-1}(x) + \pi)$

$\quad +\uparrow \qquad +\uparrow \quad \Rightarrow \uparrow$

See here in this example e^x and $(\tan^{-1}x + \pi)$ both are increasing function so y is increasing function

Tangent & Normal: Tangent



Slope of tangent $(m) = \left(\frac{dy}{dx}\bigg|_{P(x_1,y_1)}\right) = \tan\theta$

$\frac{dy}{dx}\bigg|_{(x_1, y_1)}$ = Slope of Tangent (m)

NOTE 1: *If tangent is parallel to $x-axis, \theta = 0$*

$\tan\theta = 0$

$\left(\frac{dy}{dx}\right)_{x_1,y_1} = 0$

$m = 0$

NOTE 2: *If tangent is $\perp$ to $x-axis$ or parallel to $y-axis, \theta = 90$*

$\tan\theta = \infty$

$\left(\frac{dx}{dy}\right)_{x_1,y_1} = 0$

$m = \infty$

$m \rightarrow \oplus$ $f \uparrow$

$m \rightarrow = 0$ $f \leftrightarrow$

$m \rightarrow \ominus$ $f \downarrow$

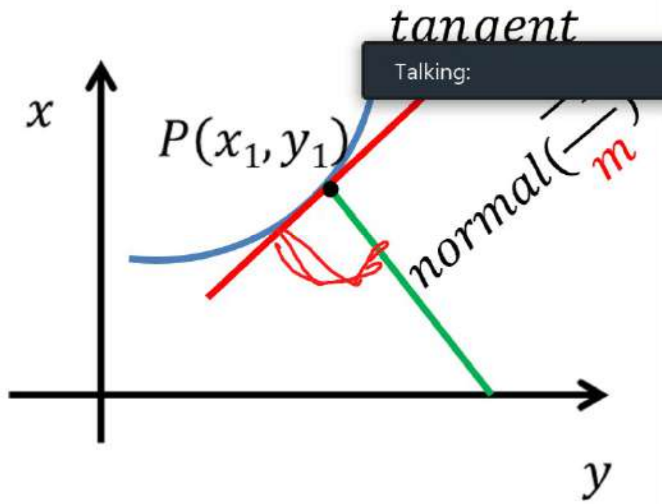
Equation Of Tangent and Normal

Equation of tangent:

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \left(\frac{dy}{dx}\bigg|_{P(x_1,y_1)}\right)(x - x_1)$$

m



Normal ⊥ tangent at that point

Equation of normal:

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - y_1 = \left(\frac{-dx}{dy}\bigg|_{P(x_1,y_1)}\right)(x - x_1)$$

Slopes

$m = \frac{dy}{dx}$ Tangent

$-\frac{1}{m}$ ← Normal

Equation Of Tangent and Normal

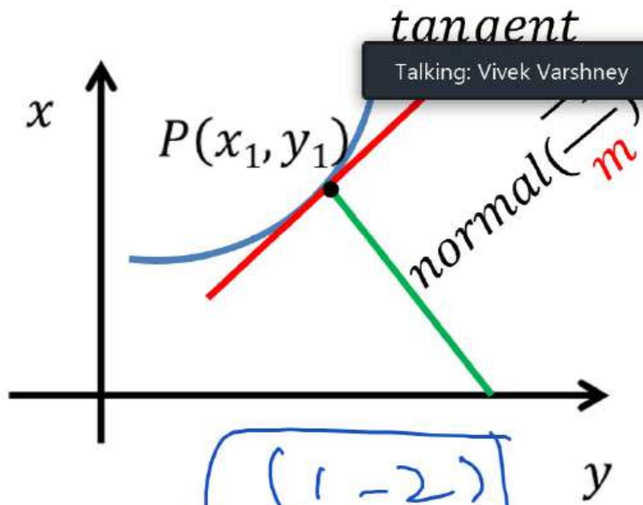
Equation of tangent:

$y-(-2)=(-1)(x-1)$

$$y - y_1 = m(x - x_1)$$

$\boxed{y=-x-1}$

$$y - y_1 = \left(\frac{dy}{dx}\bigg|_{P(x_1,y_1)}\right)(x - x_1)$$



$\boxed{(1,-2)}$

Ex $y = x^2 - 3x$

$x_1 = 1$ eq^n of
① Normal &
② Tangent

$y = (1)^2 - 3(1)$

$\boxed{y_1 = -2}$

$y' = 2x - 3$

✓ $\boxed{m = 2(1) - 3 = -1}$

Tangent

✓ $\boxed{\text{Normal} = \frac{-1}{(-1)} = +1}$

Equation of normal:

$y-(-2)=(1)(x-1)$

$\boxed{y = x - 3}$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - y_1 = \left(\frac{-dx}{dy}\bigg|_{P(x_1,y_1)}\right)(x - x_1)$$

Second Degree Curve

The equation of curve is: $\boldsymbol{ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0}$

We replace equation by:

$x^2 \rightarrow xx_1$

$y^2 \rightarrow yy_1$

$2x \rightarrow x + x_1$

$2y \rightarrow y + y_1$

$xy \rightarrow \frac{xy_1 + x_1y}{2}$

Tangent (directly)

To find Tangent directly

$x \Rightarrow \frac{x+x_1}{2}$

$y \Rightarrow \frac{y+y_1}{2}$

Ex: $2y = x^2 + 3$, *Find the eqn of tangent at* (x_1, y_1)

$\boldsymbol{(y + y_1) = xx_1 + 3}$

Tangent

Second Degree Curve

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Talking: Vivek Varshney

The equation of curve is: $\boldsymbol{ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0}$

We replace equation by:

$y = 3x - 5$

$$x^2 \to xx_1$$

$$y^2 \to yy_1$$

$$2x \to x + x_1$$

$$2y \to y + y_1$$

$$xy \to \frac{xy_1 + x_1y}{2}$$

Ex: $2y = x^2 + 3$, *Find the eqn of tangent at* (x_1, y_1)

$$\boldsymbol{(y + y_1) = xx_1 + 3}$$

$y = 2x^2 + 3x - 5$

$\boxed{x = 0 \ , \ y = -5}$

point

Tangent eqn

$\frac{y + y_1}{2} = 2(xx_1) + 3\left(\frac{x + x_1}{2}\right) - 5$

$\frac{y - 5}{2} = \frac{3x}{2} - 5$

The function $f(x) = \dfrac{2}{x} + \dfrac{x}{2}$ has a local minimum at $x =$

(a) -2 (b) 0 (c) 1 (d) 2

$y = \sin x$

$[0, \pi]$

$y(0) = \sin 0 = 0$

$y(\pi) = \sin \pi = 0$

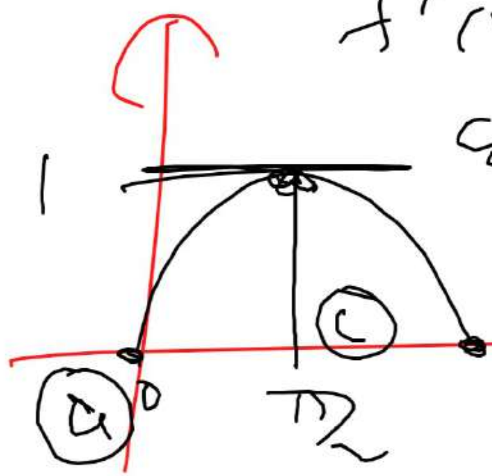
Rolle's

$f'(x) = \cos x$

$f'(x) = 0$

$\cos x = 0$

$x = \pi/2$



$\frac{\pi}{2} \in [0, \pi]$

Lagrange's Mean Value Theorem

Let (i) $F(x)$ be continuous in $[a, b]$

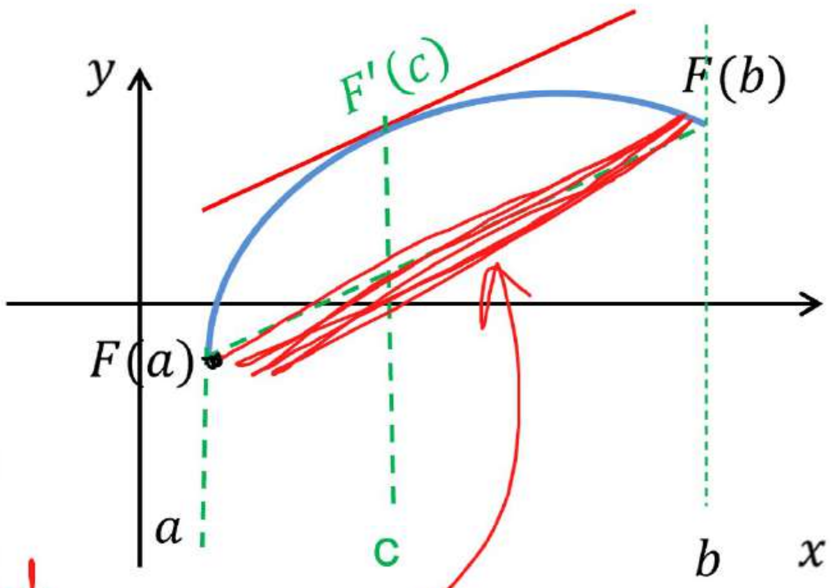
(ii) $F(x)$ be differentiable on (a, b)

$$\frac{y_2 - y_1}{x_2 - x_1}$$

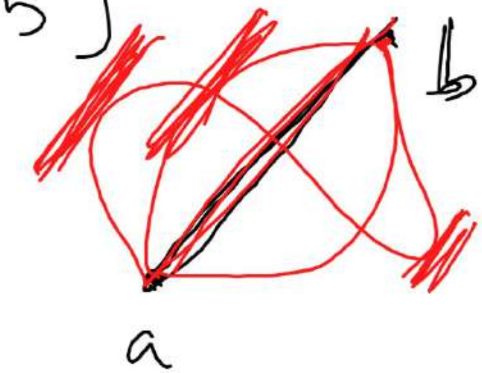
m

Then their exists at least one c in (a, b) such that $\frac{F(b)-F(a)}{b-a} = F'(c)$

Tangent at a point will be parallel



$x \in [a, b]$



$F'(c)$ is slope of tangent at c

Lagrange's Mean Value Theorem

Let (i) $F(x)$ *be continuous in* $[a, b]$

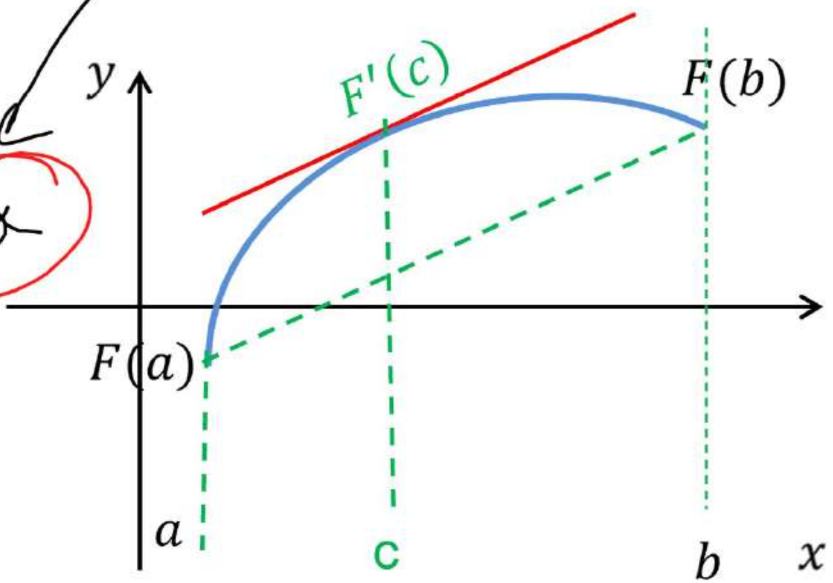
(ii) $F(x)$ *be differentiable on* (a, b)

Then their exists at least one c *in* (a, b) *such that* $\frac{F(b)-F(a)}{b-a} = F'(c)$

$\frac{4-1}{2-1} = 2x$

$\frac{3}{1} = 2x$

$\Rightarrow x = \frac{3}{2}$ or $\underline{1.5}$ c



$y = x^2$

$x \in [1, 2]$

(1) $y' = 2x$ 1.5

at $x = 1$ $y = 1^2 = 1$

$x = 2$ $y = 2^2 = 4$

F'(c) is slope of tangent at c

Lagrange's Mean Value Theo

Let (i)$F(x)$ *be continuous in* $[a, b]$

(ii)$F(x)$ *be differentiable on* (a, b)

Then their exists at least one c *in* (a, b) *such that* $\dfrac{F(b)-F(a)}{b-a} = F'(c)$

$$\frac{-2-0}{2-0} = 2x - 3$$

$$\Rightarrow -1 = 2x - 3 \Rightarrow x = 1$$

c

$y = x^2 - 3x$

$x \in [0, 2]$

a b

$c = ?$

$y' = 2x - 3$

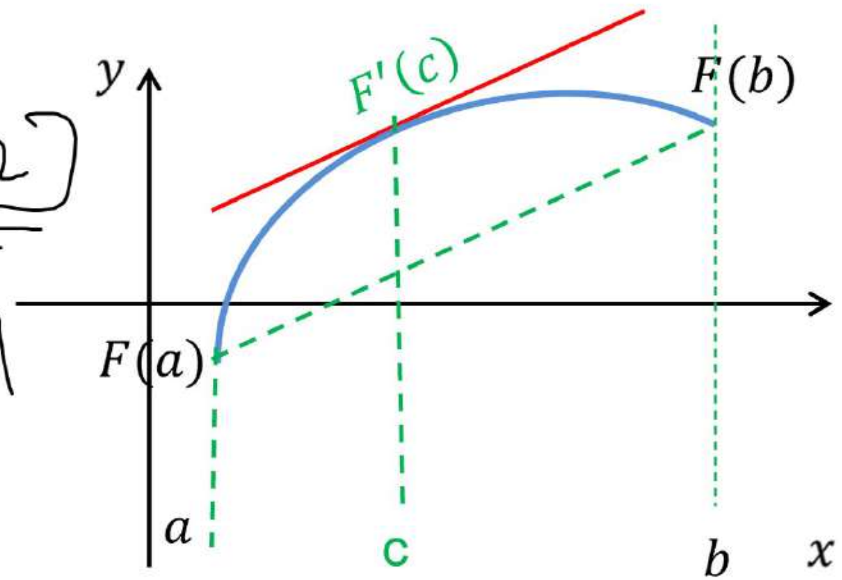
$y(0) = 0$

$y(2) = 2^2 - 3(2)$

$= -2$

$1 \in [0, 2]$

verified



F'(c) is slope of tangent at c

Maximum and Minima: Local (Relative) Maximum

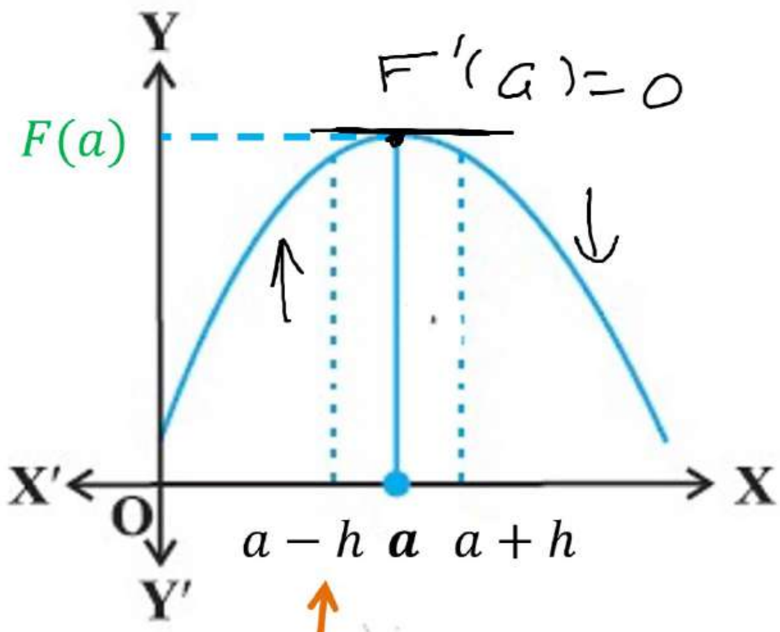
Extremum at an interior point

Local (relative) maximum

$At\ x = \boldsymbol{a}\ ;\ h = +ve\ small\ quantity$

$$F(x) \leq F(a)\ \forall\, x \in (a-h, a+h)$$

$$F(a-h) \leq F(a) \geq F(a+h)$$



$x < a \quad F(x) \uparrow$

$x > a \quad F(x) \downarrow$

$x = a \Rightarrow F(x)$ maxi;

$x = \boldsymbol{a} \rightarrow point\ of\ local\ maximum$

$\boldsymbol{F(a)} \rightarrow local\ maximum\ value$

$F'(x) = 0$

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The function $f(x) = \frac{2}{x} + \frac{x}{2}$ *has a local minimum at* $x =$

(a) -2 (b) 0 (c) 1 (d) 2

max

min

$$f'(x) = \frac{-2}{x^2} + \frac{1}{2}$$

$$f'(x) = 0$$

$$-\frac{2}{x^2} + \frac{1}{2} = 0 \rightarrow x^2 = 4$$

$$x = 2, -2$$

$$f''(x) = -\frac{4}{x^3} + 0$$

$$f''(2) < 0$$

$$f''(-2) > 0$$

$f'(x) = 0$

↳ $f''(x) > 0$ min

↳ $f''(x) < 0$ max

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Strictly Increasing Function

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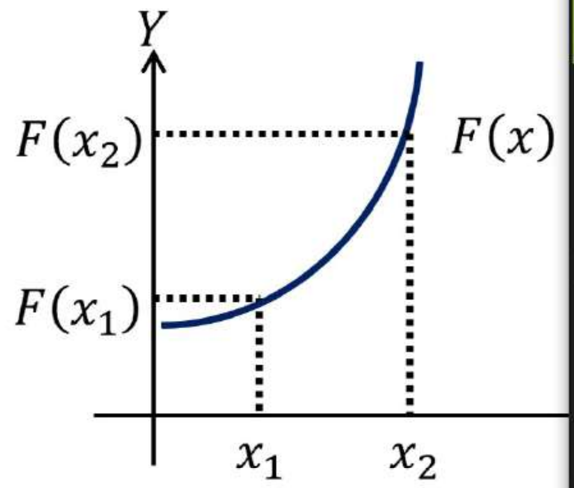
$$F'(x) > 0$$



Ex $F(x) = x^2 + 3x$

$(-1 \text{ to } +3)$

$F'(x) = 2x + 3$

$F'(x) > 0$ $\quad x \in [-1, 3]$ $\rightarrow$ $\boxed{x > -1.5}$

$F'(x) > 0$

$2x + 3 > 0$

$\Rightarrow 2x > -3$