MATHEMATICS

TOPIC- CONTINUITY AND DIFFERENTIABILITY

1. Let [x] denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then,

   a) $\lim_{x\to 0} f(x)$ does not exist

   b) $f(x)$ is continuous at $x = 0$

   c) $f(x)$ is not differentiable at $x = 0$

   d) $f'(0) = 1$

2. The value of $f(0)$ so that $\frac{(-e^x+2^x)}{x}$ may be continuous at $x = 0$ is

   a) $\log\left(\frac{1}{2}\right)$ b) 0 c) 4 d) $-1 + \log 2$

3. Let $f(x)$ be an even function. Then $f'(x)$

   a) Is an even function b) Is an odd function c) May be even or odd d) None of these

4. If $f(x) = \begin{cases} [\cos \pi\ x], x < 1 \\ |x-2|,\ 2 > x \geq 1 \end{cases}$, then $f(x)$ is

   a) Discontinuous and non-differentiable at $x = -1$ and $x = 1$

   b) Continuous and differentiable at $x = 0$

   c) Discontinuous at $x = 1/2$

   d) Continuous but not differentiable at $x = 2$

5. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, x \neq -2 \\ 2, \quad x = -2 \end{cases}$, then $f(x)$ is

   a) Continuous at $x = -2$

   b) Not continuous $x = -2$

   c) Differentiable at $x = -2$

   d) Continuous but not derivable at $x = -2$

6. If $f(x) = |\log|x||$, then

   a) $f(x)$ is continuous and differentiable for all x in its domain

   b) $f(x)$ is continuous for all x in its domain but not differentiable at $x = \pm 1$

   c) $f(x)$ is neither continuous nor differentiable at $x = \pm 1$

   d) None of the above

7. If $f'(a) = 2$ and $f(a) = 4$, then $\lim_{x\to a} \frac{xf(a)-af(x)}{x-a}$ equals

   a) $2a - 4$ b) $4 - 2a$ c) $2a + 4$ d) None of these

8. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then

a) f(x) is continuous but not differentiable at $x = 0$ b) f(x) is differentiable at $x = 0$

c) f(x) is not differentiable at $x = 0$ d) None of the above

9. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0, x \leq 1 \\ x^2 b + ax + c, & x > 1 \end{cases}$, then, f(x) is continuous and differentiable at $x = 1$, if

a) $c = 0, a = 2b$ b) $a = b, c \in R$ c) $a = b, c = 0$ d) $a = b, c \neq 0$

10. For the function $f(x) = \begin{cases} |x - 3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ which one of the following is incorrect?

a) Continuous at $x = 1$ b) Derivable at $x = 1$ c) Continuous at $x = 3$ d) Derivable at $x = 3$

11. If $f: R \to R$ is defined by

$$f(x) = \begin{cases} \frac{2\sin x - \sin 2x}{2x \cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Then the value of a so that f is continuous at 0 is

a) 2 b) 1 c) -1 d) 0

12. $f(x) = x + |x|$ is continuous for

a) $x \in (-\infty, \infty)$ b) $x \in (-\infty, \infty) - \{0\}$ c) Only $x > 0$ d) No value of x

13. If the function

$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$$

Is continuous at $x = 0$

a) $a = \log_e b, b = \frac{2}{3}$ b) $b = \log_e a, a = \frac{2}{3}$ c) $a = \log_e b, b = 2$ d) None of these

14. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \cdots + \frac{x^2}{(1+x^2)^n} + \cdots$, then at $x = 0$, f(x)

a) Has no limit

b) Is discontinuous

c) Is continuous but not differentiable

d) Is differentiable

15. Let$f(x) = \begin{cases} 1, & \forall \quad x < 0 \\ 1 + \sin x, & \forall \quad 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?

a) 1 b) -1 c) ∞ d) Does not exist

16. The function $f(x) = x - |x - x^2|$ is

a) Continuous at $x = 1$ b) Discontinuous at $x = 1$

c) Not defined at $x = 1$ d) None of the above

17. If $f(x + y + z) = f(x).f(y).f(z)$ for all x, y, z and $f(2) = 4, f'(0) = 3$, then $f'(2)$ equals

a) 12 b) 9 c) 16 d) 6

18. If $f(x) = |\log_e |x||$, then $f'(x)$ equals

a) $\frac{1}{|x|}, x \neq 0$

b) $\frac{1}{x}$ for $|x| > 1$ and $\frac{-1}{x}$ for $|x| < 1$

c) $\frac{-1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

d) $\frac{1}{x}$ for $|x| > 0$ and $-\frac{1}{x}$ for $x < 0$

19. If the function $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) 0 c) $\frac{1}{2}$ d) -1

20. Function $f(x) = |x-1| + |x-2|, x \in R$ is

a) Differentiable everywhere in R

b) Except $x = 1$ and $x = 2$ differentiable everywhere in R

c) Not continuous at $x = 1$ and $x = 2$

d) Increasing in R

21. The set of points where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is

a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-1, \infty)$ d) None of these

22. If $f(x) = x \sin\left(\frac{1}{x}\right), x \neq 0$, then the value of function at $x = 0$, so that the function is continuous at $x = 0$ is

a) 1 b) -1 c) 0 d) Indeterminate

23. The value of $f(0)$ so that the function $f(x) = \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}$ $(x \neq 0)$ is continuous everywhere, is given by

a) -1 b) 1 c) 26 d) None of these

24. The derivative of $f(x) = |x|^3$ at $x = 0$ is

a) -1 b) 0 c) Does not exist d) None of these

25. If $f(x) = \begin{cases} \frac{(4^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}, & x \neq 0 \\ 9(\log 4)^3, & x = 0 \end{cases}$ is continuous function at $x = 0$, then the value of a is equal to

a) 3 b) 1 c) 2 d) 0

26. $f(x) = |[x] + x|$ in $-1 < x \leq 2$ is

a) Continuous at $x = 0$

b) Discontinuous at $x = 1$

c) Not differentiable at $x = 2, 0$

d) All the above

27. Let$f(x) = [x^3 - x]$, where $[x]$the greatest integer function is. Then the number of points in the interval (1, 2), where function is discontinuous is

a) 4 b) 5 c) 6 d) 7

28. If $y = \cos^{-1}\cos(|x| - f(x))$, where

$f(x) = \begin{cases} 1, \text{if } x > 0 \\ -1, \text{if } x < 0 \\ 0, \text{if } x = 0 \end{cases}$. Then, $(dy/dx)\ x = \frac{5\pi}{4}$ is equal to

a) -1
b) 1
c) 0
d) Cannot be determined

29. Let $f(x+y) = f(x) + f(y)$ and $f(x) = x^2 g(x)$ for all $x, y \in R$, where $g(x)$ is continuous function. Then, $f'(x)$ is equal to

a) $g'(x)$
b) $g(0)$
c) $g(0) + g'(x)$
d) 0

30. Let a function $f(x)$ be defined by $f(x) = \begin{cases} x, & x \in Q \\ 0, & x \in R - Q \end{cases}$ Then, $f(x)$ is

a) Everywhere continuous
b) Nowhere continuous
c) Continuous only at some points
d) Discontinuous only at some points

31. The function $f(x) = \begin{cases} 1 - 2x + 3x^2 - 4x^3 + \cdots \text{to } \infty, & x \neq -1 \\ 1, & x = -1 \end{cases}$ is

a) Continuous and derivable at $x = -1$
b) Neither continuous nor derivable at $x = -1$
c) Continuous but not derivable at $x = -1$
d) None of these

32. $f(x) = \begin{cases} 2a - x & \text{in } -a < x < a \\ 3x - 2a & \text{in } a \leq x \end{cases}$. Then, which of the following is true?

a) $f(x)$ is discontinuous at $x = a$
b) $f(x)$ is not differentiable at $x = a$
c) $f(x)$ is differentiable at $x \geq a$
d) $f(x)$ is continuous at all $x < a$

33. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

a) $f(x)g(0)$
b) $2f(x)g(0)$
c) $2g(0)$
d) None of these

34. If $f(x) = [x \sin \pi\ x]$, then which of the following is incorrect?

a) $f(x)$ is continuous at $x = 0$
b) $f(x)$ is continuous in $(-1, 0)$
c) $f(x)$ is differentiable at $x = 1$
d) $f(x)$ is differentiable in $(-1, 1)$

35. If $f(x) = \begin{cases} 1, x < 0 \\ 1 + \sin x, 0 \leq x \leq \frac{\pi}{2} \end{cases}$ then derivative of $f(x)$ at $x = 0$

a) Is equal to 1
b) Is equal to 0
c) Is equal to -1
d) Does not exist

36. If the derivative of the function $f(x)$ is everywhere continuous and is given by

$f(x) = \begin{cases} bx^2 + ax + 4; & x \geq -1 \\ ax^2 + b; & x < -1 \end{cases}$, then

a) $a = 2, b = -3$
b) $a = 3, b = 2$
c) $a = -2, b = -3$
d) $a = -3, b = -2$

37. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

a) $f(x)$ is not continuous at $x = 0$

b) $f(x)$ is not continuous and differentiable at $x = 0$

c) $f(x)$ is not continuous at $x = 0$ but not differentiable at $x = 0$

d) None of these

38. If the function $f(x) = \begin{cases} Ax - B, & x \leq 1 \\ 3x, & 1 < x < 2 \\ B x^2 - A, & x \geq 2 \end{cases}$ be continuous at $x = 1$ and discontinuous at $x = 2$, then

a) $A = 3 + B, B \neq 3$
b) $A = 3 + B, B = 3$
c) $A = 3 + B$
d) None of these

39. If $f(x) = \begin{cases} |x - 4|, \text{for } x \geq 1 \\ (x^3/2) - x^2 + 3x + (1/2), \text{for } x < 1 \end{cases}$, then

a) $f(x)$ is continuous at $x = 1$ and $x = 4$

b) $f(x)$ is differentiable at $x = 4$

c) $f(x)$ is continuous and differentiable at $x = 1$

d) $f(x)$ is not continuous at $x = 1$

40. The function $f(x) = a[x + 1] + b[x - 1]$, where $[x]$ is the greatest integer function, is continuous at $x = 1$, is

a) $a + b = 0$
b) $a = b$
c) $2a - b = 0$
d) None of these

41. Let $f(x) = \begin{cases} 5^{1/x}, & x < 0 \\ \lambda[x], & x \geq 0 \end{cases}$ and $\lambda \in R$, then at $x = 0$

a) f is discontinuous
b) f is continuous only, if $\lambda = 0$
c) f is continuous only, whatever λ may be
d) None of the above

42. If for a continuous function f, $f(0) = f(1) = 0, f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to

a) 1
b) 2
c) 0
d) None of these

43. If $f(x) = \begin{cases} ax^2 - b, & |x| < 1 \\ \frac{1}{|x|}, & |x| \geq 1 \end{cases}$ is differentiable at $x = 1$, then

a) $a = \frac{1}{2}, b = -\frac{1}{2}$
b) $a = -\frac{1}{2}, b = -\frac{3}{2}$
c) $a = b = \frac{1}{2}$
d) $a = b = -\frac{1}{2}$

44. Let $f(x) = \frac{\sin 4\pi [x]}{1+[x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then

a) $f(x)$ is not differentiable at some points

b) $f'(x)$ exists but is different from zero

c) $f'(x) = 0$ for all x

d) $f'(x) = 0$ but f is not a constant function

45. The value of k which makes $f(x) = \begin{cases} \sin(1/k), & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at $x = 0$ is

a) 8
b) 1
c) -1
d) None of these

46. The function $f(x) = \max[(1-x), (1+x), 2], x \in (-\infty, \infty)$ is

a) Continuous at all points
b) Differentiable at all points
c) Differentiable at all points except at $x = 1$ and $x = -1$
d) None of the above

47. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then,

a) $f(x)$ is bounded
b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$
c) $xf(x) \to 1$ as $x \to 0$
d) $f(x) = \ln x$

48. Suppose a function $f(x)$ satisfies the following conditions for all x and y: (i) $f(x+y) = f(x)f(y)$ (ii) $f(x) = 1 + x\,g(x)\log a$, where $a > 1$ and $\lim_{x\to 0} g(x) = 1$. Then, $f'(x)$ is equal to

a) $\log a$
b) $\log a^{f(x)}$
c) $\log(f(x))^a$
d) None of these

49. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then, $g'(x)$ is equal to

a) $\frac{1}{1+(g(x))^3}$
b) $\frac{1}{1+(f(x))^3}$
c) $1+(g(x))^3$
d) $1+(f(x))^3$

50. If $f(x) = |x^2 - 4x + 3|$, then

a) $f'(1) = -1$ and $f'(3) = 1$
b) $f'(1) = -1$ and $f'(3)$ does not exist
c) $f'(1) = -1$ does not exist and $f'(3) = 1$
d) Both $f'(1)$ and $f'(3)$ do not exist

51. The points of discontinuity of $\tan x$ are

a) $n\pi, n \in I$
b) $2n\pi, n \in I$
c) $(2n+1)\frac{\pi}{2}, n \in I$
d) None of these

52. Let $f(x) = ||x| - 1|$, then points where $f(x)$ is not differentiable, is/(are)

a) $0, \pm 1$
b) ± 1
c) 0
d) 1

53. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$. Then

a) $f(x)$ is continuous at $x = 0$
b) $f(|x|)$ is continuous at $x = 0$
c) $f(x)$ is discontinuous at $x = 0$
d) None of the above

54. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,

a) $f(x)$ is continuous on R^+
b) $f(x)$ is continuous on R
c) $f(x)$ is continuous on $R - Z$
d) None of these

55. The function $f(x) = \frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is

a) -1/2
b) ½
c) -1
d) 1

56. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$. Then, which one of the following is true?

a) f is differentiable at $x = 1$ but not at $x = 0$

b) f is neither differentiable at $x = 0$ nor at $x = 1$

c) f is differentiable at $x = 0$ and at $x = 1$

d) f is differentiable at $x = 0$ but not at $x = 1$

57. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = \frac{m\pi}{2}$ d) $m = n = \frac{\pi}{2}$

58. Let f be differentiable for allx. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

a) $f(6) = 5$ b) $f(6) < 5$ c) $f(6) < 8$ d) $f(6) \geq 8$

59. If $\lim_{x\to a^+} f(x) = l = \lim_{x\to a^-} g(x)$ and $\lim_{x\to a^-} f(x) = m \lim_{x\to a^+} g(x)$, then the function $f(x)\, g(x)$

a) Is not continuous at $x = a$

b) Has a limit when $x \to a$ and it is equal to lm

c) Is continuous at $x = a$

d) Has a limit when $x \to a$ but it is not equal to lm

60. Let $f(x)$ be a function satisfying $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) = 1 + x\, g(x)$ where $\lim_{x\to 0} g(x) = 1$. Then, $f'(x)$ is equal to

a) $g'(x)$ b) $g(x)$ c) $f(x)$ d) None of these

61. The set of points where the function $f(x) = x|x|$ is differentiable is

a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(0, \infty)$ d) $[0, \infty)$

62. If $f(x + y) = f(x)f(y)$ for all real x and y, $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is

a) 30 b) 13 c) 10 d) 0

63. If $f(x) = |x - a|\phi(x)$, where $\phi(x)$ is continuous function, then

a) $f'(a^+) = \phi(a)$ b) $f'(a^-) = \phi(a)$ c) $f'(a^+) = f'(a^-)$ d) None of these

64. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is

a) Continuous as well as differentiable for all x

b) Continuous for all x but not differentiable at $x = 0$

c) Neither differentiable nor continuous at $x = 0$

d) Discontinuous everywhere

65. If $f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$, then

a) Both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$

b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x = 0$

c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x = 0$

d) Both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$

66. If $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ exists finitely, then

a) $\lim_{x\to c} f(x) = f(c)$

b) $\lim_{x\to c} f'(x) = f'(c)$

c) $\lim_{x\to c} f(x)$ does not exist

d) $\lim_{x\to c} f(x)$ may or may not exist

67. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$, is

a) 1 b) 2 c) 3 d) 4

68. If $f(x) = \begin{cases} \log_{(1-3x)}(1 + 3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to

a) -2 b) 2 c) 1 d) -1

69. Let $f(x)$ be a function differentiable at $x = c$. Then, $\lim_{x\to c} f(x)$ equals

a) $f'(c)$ b) $f''(c)$ c) $\frac{1}{f(c)}$ d) None of these

70. If $f(x) = ae^{|x|} + b|x|^2$; $a, b \in R$ and $f(x)$ is differentiable at $x = 0$. Then a and b are

a) $a = 0, b \in R$ b) $a = 1, b = 2$ c) $b = 0, a \in R$ d) $a = 4, b = 5$

ANSWERS - KEY

1)	B	2)	D	3)	B	4)	C	5)	B	6)	B	7)	B
8)	C	9)	A	10)	D	11)	D	12)	A	13)	A	14)	B
15)	D	16)	A	17)	A	18)	B	19)	C	20)	B	21)	B
22)	C	23)	D	24)	B	25)	A	26)	D	27)	C	28)	B
29)	D	30)	B	31)	B	32)	B	33)	B	34)	C	35)	D
36)	C	37)	B	38)	A	39)	A	40)	A	41)	C	42)	B
43)	B	44)	C	45)	D	46)	C	47)	D	48)	B	49)	C
50)	D	51)	C	52)	A	53)	C	54)	B	55)	C	56)	D
57)	C	58)	D	59)	B	60)	C	61)	A	62)	A	63)	A
64)	B	65)	D	66)	A	67)	C	68)	D	69)	D	70)	A

1 (b)

We have,

$-\pi/4 < x < \pi/4$

$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \le \tan^2 x < 1 \Rightarrow [\tan^2 x]$

$= 0$

$\therefore f(x) = [\tan^2 x] = 0$ for all $x \in (-\pi/4, \pi/4)$

Thus, f(x) is a constant function on $\in (-\pi/4, \pi/4)$

So, it is continuous on $\in (-\pi/4, \pi/4)$ and $f'(x) =$

0 for all $x \in (-\pi/4, \pi/4)$

2 (d)

Since, f(x) is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x \log 2}{1} = f(0)$ [by L 'Hospital's rule]

$\Rightarrow f(0) = -1 + \log 2$

3 (b)

Since f(x) is an even function

$\therefore f(-x) = f(x)$ for all x

$\Rightarrow -f'(-x) = f'(x)$ for all x

$\Rightarrow f'(-x) = -f'(x)$ for all x

$\Rightarrow f'(x)$ is an odd function

4 (c)

We have,

$$f(x) = \begin{cases} [\cos \pi x], x < 1 \\ |x-2|, 1 \le x < 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2-x, & 1 \le x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \le 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \le x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$$

It is evident from the definition that f(x) is

discontinuous at $x = 1/2$

5 (b)

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} \frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1}(-h)} = \lim_{h\to 0} \frac{-h}{\tan^{-1} h} = -1$$

and,

$$\lim_{x\to -2^+} f(x) = \lim_{h\to 0} f(-2+h)$$

$$= \lim_{h\to 0} \frac{|-2+h+2|}{\tan^{-1}(-2+h+2)}$$

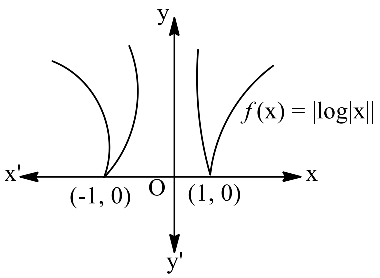
$$\Rightarrow \lim_{x\to -2^+} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1} h} = 1$$

$$\therefore \lim_{x\to -2^-} f(x) \neq \lim_{\to -2^+} f(x)$$

So, f(x) is neither continuous nor differentiable at

$x = -2$

6 (b)



From the graph of $f(x) = |\log|x||$ it is clear that

f(x) is everywhere continuous but not

differentiable at $x = \pm 1$, due to sharp edge

7 (b)

We have,

$$\lim_{x\to a} \frac{xf(a) - a\,f(x)}{x-a}$$

$$= \lim_{x\to a} \frac{x\,f(a) - a\,f(a) - a\big(f(x) - f(a)\big)}{x-a}$$

$$\Rightarrow \lim_{x\to a} \frac{x\,f(a) - a\,f(x)}{x-a}$$

$$= \lim_{x\to a} \frac{f(a)(x-a)}{x-a}$$

$$- a \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$\Rightarrow \lim_{x\to a} \frac{x\,f(a) - a\,f(x)}{x-a} = f(a) - a\,f'(a) = 4 - 2a$$

8 (c)

Given,$f(x) = x\left(\sqrt{x} + \sqrt{x+1}\right)$. At $x = 0$ LHL of $\sqrt{x}$ is

not defined, therefore it is not continuous at $x = 0$

Hence, it is not differentiable at $x = 0$

9 (a)

Here, $f'(x) = \begin{cases} 2ax, & b \neq 0, x \leq 1 \\ 2bx + a, & x > 1 \end{cases}$

Since, f(X) is continuous at x = 1

$\therefore \lim_{h\to 0} f(x) = \lim_{h\to 1^+} f(x)$

$\Rightarrow \quad a + b = b + a + c \Rightarrow c = 0$

Also, f(x) is differentiable at x = 1

$\therefore$ (LHD at x = 1)=(RHD at x = 1)

$\Rightarrow \quad 2a = 2b(1) + a \quad \Rightarrow \quad a = 2b$

10 (d)

We have,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left\{\frac{x^2}{4} - \frac{3x}{4} + \frac{13}{4}\right\} = \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = 2$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} |x - 3| = 2$$

and, $f(1) = |1 - 3| = 2$

$\therefore \lim_{x\to 1^-} f(x) = f(1) = \lim_{x\to 1^+} f(x)$

So, f(x) is continuous at x = 1

We have,

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3} |x - 3| = 0, \lim_{x\to 3^+} f(x) = \lim_{x\to 3} |x - 3|$$
$$= 0$$

and, $f(3) = 0$

$\therefore \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

So, f(x) is continuous at x = 3

Now,

(LHD at x = 1)

$$= \left\{\frac{d}{dx}\left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\right)\right\}_{x=1} = \left\{\frac{x}{2} - \frac{3}{2}\right\}_{x=1} = \frac{1}{2} - \frac{3}{2}$$
$$= -1$$

(RHD at x = 1) $= \left\{\frac{d}{dx}\left(-(x - 3)\right)\right\}_{x=1} = -1$

$\therefore$ (LHD at x = 1) = (RHD at x = 1)

So, f(x) is differentiable at x = 1

11 (d)

$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{2\sin x - \sin 2x}{2x\cos x} \quad \left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x\to 0} \frac{2\cos x - 2\cos 2x}{2(\cos x - x\sin x)}$$

$$= \lim_{x \to 0} \frac{2-2}{2(1-0)} = 0$$

Since, f(x) is continuous at $x = 0$

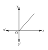
$\therefore \quad f(0) = \lim_{x \to 0} f(x)$

$\Rightarrow a = 0$

12 (a)

Given, $f(x) = x + |x|$

$$\therefore \quad f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph of f(x) is continuous for every value of x

Alternate

Since, x and |x| is continuous for every value of x, so their sum is also continous for every value of x

13 (a)

Since f(x) is continuous at $x = 0$

$\therefore \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$

$\Rightarrow \lim_{x \to 0} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}} = b = \lim_{x \to 0} e^{\frac{\tan 2x}{\tan 3x}}$

$\Rightarrow e^a = b = e^{2/3} \Rightarrow a = \frac{2}{3}$ and $a = \log_e b$

14 (b)

We have,

$$f(x) = \begin{cases} x^2 + \dfrac{(x^2/1+x^2)}{1-(1/1+x^2)} = x^2 + 1, x \neq 0 \\ 0, \quad x = 0 \end{cases}$$

Clearly, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 1 \neq f(0)$

So, f(x) is discontinuous at $x = 0$

15 (d)

LHD$= \lim_{h \to 0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h \to 0} \frac{1-1}{-h} = 0$

RHD$= \lim_{h \to 0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h \to 0} \frac{1+\sin(0+h)-1}{h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$

$\Rightarrow$ LHD$\neq$RHD

16 (a)

Given, $f(x) = x - |x - x^2|$

At $x = 1$, $f(1) = 1 - |1 - 1| = 1$

$$\lim_{x \to 1^{-1}} f(x) = \lim_{h \to 0}[(1-h) - |(1-h) - (1-h)^2|]$$

$$= \lim_{h \to 0} [(1-h) - |h - h^2|] = 1$$

$$\lim_{x \to 1^{+}} f(x) = \lim_{h \to 0}[(1+h) - |(1+h) - (1+h)^2|]$$

$$= \lim_{h \to 0} [1 + h - |-h^2 - h|] = 1$$

$$\because \lim_{x \to 1^{-1}} f(x) = \lim_{x \to 1^{+}} = f(1)$$

17 (a)

We have,

$f(x + y + z) = f(x)f(y)f(z)$ for all x, y, z ...(i)

$\Rightarrow f(0) = f(0)f(0)f(0)$ [Putting $x = y = z = 0$]

$\Rightarrow f(0)\{1 - f(0)^2\} = 0$

$\Rightarrow f(0) = 1$ [$\because f(0) = 0 \Rightarrow f(x) = 0$ for all x]

Putting $z = 0$ and $y = 2$ in (i), we get

$f(x + 2) = f(x)f(2)f(0)$

$\Rightarrow f(x + 2) = 4f(x)$ for all x

$\Rightarrow f'(2) = 4f'(0)$ [Putting $x = 0$]

$\Rightarrow f'(2) = 4 \times 3 = 12$

18 (b)

For $x > 1$, we have

$$f(x) = |\log|x|| = \log x \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $x < -1$, we have

$$f(x) = |\log|x|| = \log(-x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $0 < x < 1$, we have

$$f(x) = |\log|x|| = -\log x \quad \Rightarrow \quad f'(x) = \frac{-1}{x}$$

For $-1 < x < 0$, we have

$$f(x) = -\log(-x) \quad \Rightarrow \quad f'(x) = -\frac{1}{x}$$

$$\text{Hence, } f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$$

19 (c)

Since, $\lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x \to 0} \frac{1 - \cos x}{x^2} = k$$

$$\Rightarrow \lim_{x \to 0} \frac{-(-\sin x)}{2x} = k \quad \text{[using L 'Hospital's rule]}$$

$$\Rightarrow \quad \frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x} = k \quad \Rightarrow \quad k = \frac{1}{2}$$

20 (b)

Given, $f(X) = |x-1| + |x-2|$

$$= \begin{cases} x-1+x-2, & x \geq 2 \\ x-1+2-x, & 1 \leq x < 2 \\ 1-x+2-x, & x < 1 \end{cases}$$

$$= \begin{cases} 2x-3, & x \geq 2 \\ 1, & 1 \leq x < 2 \\ 3-2x, & x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x > 2 \\ 0, & 1 < x < 2 \\ -1, & x < 1 \end{cases}$$

Hence, except $x = 1$ and $x = 2, f(x)$ is differentiable everywhere in R

21 (b)

Clearly, f(x) is differentiable for all non-zero values of x. For $x \neq 0$, we have

$$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

Now,

$$(\text{LHD at } x = 0) = \lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}$$

$$= \lim_{h\to 0}\frac{f(0-h)-f(0)}{x-0}$$

$$\Rightarrow (\text{LHD at } x = 0) = \lim_{h\to 0}\frac{\sqrt{1-e^{-h^2}}}{-h}$$

$$= \lim_{h\to 0} -\frac{\sqrt{1-e^{-h^2}}}{h}$$

$$\Rightarrow (\text{LHD at } x = 0) = -\lim_{h\to 0}\sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = -1$$

and, $(\text{RHD at } x = 0) = \lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} =$

$$\lim_{h\to 0}\frac{\sqrt{1-e^{-h^2}}-0}{h}$$

$$\Rightarrow (\text{RHD at } x = 0) = \lim_{h\to 0}\sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = 1$$

So, f(x) is not differentiable at $x = 0$

Hence, the set of points of differentiability of f(x) is $(-\infty, 0) \cup (0, \infty)$

22 (c)

Since f(x) is continuous at $x = 0$

$\therefore f(0) = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

23 (d)

For f(x) to be continuous everywhere, we must have,

$f(0) = \lim_{x \to 0} f(x)$

$\Rightarrow f(0) = \lim_{x \to 0} \frac{2 - (256 - 7x)^{1/8}}{(5x + 32)^{1/5} - 2} \quad \left[\text{Form} \frac{0}{0}\right]$

$\Rightarrow f(0) = \lim_{x \to 0} \frac{\frac{7}{8}(256 - 7x)^{-\frac{7}{8}}}{(5x + 32)^{-4/5}} = \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$

24 (b)

We have,

$f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$

$\therefore (\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} -\frac{x^3}{x}$

$= 0$

and,

$\therefore (\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x^3}{x} = 0$

Clearly, (LHD at $x = 0$) = (RHD at $x = 0$)

Hence, f(x) is differentiable at $x = 0$ and its derivative at $x = 0$ is 0

25 (a)

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{4^x - 1}{x}\right)^3 \times \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{ax^2}{\log\left(1 + \frac{1}{3}x^2\right)}$

$= (\log 4)^3 . 1. a \lim_{x \to 0} \left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \ldots}\right)$

$= 3a\,(\log 4)^3$

$\because \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow 3a\,(\log 4)^3 = 9(\log 4)^3$

$\Rightarrow a = 3$

26 (d)

We have,

$f(x) = |[x]x| \text{ for } -1 < x \leq 2$

$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \leq x < 1 \\ x, & 1 \leq x < 2 \\ 2x, & x = 2 \end{cases}$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non-differentiable at $x = 2$

27 (c)

Given, $f(x) = [x^3 - 3]$

Let $g(x) = x^3 - x$ it is in increasing function

$\therefore\ g(1) = 1 - 3 = -2$

and $g(2) = 8 - 3 = 5$

Here, f(x) is discontinuous at six points

28 (b)

Given, $y = \cos^{-1}\cos(x-1),\ \ x > 0$

$\Rightarrow\ \ y = x - 1, \qquad 0 \le x - 1 \le \pi$

$\therefore\ y = x - 1, \qquad 1 \le x \le \pi + 1$

At $x = \frac{5\pi}{4} \in [1, \pi + 1]$

$$\Rightarrow \frac{dy}{dx} = 1 \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=\frac{5\pi}{4}} = 1$$

29 (d)

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x) + f(h) - f(x)}{h} \qquad [\because f(x+y) = f(x) + f(y)]$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(h)}{h} = \lim_{h\to 0}\frac{h^2 g(h)}{h}$$

$$\Rightarrow f'(x) = 0 \times g(0) = 0 \qquad \left[\begin{array}{l}\because g \text{ is continuous} \\ \therefore \lim_{h\to 0} g(h) = g(0)\end{array}\right]$$

30 (b)

Using Heine's definition of continuity, it can be shown that f(x) is everywhere discontinuous

31 (b)

For $x \ne -1$, we have

$f(x) = 1 - 2x + 3x^2 - 4x^3 + \cdots \infty$

$$\Rightarrow f(x) = (1+x)^{-2} = \frac{1}{(1+x)^2}$$

Thus, we have

$$f(x) = \begin{cases} \frac{1}{(1+x)^2}, & x \ne -1 \\ 1, & x = -1 \end{cases}$$

We have, $\lim_{x\to -1^-} f(x) \to \infty$ and $\lim_{x\to -1^-} f(x) \to \infty$

So, f(x) is not continuous at $x = -1$

Consequently, it is not differentiable there at

32 (b)

At $x = a$,

LHL= $\lim_{x\to a^-} f(x) = \lim_{x\to a} 2a - x = a$

And RHL= $\lim_{x\to a^+} f(x) = \lim_{x\to a} 3x - 2a = a$

And $f(a) = 3(a) - 2a = a$

∴ LHL=RHL= f(a)

Hence, it is continuous at $x = a$

Again, at $x = a$

LHD= $\lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$

$= \lim_{h\to 0} \frac{2a - (a-h) - a}{-h} = -1$

and RHD= $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$= \lim_{h\to 0} \frac{3(a+h) - 2a - a}{h} = 3$

∴ LHD≠RHD

Hence, it is not differentiable at $x = a$

33 (b)

We have,

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h}$

$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{f(h) - 1}{h}$

$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{1 + (\sin 2h)g(h) - 1}{h}$

$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{\sin 2h}{h} \times \lim_{h\to 0} g(h) = 2f(x)g(0)$

34 (c)

If $-1 \le x \le 1$, then $0 \le x \sin \pi x \le 1/2$

$\therefore f(x) = [x \sin \pi x] = 0$, for $-1 \le x \le 1$

If $1 < x < 1 + h$, where h is a small positive real number, then

$\pi < \pi x < \pi + \pi h \Rightarrow -1 < \sin \pi x < 0 \Rightarrow -1 < x \sin \pi x < 0$

$\therefore f(x) = [x \sin \pi x] = -1$ in the right

neighbourhood of $x = 1$

Thus, f(x) is constant and equal to zero in $[-1, 1]$

and so f(x) is differentiable and hence continuous

on $(-1,1)$

At $x = 1$, f(x) is discontinuous because

$\Rightarrow \lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = -1$

Hence, f(x) is not differentiable at $x = 1$

35 (d)

We have,

$$(\text{LHD at } x = 0) = \left\{\frac{d}{dx}(1)\right\}_{x=0} = 0$$

$$(\text{RHD at } x = 0) = \left\{\frac{d}{dx}(1 + \sin x)\right\}_{x=0} = \cos 0 = 1$$

Hence, $f'(x)$ at $x = 0$ does not exist

36 (c)

$$\text{Here, } f'(x) = \begin{cases} 2bx + a, & x \geq -1 \\ 2a, & x < -1 \end{cases}$$

Given, $f'(x)$ is continuous everywhere

$$\therefore \lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$$

$$\Rightarrow -2b + a = -2a$$

$$\Rightarrow 3a = 2b$$

$$\Rightarrow a = 2, \quad b = 3$$

or $a = -2$, $b = -3$

37 (b)

We have,

$$\lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \frac{\log \cos x}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x\to 0} \frac{\log(1 - 1 + \cos x)}{\log(1 + x^2)} \cdot \frac{1 - \cos x}{1 - \cos x}$$

$$\Rightarrow \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x\to 0} \frac{\log\{1 - (1 - \cos x)\}}{1 - \cos x}$$

$$\cdot \frac{1 - \cos x}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$$= -\lim_{x\to 0}\log\frac{[1-(1-\cos x)]}{-(1-\cos x)}$$

$$\times \frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\times\frac{x^2}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = -\frac{1}{2}$$

Hence, f(x) is differentiable and hence continuous at $x = 0$

38 (a)

Since f(x) is continuous at $x = 1$. Therefore,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow A - B = 3 \Rightarrow A = 3 + B$

...(i)

If f(x) is continuous at $x = 2$, then

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \Rightarrow 6 = 4B - A$ ...(ii)

Solving (i) and (ii) we get $B = 3$

As f(x) is not continuous at $x = 2$. Therefore, $B \neq 3$

Hence, $A = 3 + B$ and $B \neq 3$

39 (a)

We have,

$$f(x) = \begin{cases} x-4, & x \geq 4 \\ -(x-4), & 1 \leq x < 4 \\ (x^3/2) - x^2 + 3x + (1/2), & x < 1 \end{cases}$$

Clearly, f(x) is continuous for all x but it is not differentiable at $x = 1$ and $x = 4$

40 (a)

It is given that f(x) is continuous at $x = 1$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$$\Rightarrow \lim_{x\to 1^-} a[x+1] + b[x-1]$$

$$= \lim_{x\to 1^+} a[x+1] + b[x-1]$$

$\Rightarrow a - b = 2a + 0 \times b$

$\Rightarrow a + b = 0$

41 (c)

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \lambda[x] = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 5^{1/x} = 0$

And $f(0) = \lambda[0] = 0$

$\therefore$ f is continuous only whatever λ may be

42 (b)

We have,

$y(x) = f(e^x)\, e^{f(x)}$

$\Rightarrow y'(x) = f'(e^x) \cdot e^x \cdot e^{f(x)} + f(e^x)\, e^{f(x)}\, f'(x)$

$\Rightarrow y'(0) = f'(1)e^{f(0)} + f(1)e^{f(0)}f'(0)$

$\Rightarrow y'(0) = 2 \qquad [\because f(0) = f(1) = 0, f'(1) = 2]$

43 (b)

Since f(x) is differentiable at $x = 1$. Therefore,

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{a(1-h)^2 - b - 1}{-h} = \lim_{h\to 0} \frac{\frac{1}{|1+h|} - 1}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{(a-b-1) - 2\,ah + ah^2}{-h} = \lim_{h\to 0} \frac{-h}{h(1+h)}$$

$$\Rightarrow \lim_{h\to 0} \frac{-(a-b-1) - 2\,ah - ah^2}{h} = -1$$

$\Rightarrow -(a-b-1) = 0$ and so $\lim_{h\to 0} \frac{2ah - ah^2}{h} = -1$

$\Rightarrow a - b - 1 = 0$ and $2a = -1 \Rightarrow a = -\frac{1}{2}, b = -\frac{3}{2}$

44 (c)

We have,

$f(x) = \frac{\sin 4\,\pi[x]}{1+[x]^2} = 0$ for all x $[\because$

$4\pi[x]$is a multiple of $\pi]$

$\Rightarrow f'(x) = 0$ for all x

45 (d)

We have,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \sin\frac{1}{x}$$

$\Rightarrow \lim_{x\to 0} f(x) =$ An oscillating number which

oscillates between -1 and 1

Hence, $\lim_{x\to 0} f(x)$ does not exist

Consequently, f(x) cannot be continuous at $x = 0$

for any value of k

46 (c)

It is clear from the graph that f(x) is continuous everywhere and also differentiable everywhere except $\{-1, 1\}$ due to sharp edge

47 (d)

We have,

$\log\left(\frac{x}{y}\right) = \log x - \log y$ and $\log(e) = 1$

$\therefore f(x) = \log x$

Clearly, f(x) is unbounded because $f(x) \to -\infty$ as $x \to 0$ and $f(x) \to +\infty$ as $x \to \infty$

We have,

$$f\left(\frac{1}{x}\right) = \log\left(\frac{1}{x}\right) = -\log x$$

As $x \to 0, f\left(\frac{1}{x}\right) \to \infty$

Also,

$$\lim_{x\to 0} xf(x) = \lim_{x\to 0} x\log x = \lim_{x\to 0}\frac{\log x}{1/x}$$

$$\Rightarrow \lim_{x\to 0} xf(x) = \lim_{x\to 0}\frac{1/x}{-1/x^2} = -\lim_{x\to 0} x = 0$$

49 (c)

Since g(x) is the inverse of f(x). Therefore,

fog(x) = x, for all x

$\Rightarrow \frac{d}{dx}\{fog\ (x)\} = 1$, for all x

$\Rightarrow f'(g(x))\, g'(x) = 1$, for all x

$\Rightarrow \frac{1}{1+\{g(x)\}^3} \times g'(x) = 1$ for all x $\quad \left[\because f'(x) = \frac{1}{1+x^3}\right]$

$\Rightarrow g'(x) = 1 + \{g(x)\}^3$, for all x

50 (d)

We have,

$f(x) = |x^2 - 4x + 3|$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, & \text{if } x^2 - 4x + 3 \geq 0 \\ -(x^2 - 4x + 3), & \text{if } x^2 - 4x + 3 < 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, & \text{if } x \leq 1 \text{ or } x \geq 3 \\ -x^2 + 4x - 3, & \text{if } 1 < x < 3 \end{cases}$$

Clearly, f(x) is everywhere continuous

Now,

$$(\text{LHD at } x = 1) = \left(\frac{d}{dx}(x^2 - 4x + 3)\right)_{\text{at } x=1}$$

$\Rightarrow$ (LHD at $x = 1$) $= (2x - 4)_{\text{at } x=1} = -2$

and,

$$(\text{RHD at } x = 1) = \left(\frac{d}{dx}(-x^2 + 4x - 3)\right)_{\text{at } x=1}$$

$\Rightarrow$ (RHD at $x = 1$) $= (-2x + 4)_{\text{at } x=1} = 2$

Clearly, (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$

Similarly, it can be checked that $f(x)$ is not

differentiable at $x = 3$ also

<u>ALITER</u> We have,

$f(x) = |x^2 - 4x + 3| = |x - 1|\,|x - 3|$

Since, $|x - 1|$ and $|x - 3|$ are not differentiable at

1 and 3 respectively

Therefore, $f(x)$ is not differentiable at $x = 1$ and

$x = 3$

51 (c)

The point of discontinuity of $f(x)$ are those points

where $\tan x$ is infinite.

ie, $\tan x = \tan \infty$

$\Rightarrow \quad x = (2n + 1)\frac{\pi}{2}, \qquad n \in I$

52 (a)

Using graphical transformation

As, we know the function is not differentiable at6

sharp edges and in figure (iii) $y = \big||x| - 1\big|$ we

have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

53 (c)

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} 2(0 - h) = 0$

And $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} 2(0 + h) + 1 = 1$

$\because \quad \lim_{x \to 0^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\therefore \quad f(x)$ is discontinuous at $x = 0$

54 (b)

Draw a rough sketch of $y = f(x)$ and observe its

properties

55 (c)

$$\lim_{x\to\pi}\frac{(1+\cos x)-\sin x}{(1+\cos x)+\sin x}$$

$$=\lim_{x\to\pi}\frac{2\cos^2 x/2-2(\sin x/2)\cos\ x/2}{2\cos^2 x/2+2(\sin x/2)\cos\ x/2}$$

$$=\lim_{x\to\pi}\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)=-1$$

Since, f(x) is continuous at $x=\pi$

$\therefore\ \ f(\pi)=\lim_{x\to\pi}f(x)=-1$

56 (d)

$$f'(1^-)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$=\lim_{h\to 0}\frac{(1-h-1).\sin\left(\frac{1}{1-h-1}\right)-0}{-h}$$

$$=-\lim_{h\to 0}\ \sin\frac{1}{h}$$

And $f'(1^+)=\lim_{h\to 0}\ \frac{f(1+h)-f(1)}{h}$

$$=\lim_{h\to 0}\frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right)-0}{h}$$

$$=\lim_{h\to 0}\ \sin\frac{1}{h}$$

$\therefore\ \ f'(1^-)\neq f'(1^+)$

f is not differentiable at $x=1$

Again, now

$$f'(0^+)=\lim_{h\to 0}\frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right)-\sin 1}{h}$$

$$=\lim_{h\to 0}\frac{\left[-\left\{(h-1)\cos\left(\frac{1}{h-1}\right)\times\left(\frac{1}{(h-1)^2}\right)\right\}+\sin\right.}{1}$$

[using L 'Hospital's rule]

$=\cos 1-\sin 1$

And $f'(0^-)=\lim_{h\to 0}\frac{(0-h-1)\sin\left(\frac{1}{0-h-1}\right)-\sin 1}{-h}$

$$=\lim_{h\to 0}\frac{(-h-1)\cos\left(\frac{1}{-h-1}\right)\left(\frac{1}{(-h-1)^2}\right)-\sin\left(\right.}{-1}$$

[using L 'Hospital's rule]

$=\cos 1-\sin 1$

$\Rightarrow\ \ f'(0^-)=f'(0^+)$

$\therefore\ \ $ f is differentiable at $x=0$

57 (c)

As f(x) is continuous at $x=\frac{\pi}{2}$

$\therefore \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x)$

$\Rightarrow m\frac{\pi}{2} + 1 = \sin\frac{\pi}{2} + n \Rightarrow m\frac{\pi}{2} + 1 = 1 + n \Rightarrow n$

$= \frac{m\,\pi}{2}$

58 (d)

Since, $\frac{f(6)-f(1)}{6-1} \geq 2 \quad \left[\because \quad f'(x) = \frac{y_2-y_1}{x_2-x_1}\right]$

$\Rightarrow \quad f(6) - f(1) \geq 10$

$\Rightarrow f(6) + 2 \geq 10$

$\Rightarrow \quad f(6) \geq 8$

59 (b)

We have,

$\lim_{x \to a^-} f(x)\ g(x) = \lim_{x \to a^-} f(x) \, . \lim_{x \to a^-} g(x) = m \times l = ml$

and,

$\lim_{x \to a^+} f(x)\ g(x) = \lim_{x \to a^+} f(x) \lim_{x \to a^+} g(x) = lm$

$\therefore \lim_{x \to a^-} f(x)\ g(x) = \lim_{x \to a^+} f(x)\ g(x) = lm$

Hence, $\lim_{x \to a} f(x)\ g(x)$ exists and is equal to lm

60 (c)

We have,

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x)f(h) - f(x)}{h}$

$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{f(h) - 1}{h} \qquad [\because f(x+y)$

$= f(x)f(y)]$

$\Rightarrow f'(x) = f(x) \left\{\lim_{h \to 0} \frac{1 + h\,g(h) - 1}{h}\right\} \qquad [\because f(x)$

$= 1 + x\,g(x)]$

$\Rightarrow f'(x) = f(x) \lim_{h \to 0} g(h) = f(x) \cdot 1 = f(x)$

61 (a)

We have, $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Clearly, f(x) is differentiable for all $x > 0$ and for

all$x < 0$. So, we check the differentiable at $x = 0$

Now, (RHD at $x = 0$)

$\left(\frac{d}{dx}(x)^2\right)_{x=0} = (2x)_{x=0} = 0$

And (LHD at $= 0$)

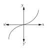
$\left(\frac{d}{dx}(-x)^2\right)_{x=0} = (-2x)_{x=0} = 0$

$\therefore$ (LHD at $x = 0$)=(RHD at $x = 0$)

So, $f(x)$ is differentiable for all x ie, the set of all points where $f(x)$ is differentiable is $(-\infty, \infty)$

Alternate

It is clear from the graph $f(x)$ is differentiable everywhere.



62 (a)

Since, $f'(0) = \lim\limits_{x \to 0} \frac{f(x)-f(0)}{x-0} = 10$

$\Rightarrow \lim\limits_{h \to 0} \frac{f(0+h) - f(0)}{h} = 10$

$\Rightarrow f(0)\left(\lim\limits_{h \to 0} \frac{f(h)-1}{h}\right) = 10$ ...(i)

$[\because \ f(0+h) = f(0)f(h), \text{given}]$

Now, $f(0) = f(0)f(0)$

$\Rightarrow f(0) = 1$

$\therefore$ From Eq. (i)

$\lim\limits_{h \to 0} \frac{f(h)-1}{h} = 10$ ...(ii)

Now, $f'(6) = \lim\limits_{h \to 0} \frac{f(6+h)-f(6)}{h}$

$= \lim\limits_{x \to 0} \left(\frac{f(h)-1}{h}\right) f(6)$ [from Eq. (ii)]

$= 10 \times 3 = 30$

63 (a)

We have,

$f'(a^+) = \lim\limits_{x \to a^+} \frac{f(x) - f(0)}{x - a}$

$\Rightarrow f'(a^+) = \lim\limits_{x \to a^+} \frac{|x - a|\phi(x)}{x - a}$

$\Rightarrow f'(a^+) = \lim\limits_{x \to a} \frac{(x-a)}{(x-a)}\phi(x)$ $[\because x > a \ \therefore |x - a| = x - a]$

$\Rightarrow f'(a^+) = \lim\limits_{x \to a} \phi(x)$

$\Rightarrow f'(a^+) = \phi(a)$ $[\because \phi(x)$ is continuous at $x = a]$

and,

$f'(a^-) = \lim\limits_{x \to a^-} \frac{f(x) - f(0)}{x - a}$

$\Rightarrow f'(a^-) = \lim_{x\to a^-} \frac{|x-a|\phi(x)}{x-a}$

$\Rightarrow f'(a^-) = \lim_{x\to a} \frac{(x-a)\phi(x)}{(x-a)}$ $\quad [\because x < a \therefore |x-a|$

$= -(x-a)]$

$\Rightarrow f'(a^-) = -\lim_{x\to a} \phi(x)$

$\Rightarrow f'(a^-) = -\phi(a)$ $\quad [\because \phi(x)$ is continuous at $x =$

$a]$

64 (b)

LHL$= \lim_{h\to 0}(0-h)_e{}^{-\left(\frac{1}{|-h|}+\frac{1}{(-h)}\right)} = \lim_{h\to 0}(-h) = 0$

RHL$= \lim_{h\to 0}(0+h)_e{}^{-\left(\frac{1}{|h|}+\frac{1}{(h)}\right)} = \lim_{h\to 0}\frac{h}{e^{2/h}} = 0$

LHL=RHL$= f(0)$

Therefore, $f(x)$ is continuous for all x

Differentiability at $x = 0$

$Lf'(0) = \lim_{h\to 0}\frac{(-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}}{(-h)-0} = 1$

$Rf'(0) = \lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{h-0}$

$= \lim_{h\to 0}\frac{1}{e^{2/h}} = 0$

$\Rightarrow \quad Rf'(0)Lf'(0)$

Therefore, $f(x)$ is not differentiable at $x = 0$

65 (d)

We have,

$f(x) = \begin{cases} 3, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$

Clearly, f is continuous but not differentiable at

$x = 0$

Now,

$f(|x|) = 2|x| + 1$ for all x

Clearly, $f(|x|)$ is everywhere continuous but not

differentiable at $x = 0$

67 (c)

We have,

$f(x) = |x-0.5| + |x-1| + \tan x, 0 < x < 2$

$\Rightarrow f(x) = \begin{cases} -2x + 1.5 + \tan x, & 0 < x < 0.5 \\ 0.5 + \tan x, & 0.5 \leq x < 1 \\ 2x - 1.5 + \tan x, & 1 \leq x < 2 \end{cases}$

It is evident from the above definition that

$Lf'(0.5) \neq Rf'(0.5)$ and $Lf'(1) \neq Rf'(1)$

Also, the function is not continuous at $= \pi/2$. So,

it cannot be differentiable thereat

68 (d)

Given, $f(x) = \begin{cases} \log_{(1-3x)}(1+3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\log(1+3x)}{\log(1-3x)}$

$= -\lim_{x \to 0} \frac{\log(1+3x)}{3x} \cdot \frac{(-3x)}{\log(1-3x)}$

$= -1$

And $f(0) = k$

$\because f(x)$ is continuous at $x = 0$

$\therefore \quad k = -1$

69 (d)

Since $f(x)$ is differentiable at $x = c$. Therefore, it is

continuous at $x = c$

Hence, $\lim_{x \to c} f(x) = f(c)$

70 (a)

Given, $f(x) = ae^{|x|} + b\,|x|^2$

We know $e^{|x|}$ is not differentiable at $x = 0$ and $|x|^2$

is differentiable at $x = 0$

$\therefore$ $f(x)$ is differentiable at $x = 0$, if $a = 0$ and $b \in R$