Distance and Displacement

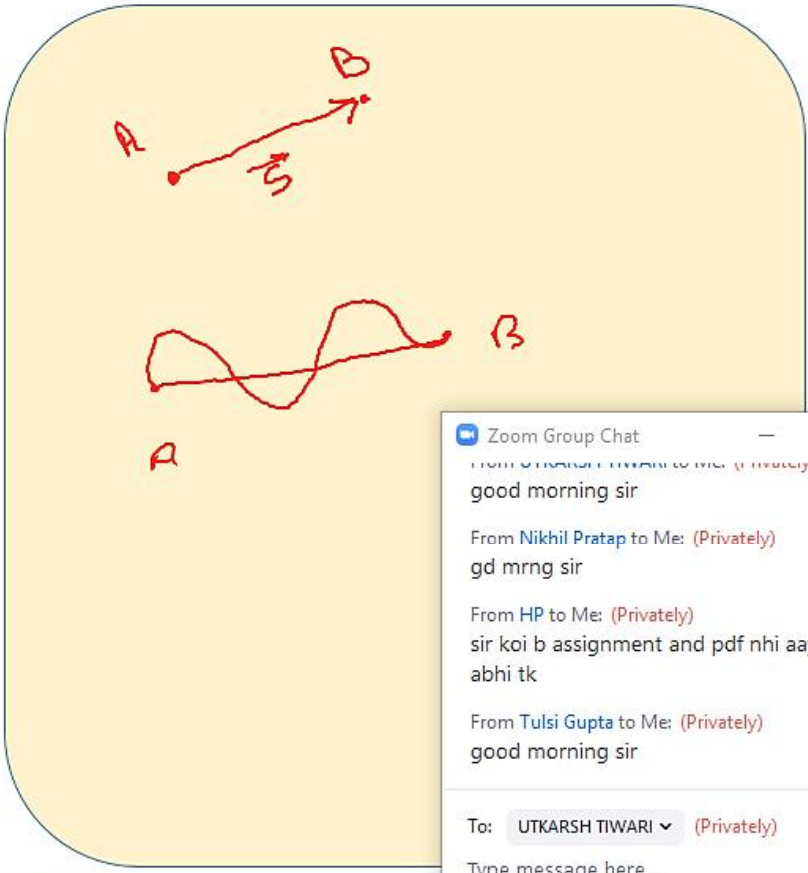
Displacement

- Vector joining the initial and final position of the particle.
- Can be negative, positive or zero.

Distance

- It is the length of the actual path travelled by a particle.
- Scalar quantity

Distance travelled ≥ |Displacement|



Average velocity and speed

Average Velocity

$$\text{Average Velocity} = \frac{\text{Displacement}}{\text{Time Interval}} = \frac{\text{Change in position}}{\text{Time Interval}}$$

$$= \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- It is vector in the direction of displacement.

Average Speed

$$\text{Average Speed} = \frac{\text{Distance travelled}}{\text{Time Interval}}$$

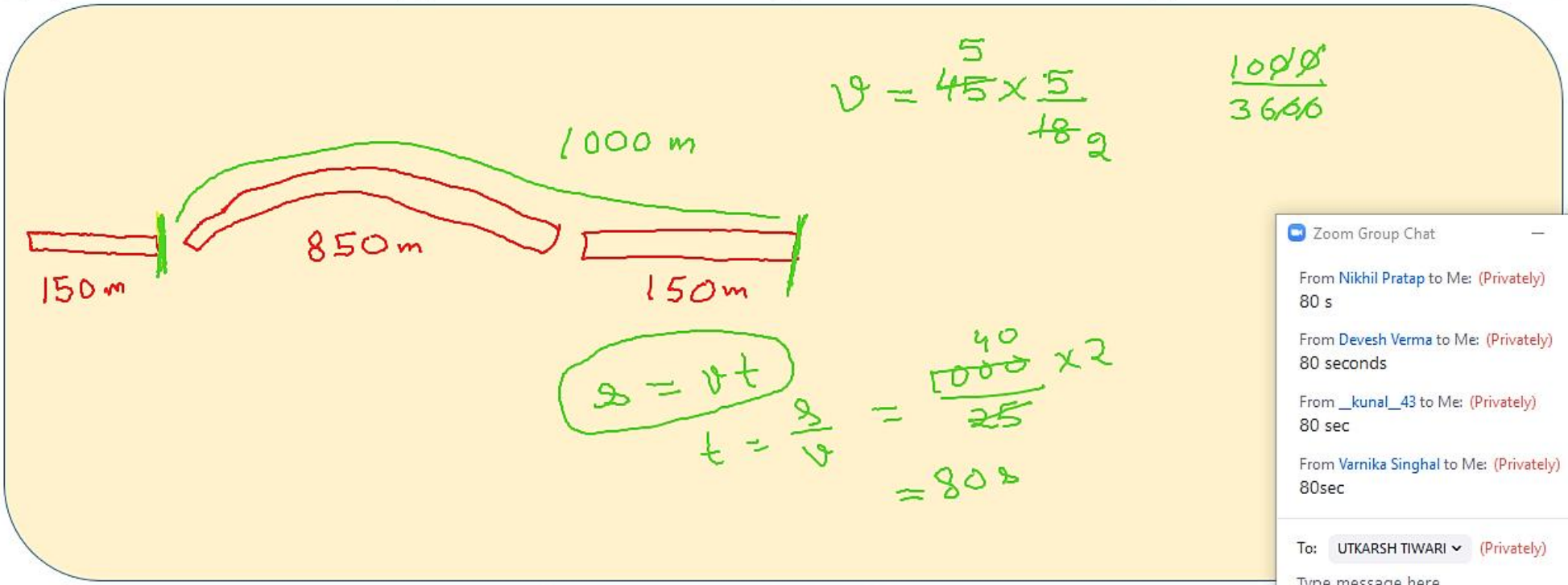
- Scalar quantity

Average Speed ≥ |Average Velocity|

Example

A 150 m long train is moving with a uniform velocity of 45 km / h. The time taken by the train to cross a bridge of length 850 metres is : **[2001]**

(1) 56 sec (2) 68 sec (3) 80 sec (4) 92 sec



Example

A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

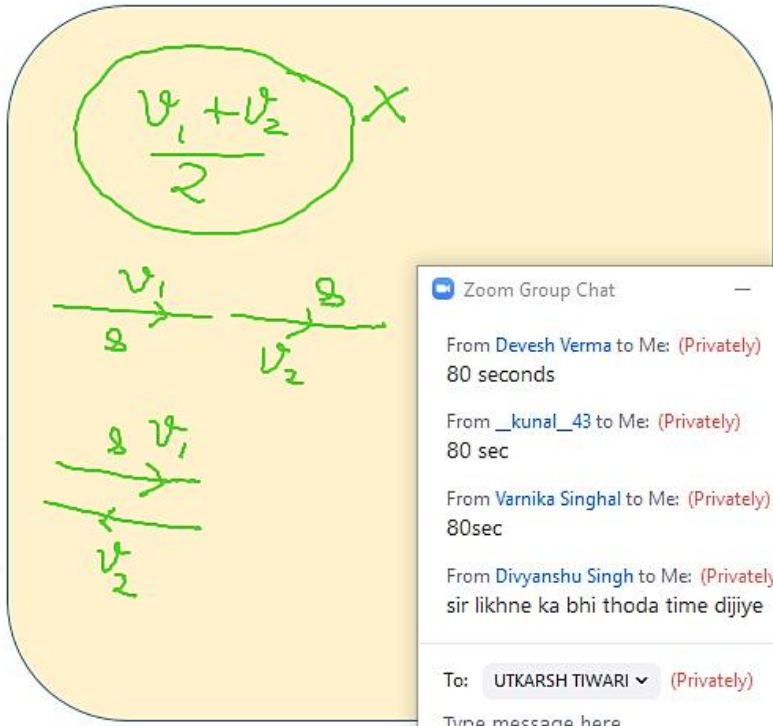
Sol.

Let total distance travelled by the particle be 2s.

Time taken to travel first half = $\frac{s}{v_1}$

Time taken to travel next half = $\frac{s}{v_2}$

Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \boxed{\frac{2v_1v_2}{v_1+v_2}}$



Example

A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by

Sol.

As shown, the person travels from A to B through a distance S, where first part S_1 is travelled in time t/2 and next S_2 also in time t/2.

So, according to the condition : $v_1 = \frac{S_1}{t/2}$ and $v_2 = \frac{S_2}{t/2}$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}} = \frac{S_1 + S_2}{t} = \frac{\frac{v_1 t}{2} + \frac{v_2 t}{2}}{t} = \boxed{\frac{v_1 + v_2}{2}}$$

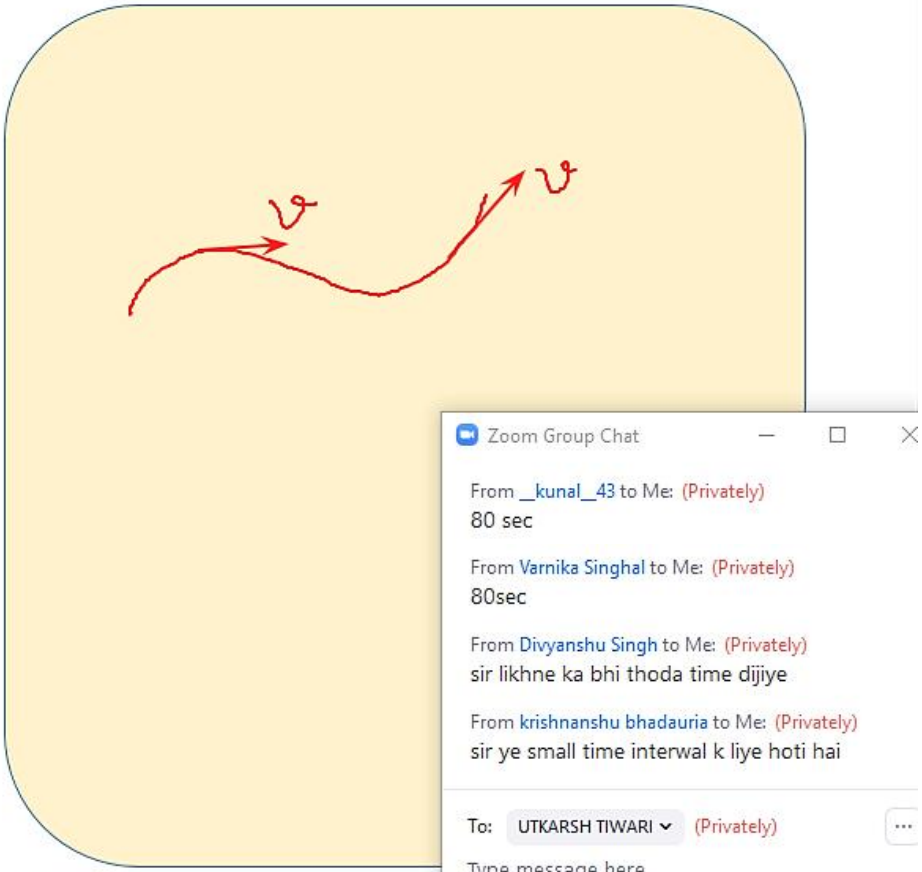
Instantaneous Velocity and Speed

Instantaneous Velocity

- The velocity at a particular instant of time is known as instantaneous velocity.

$$v = \frac{dx}{dt}$$

- It is always tangential to the path.



Instantaneous Speed

- The magnitude of instantaneous velocity is called instantaneous speed.

Instantaneous Speed = |Instantaneous Velocity|

Average and Instantaneous Acceleration

Average Acceleration

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

➡ ➢ Vector in the direction of change in velocity.

Instantaneous Acceleration

➢ It is acceleration at a particular instant of time.

$$a = \frac{dv}{dt} = v\frac{dv}{ds} = \frac{d^2x}{dt}$$

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{ds}\left(\frac{ds}{dt}\right)$$

$$= v\frac{dv}{ds}$$

Example

Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at t = 2 s?

Sol.

Velocity; $v = \frac{dx}{dt} = 10t + 4$

At $t = 2$ s

$v = 10(2) + 4$

$v = 24$ m/s

Acceleration; $a = \frac{d^2x}{dt^2} = 10$

Acceleration is constant, so at t = 2 s, $a = 10$ m/s²

$$\frac{d}{dt}(t^n) = n t^{n-1}$$

Example

Equation of displacement for any particle is $s = 3t^3 + 7t^2 + 14t + 8m$. Its acceleration at time t = 1 sec is : **[2000]**

(1) 10 m/s^2 (2) 16 m/s^2 (3) 25 m/s^2 (4) 32 m/s^2

$$v = \frac{ds}{dt} = 9t^2 + 14t + 14$$

$$a = \frac{dv}{dt} = 18t + 14$$

at $t = 1s$

$$a = 18(1) + 14$$

$$= 32 m/s^2$$

Example

The motion of a particle along a straight line is described by equation : $x = 8 + 12\,t - t^3$, where x is in metre and t in second. The retardation of the particle when its velocity becomes zero, is : **[2012]**

(1) 24 ms^{-2} (2) zero (3) 6 ms^{-2} (4) 12 ms^{-2}

$$v = \frac{dx}{dt} = 12 - 3t^2$$

$$v = 0 \Rightarrow 12 - 3t^2 = 0 \Rightarrow t^2 = 4 \Rightarrow t = 2$$

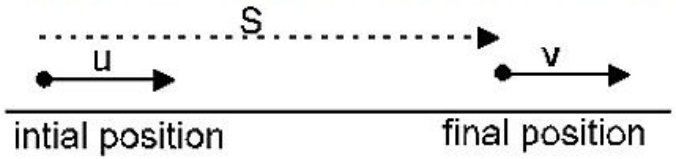
$$a = \frac{dv}{dt} = -6t$$

At $t = 2s$

$$a = -12\ m/s^2$$

Uniformly Accelerated Motion

Motion with constant acceleration



(a) $v = u + at$

(b) ➡ $s = ut + 1/2\ at^2$

➡ $s = vt - 1/2\ at^2$

$x_f = x_i + ut + 1/2\ at^2$

(c) $S = \left(\frac{v+u}{2}\right)t$

(d) $V_{av} = \frac{v+u}{2}$

(e) $v^2 = u^2 + 2as$

(f) $s_n = u + a/2\ (2n - 1)$

u = initial velocity
a = acceleration
v = final velocity
s = displacement ($x_f - x_i$)
x_f = final coordinate (position)
x_i = initial coordinate (position)
s_n = displacement during the n^{th} sec

Note: In case of motion under gravity, constant acceleration is g downwards.

(n-1) to n

4th sec

3s to 4s

Example

A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Sol.

Let the total distance be 2x.

$\therefore$ distance upto midpoint = x

Let the velocity at the mid point be v and acceleration be a.

From equations of motion

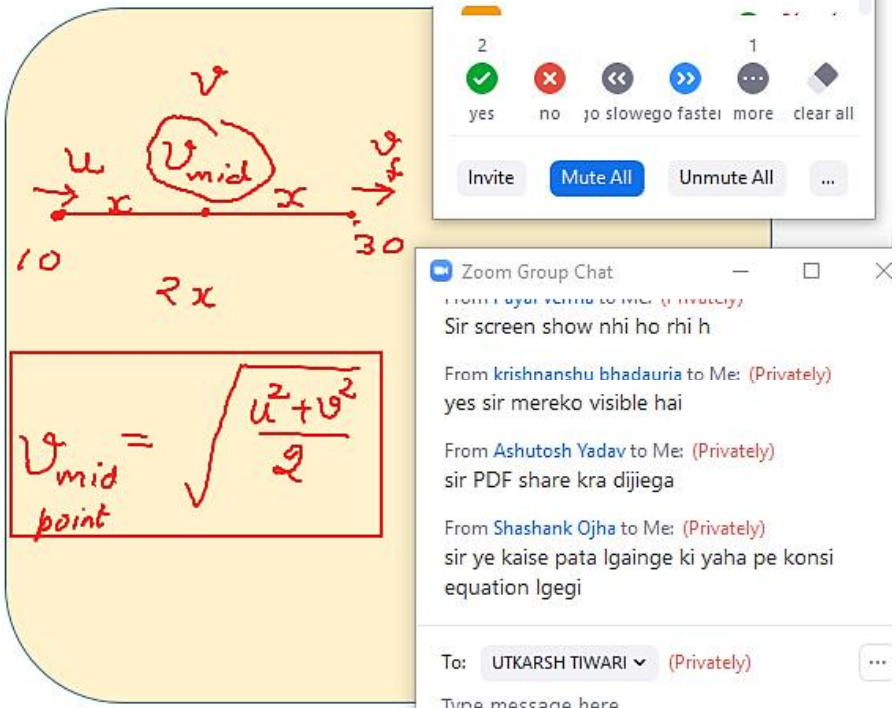
$$v^2 = 10^2 + 2ax \quad \ldots (1)$$

$$30^2 = v^2 + 2ax \quad \ldots (2)$$

(2) - (1) gives

$$v^2 - 30^2 = 10^2 - v^2$$

$$\Rightarrow \quad v^2 = 500 \quad \Rightarrow \quad v = 10\sqrt{5} \text{ m/s}$$



Example

A particle is moving in a straight line with a constant acceleration. It changes its velocity from 10 m/s to 20 m/s, while passing through a distance 135 m in t seconds. The value of t is : **[2008]**

(1) 10 (2) 1.8 (3) 12 (4) 9

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \left(\frac{u+v}{2}\right)t$

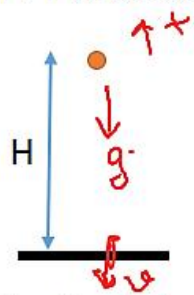
$135 = \left(\frac{10+20}{2}\right)t$

$135 = 15t$

$t = \frac{135}{15} = 9\,s$

MOTION UNDER GRAVITY

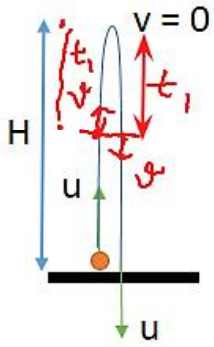
Case I : Particle is dropped at rest from height H.



$$T = \sqrt{\frac{2H}{g}}$$

$$v = \sqrt{2gH}$$

Case II : Particle is thrown with speed u from ground.



$$T = \frac{2u}{g}$$

$$H = \frac{u^2}{2g}$$

$t_0 = \frac{u}{g}$; $t_d = \frac{u}{g}$

$g = 9.8\ m/s^2$
$\approx 10\ m/s^2$

Example

Two balls are dropped from heights h and 2h respectively from the earth surface. The ratio of time of these balls to reach the earth is : **[2003]**

(1) $1 : \sqrt{2}$ (2) $\sqrt{2} : 1$ (3) $2 : 1$ (4) $1 : 4$

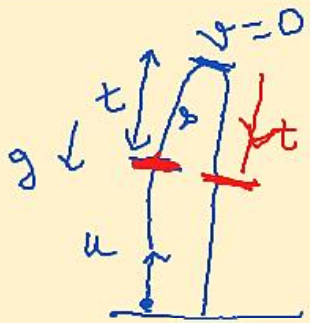
$$T = \sqrt{\frac{2h}{g}} \Rightarrow T \propto \sqrt{h}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{h}{2h}} = 1/\sqrt{2}$$

Example

If a ball is thrwon vertically upwards with speed u, the distance covered during the last t seconds of its ascent is : **[2003]**

(1) $\frac{1}{2}gt^2$ (2) $ut - \frac{1}{2}gt^2$ (3) $(u - gt)t$ (4) ut



$s = ut - \frac{1}{2}at^2$

$s = 0 - \frac{1}{2}(-g)t^2 = \frac{1}{2}gt^2$

$u = 0$

$s = ut + \frac{1}{2}at^2$

$= \frac{1}{2}gt^2$

Example

A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower? (take g = 10 m/s²)

Sol. Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n-1)$

$\Rightarrow \quad 45 = 0 + \frac{10}{2}(2n-1)$

$n = 5$ sec

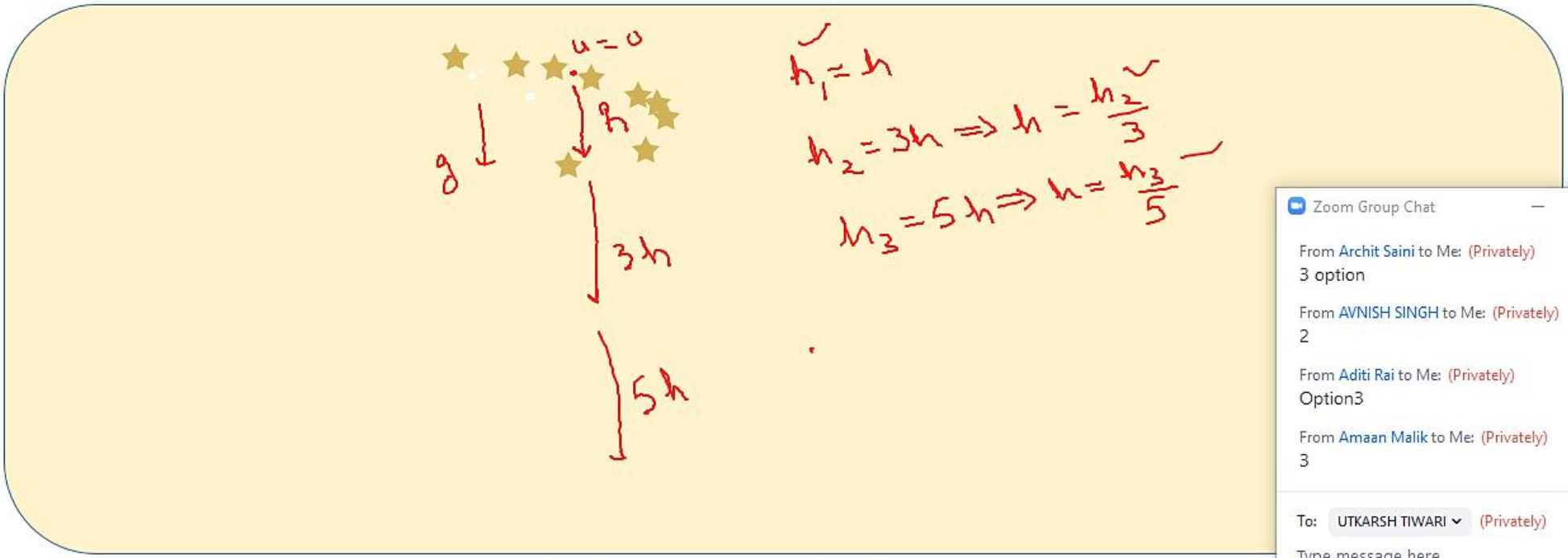
Height of tower; $h = \frac{1}{2} g t^2$

$= \frac{1}{2} \times 10 \times 5^2 = 125$ m

Example

A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is :- **[2013]**

(1) $h_1 = h_2 = h_3$ (2) $h_1 = 2h_2 = 3h_3$ (3) $h_1 = h_2/3 = h_3/5$ (4) $h_2 = 3h_1$ and $h_3 = 3h_2$



Example

From the top of a tower, a particle is thrown vertically downwards with a velocity of 10 m/s. The ratio of the distances, covered by it in the 3rd and 2nd seconds of the motion is (Take g = 10 m/s^2)

(1) 5 : 7 (2) 7 : 5 (3) 3 : 6 (4) 6 : 3 **[2002]**

$$S_n = u + \frac{a}{2}(2n-1)$$

$$S_2 = 10 + \frac{10}{2}(2\times 2-1) = 10 + 15 = 25$$

$$S_3 = 10 + \frac{10}{2}(2\times 3-1) = 10 + 25 = 35$$

$$\frac{S_3}{S_2} = \frac{35}{25} = \frac{7}{5}$$