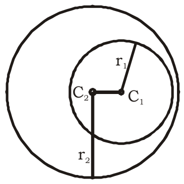


CIRCLES

Circles

A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

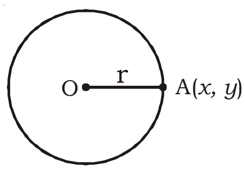
The fixed point is called centre of the circle and distance from the centre to a point on the circle is called radius of the circle.

Standard equation of a circle

Let the radius of a circle be r and O(h, k) is the centre of circle.

Let A (x, y) be any point of circle

According to definition of circle,



OA = r

$\Rightarrow OA^2 = r^2$

$\Rightarrow (x-h)^2 + (y-k)^2 = r^2$

This is standard equation of circle with radius r and centre (h, k).

General equation of a circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ always represents a circle whose centre is (–g, –f) and radius = $\sqrt{g^2 + f^2 - c}$

The length of intercepts made by circle with x-axis & y-axis are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$ respectively.

Example:- Find the eqution of circle whose centre is at point (4, 5) and passes through (3, –2)

Solution:- The general equation of circle is

$(x-h) + (y-k)^2 = r^2$

h = 4 , k = 5

$(x-4)^2 + (y-5)^2 = r^2$

Since it passes through (3, –2)

So, $(3-4)^2 + (-2-5)^2 = r^2$

$r^2 = 1 + 49$

$r^2 = 50$

So, required equation is

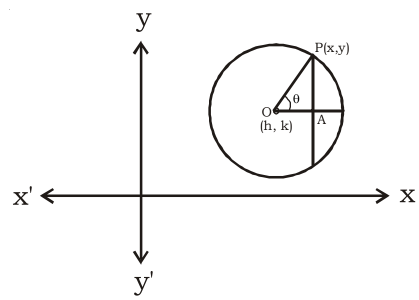
$(x-4)^2 + (y-5)^2 = 50$

Equation of circle in parametric form

If (h, k) is the centre and r is radius, then the equation of circle in parametric form is

$(x-h)^2 + (y-k)^2 = r^2$

Where $x = h + r\cos\theta$, $y = k + r\sin\theta$



and θ is the parameter.

Note:- If centre is at origin, then required parametric form is,

$x^2 + y^2 = r^2$ where $x = r\cos\theta$, $y = r\sin\theta$

Concentric Circles

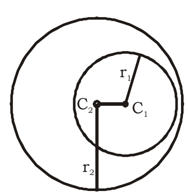
Two circles having same centre and different radius are called concentric circles.

Hence equation of two circles with centre (h, k) & radius r_1 & r_2 are given by $(x-h)^2 + (y-k)^2 = r_1^2$ and $(x-h)^2 + (y-k)^2 = r_2^2$

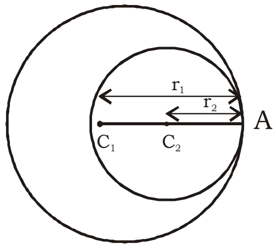
Condition of two circles touch each other

(i) If one circle lies **inside** the other,

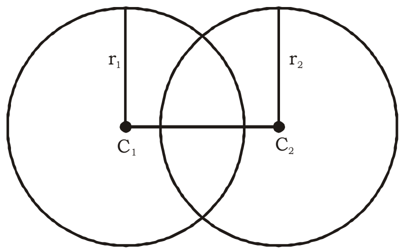
Then $C_1C_2 \geq |r_1 - r_2|$



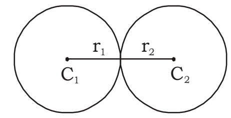
(ii) If two circle touch each other internally **at a point A, then $C_1C_2 = |r_1 - r_2|$**



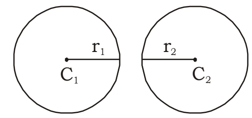
(iii) If two circles **intersect** each other,
Then $|r_1 - r_2| < C_1 C_2 < r_1 + r_2$



(iv) If two circles touch each other **externally** at a point.
Then, $C_1 . C_2 = r_1 + r_2$



(V) If two circles lies **outside** the other,
Then $C_1C_2 > |r_1 + r_2|$



Some Important Results

- If circle passes through origin, then equation is, $(x - h)^2 + (y - k)^2 = h^2 + k^2$
- If centre is at origin, then equation is $x^2 + y^2 = r^2$
- If circle touch x-axis, then equation is, $(x - h)^2 + (y - k)^2 = k^2$
- If circle touch y-axis, then equation is, $(x - h)^2 + (y - k)^2 = h^2$
- If circle touch boh x-axis and y-axis, then eqution is, $(x - a)^2 + (y - a)^2 = a^2$, [h = k = a]
- The equation of circle with given end points of diameter (x_1, y_1) & (x_2, y_2) is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$