Properties of Continuous Function

Let $f(x)$ and $g(x)$ be continuous at $x = a$

The following function are continuous at $x = a$

1) $y = kf(x)$

2) $y = (f(x))^P$

3) $y = f(x) \pm g(x)$

4) $y = f(x).\, g(x)$

5) $y = \dfrac{f(x)}{g(x)}$, $\quad g(a) \neq 0$

③ $\sin x$ ✓
$\cos x$ ✓

$\sin x + \cos x$

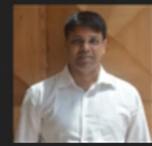
① $\sin x$

$\sin^2 x$ ✓

$2 \sin x$ ✓

② x , $\sin x$

$x + \sin x$



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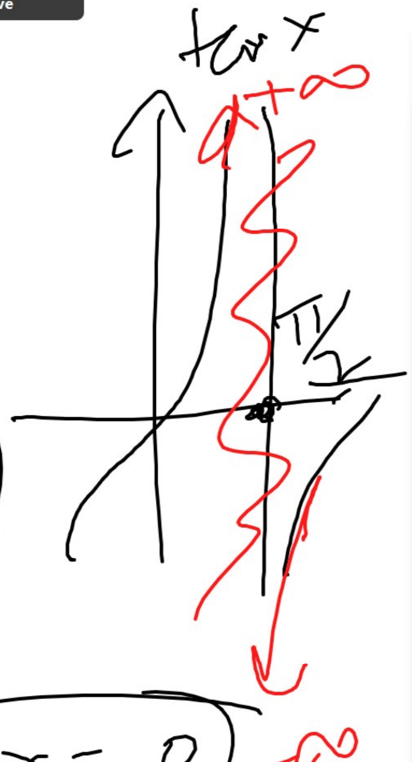
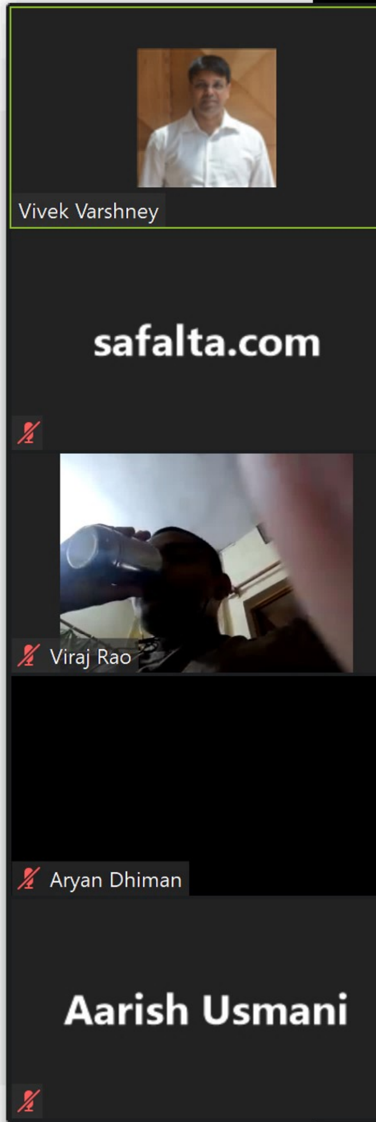
$$\tan x = \frac{\sin x}{\cos x}$$

$\cos x = 0$ where

$x = \frac{\pi}{2}$



tan x is Not continuous @ $x = \frac{\pi}{2}$



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$|x|$

LHL = RHL
= x = 0

at $x \to 0$

Properties of Continuous Function

If, at $x = a$

$f(x)$ continuous

$g(x)$ discontinuous

$\Rightarrow$ $f(x) \pm g(x)$

discontinuous . . .

ex

$\sin x + \tan x$ at $x = \frac{\pi}{2}$

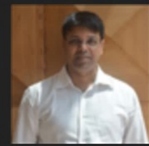
$f(x)$ discontinuous

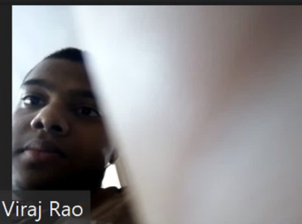
$g(x)$ discontinuous

$\Rightarrow$ $f(x) \pm g(x)$ $f(x).g(x)$, $\frac{f}{g}$

<u>may or may not be continuous</u>

? ? ? ? ?? ?





Differentiability : L.H.D & R.H.D

Rate of change

$$\frac{\Delta f}{\Delta x}$$

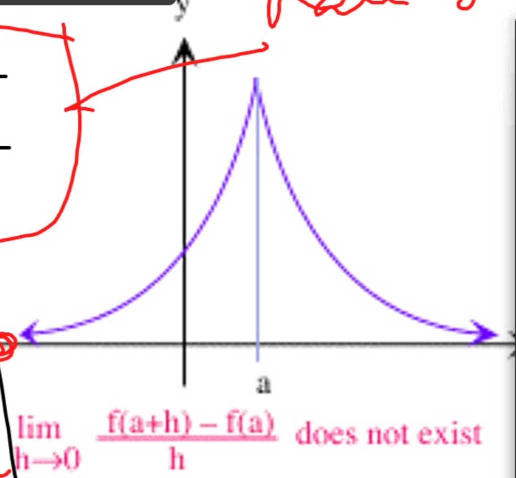
Left hand derivative:

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(a)-f(a-h)}{a-(a-h)}$$

$$f'(a^-) = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

h

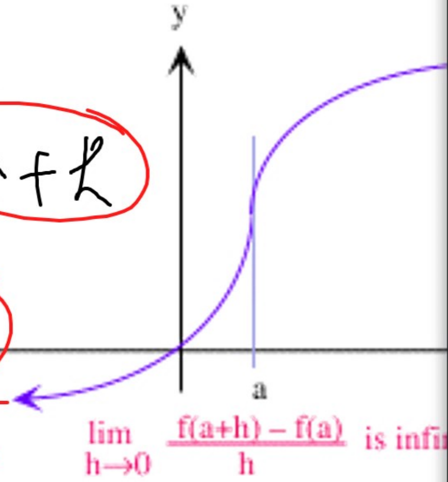
a − h a a + h



Right Hand Derivative:

$$f'(a^+) = \text{R.H.D} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$\frac{\Delta f}{\Delta x}$$



If $L.H.D = R.H.D$ then $y = f(x)$ is differentiable at $x = a$

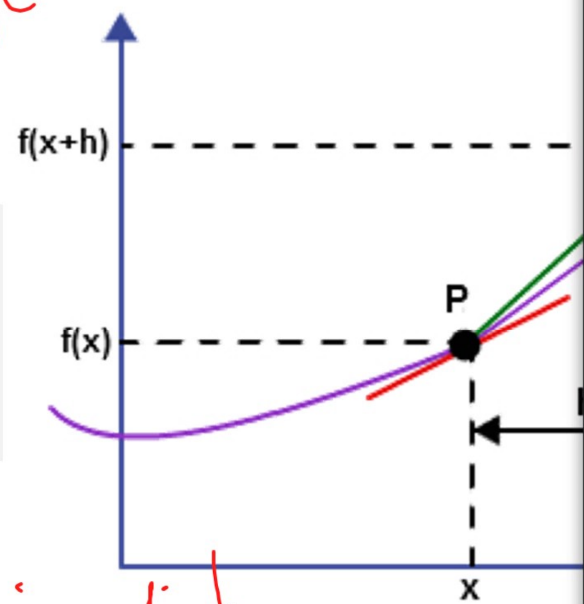
Differentiability

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\frac{\Delta f}{\Delta x}$

If you know function is differe
then use this formula

Where,
h is small positive quantity



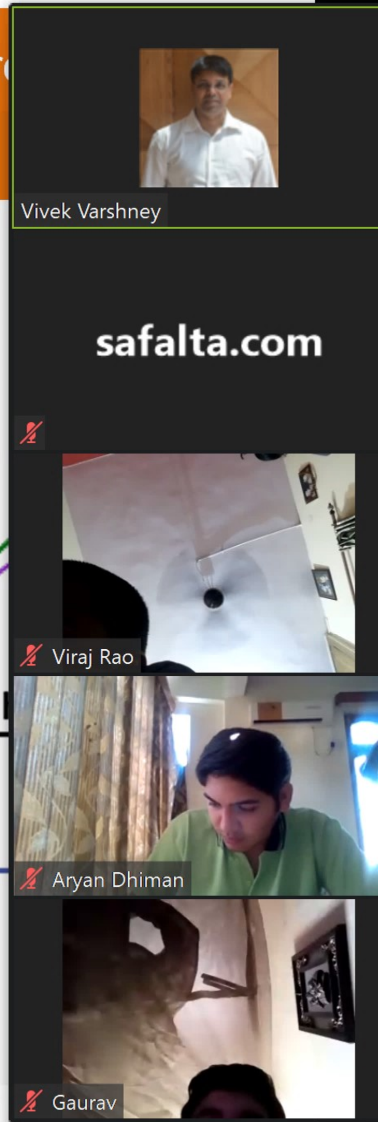
$$Ex: \; y = x^2 lnx$$

$$y' = \lim_{h \to 0} \frac{(x+h)^2 \ln(x+h) - x^2 lnx}{h}$$

Simplify & limit

$$Ex: \; y = e^{\cos x}$$

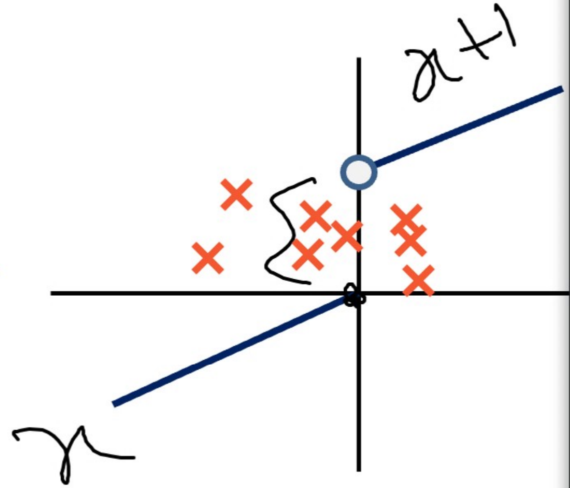
$$y' = \lim_{h \to 0} \frac{e^{\cos(x+h)} - e^{\cos x}}{h}$$



Relation Between Continuity and Differentiability Type I

1 If f(x) is **discontinuous** at $x = a$ then f(x) is **non-differentiable** at $x = a$

$$Ex: f(x) = x \quad , x \leq 0$$
$$= x + 1, x > 0$$



Ex

at $x = \frac{\pi}{2}$

tan x discont

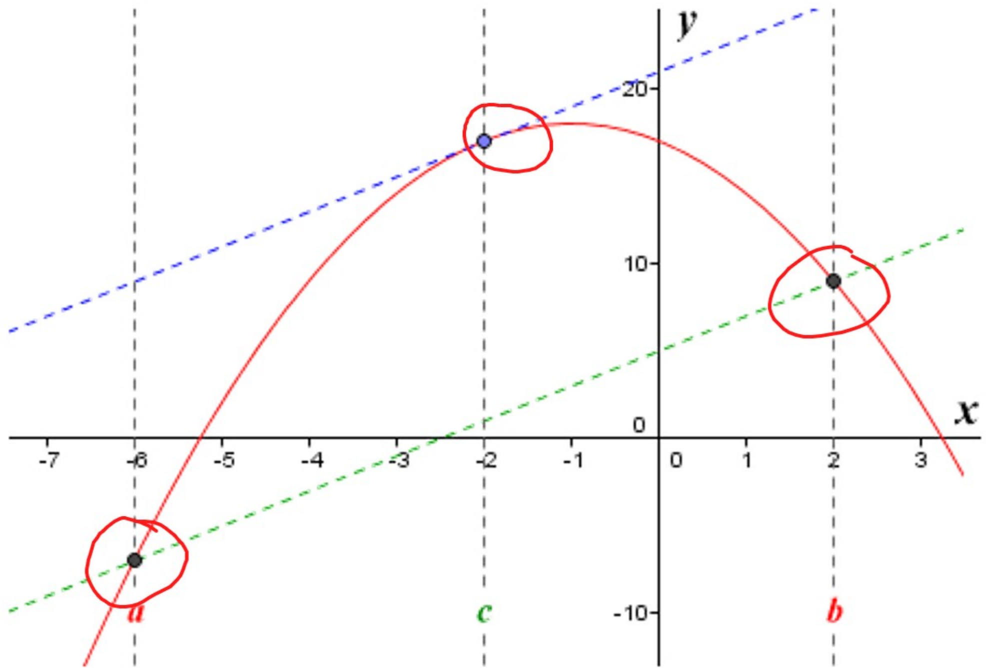
Not differentiable

Relation Betwe

2 If f(x) is **differentiable** at $x = a$, then f(x) is **continuous** at $x = a$

① LHD = RHD ✓

② function is continuous ✓

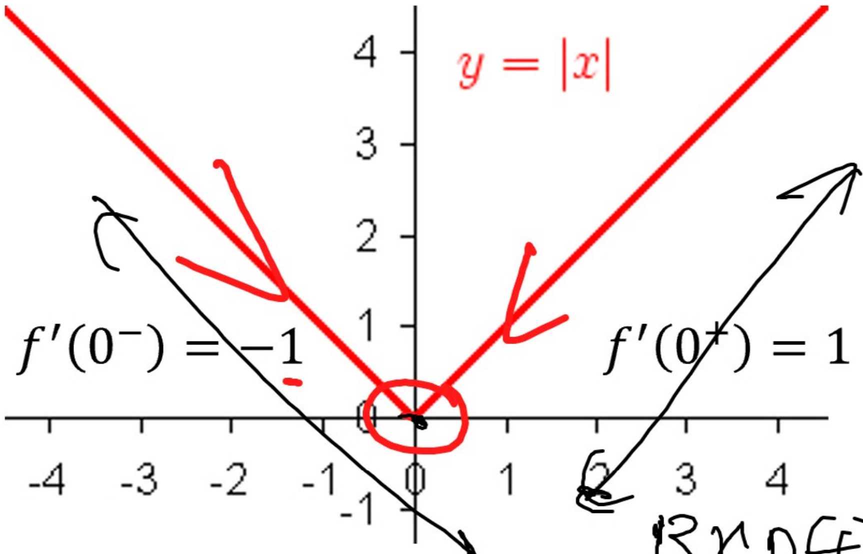


Relation Between Continuity and Differentiability Type II

If f(x) is **continuous** at $x = a$ then f(x) may or may not be **differentiable** at $x = a$

Corner

exam imp



LHD (−)

RHD (+)

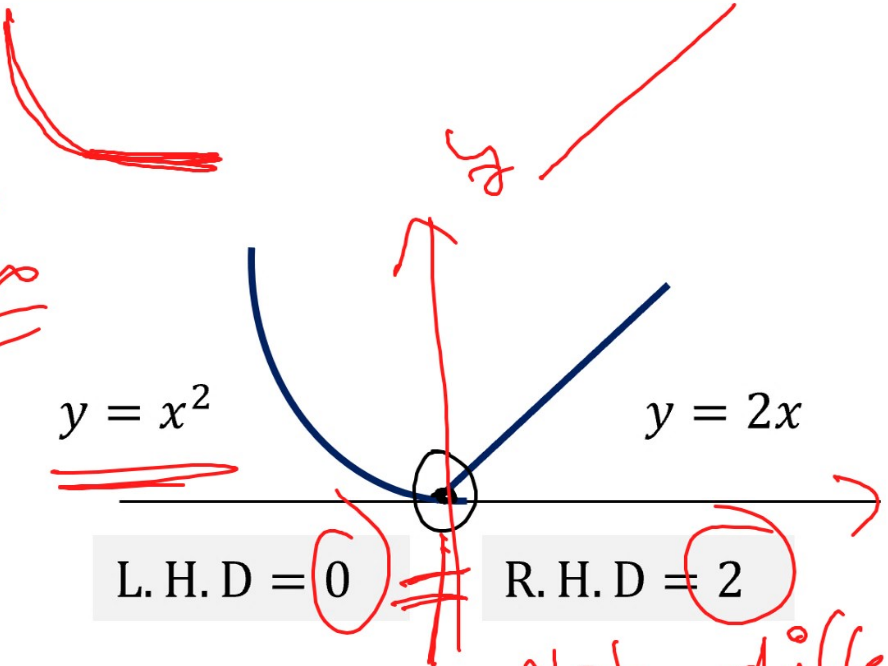
$|x| \Rightarrow$ is continuous

Corner

Continuous ✓

Not diff

L. H. D and R. H. D are different but $f(x)$ is **continuous**



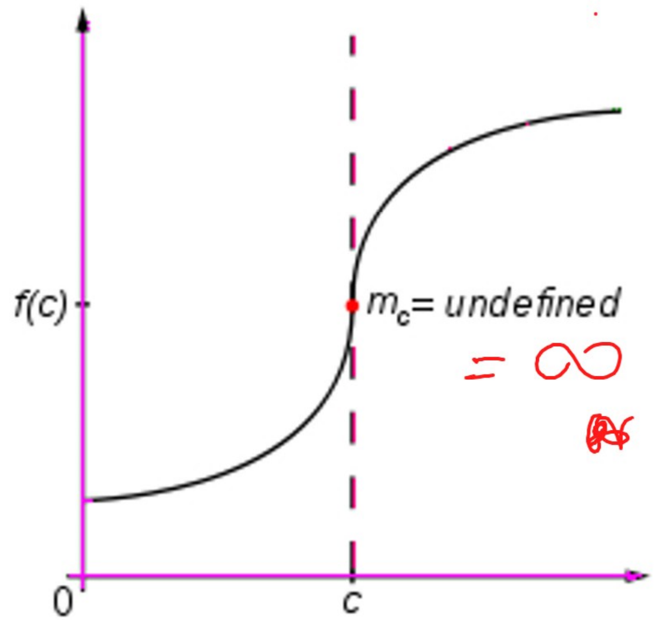
L. H. D = 0 $\neq$ R. H. D = 2

Not differentiable

Cases of Non-Differentiability: Vertical Tangent

Vertical tangent

A **tangent** line that is **vertical**, meaning it
infinite slope and function whose graph has a **vertical tangent** is not differentiable at the point of tangency.



In this case $|f'(x)| \to \infty$ at x = c

Vertical tangent on the function f(x) at x=c

Differentiability

Check following condition

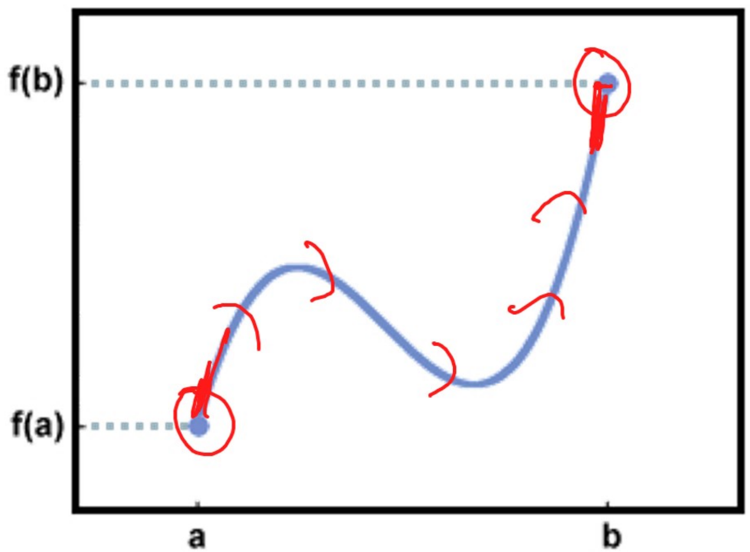
1. Differentiable on open interval (a, b)
2. Differentiable at $x = b$ (L. H. D)
3. Differentiable at $x = a$ (R. H. D)

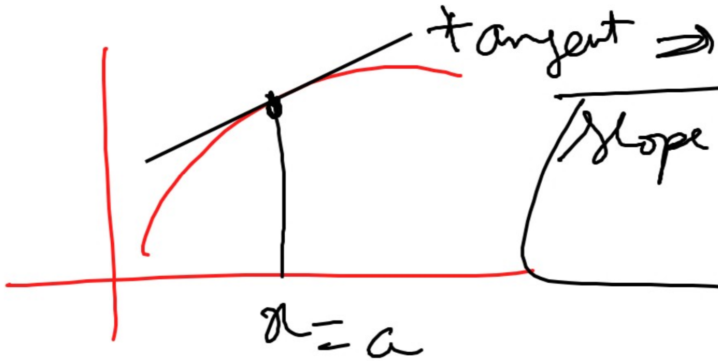
$\frac{dy}{dx}$

→ rate of change

→ slope of tangent

Check all the conditions for differentiability over a closed interval





$$\boxed{\text{slope} = \frac{dy}{dx}}$$