INVERSE TRIGONOMETRIC FUNCTION

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

Property I

(i) $\sin^{-1}(\sin x) = x$; $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos x) = x$; $x \in [0, \pi]$

(iii) $\tan^{-1}(\tan x) = x$; $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$= x - \pi$; $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(iv) $\cot^{-1}(\cot x) = x$; $x \in (0, \pi)$

(v) $\sec^{-1}(\sec x) = x$; $x \in [0, \pi], x \neq \frac{\pi}{2}$

(vi) $\text{cosec}^{-1}(\text{cosec}\, x) = x$; $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \sim \{0\}$

Property II

(i) $\sin(\sin^{-1}x) = x$; $x \in [-1, 1]$

(ii) $\cos(\cos^{-1}x) = x$; $x \in [-1, 1]$

(iii) $\tan(\tan^{-1}x) = x$; $x \in R$

(iv) $\cot(\cot^{-1}x) = x$; $x \in R$

(v) $\sec(\sec^{-1}x) = x$; $x \in R - (-1, 1)$

(vi) $\text{cosec}(\text{cosec}^{-1}x) = x$; $x \in R - (-1, 1)$

Property III

(i) $\sin^{-1}(-x) = -\sin^{-1}x$; $x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$; $x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$; $x \in R$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$; $x \in R$

(v) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$; $x \in R - (-1, 1)$

(vi) $\sec^{-1}(-x) = \pi - \sec^{-1}x$; $x \in R - (-1, 1)$

Property IV

(i) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x$; $x \in R - (-1, 1)$

(ii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$; $x \in R - (-1, 1)$

(iii) $\tan^{-1}\frac{1}{x} = \begin{cases} \cot^{-1}x & \text{for } x > 0 \\ -\pi + \cot^{-1}x & \text{for } x < 0 \end{cases}$

Property V

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$; $x \in [-1, 1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$; $x \in R$

(iii) $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$; $x \in R - (-1, 1)$

Property VI

(i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ *if* $xy < 1$

$= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ *if* $xy > -1$

(iii) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

Property VII

(i) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$; $-1 < x < 1$

(ii) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$; $-1 \le x \le 1$

(iii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $x \ge 0$

Property VIII

(i) $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$; $\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(ii) $2\cos^{-1}x = \cos^{-1}\left(2x^2 - 1\right)$; $0 \le x \le 1$

Property IX

(i) $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$; $\frac{-1}{2} \le x \le \frac{1}{2}$

(ii) $3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right)$; $\frac{1}{2} \le x \le 1$

(iii) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$; $\frac{-1}{\sqrt{3}} \le x \le \frac{1}{\sqrt{3}}$

Property X

(i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

if $x^2 + y^2 \leq 1$ or $xy < 0$

(ii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

if $x^2 + y^2 \leq 1$ or $xy > 0$

Property XI

(i) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-y^2}\sqrt{1-x^2}\right]$

if $x + y \geq 0$

(ii) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy + \sqrt{1-y^2}\sqrt{1-x^2}\right]$

if $x \leq y$

Property XII

(i) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$

$= \cot^{-1}\dfrac{\sqrt{1-x^2}}{x} = \sec^{-1}\dfrac{x}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1}\dfrac{1}{x}$

(ii) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$

$= \sec^{-1}\dfrac{1}{x} = \operatorname{cosec}^{-1}\dfrac{1}{\sqrt{1-x^2}} = \cot^{-1}\dfrac{x}{\sqrt{1-x^2}}$

(iii) $\tan^{-1}x = \sin^{-1}\dfrac{x}{\sqrt{1+x^2}} = \cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$

$= \cot^{-1}\dfrac{1}{x} = \sec^{-1}\sqrt{1+x^2} = \operatorname{cosec}^{-1}\dfrac{\sqrt{1+x^2}}{x}$