7 Vector Algebra

1. Projection of $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ along $\mathbf{i} - \mathbf{j} + \mathbf{k}$ is

   (A) $\frac{2}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$

   (C) $\sqrt{3}$ (D) $2\sqrt{3}$

2. Area of parallelogram *ABCD*, when $\mathbf{AB} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{BD} = -\mathbf{i} + \mathbf{j}$ is

   (A) 1 sq unit (B) 6 sq unit

   (C) 3 sq unit (D) None of these

3. Angle between the vectors $\mathbf{a} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = x\mathbf{i} + \mathbf{j} + (x + 1)\mathbf{k}$

   (A) is obtuse angle (B) is acute angle

   (C) is right angle (D) depends on x

4. If the points *A*, *B* and *C* with position vectors $2\mathbf{i} \times 2\mathbf{j}$, $\lambda\mathbf{i} + 8\mathbf{j}$ and $8\mathbf{i} + 32\mathbf{j}$ are collinear, then λ is equal to

   (A) $\frac{8}{5}$ (B) $\frac{16}{5}$

   (C) 4 (D) None of these

5. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are three unit vectors, then $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}$ is equal to ($\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$)

   (A) 0 (B) 1

   (C) $\frac{3}{2}$ (D) None of these

6. For what value of m are the points with position vectors $10\mathbf{i} + 3\mathbf{j}, 12\mathbf{i} - 5\mathbf{j}$ and $m\mathbf{i} + 11\mathbf{j}$ collinear?

   (A) -8 (B) 4

   (C) 8 (D) 12

7. Angle between the vector $\sqrt{3}(\mathbf{a} \times \mathbf{b})$ and $\mathbf{b} - (\mathbf{a}.\mathbf{b})\mathbf{a}$ is

   (A) $\frac{\pi}{2}$ (B) 0

   (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

8. If a, b are two unit vectors at an angle θ, then magnitude of $\mathbf{a} + \mathbf{b}$ is

   (A) 2 (B) $\sqrt{2}$

   (C) $2\cos\frac{\theta}{2}$ (D) None of these

9. A unit vector in *xy*-plane makes an angle 45° with the vector $\mathbf{i} + \mathbf{j}$ is

   (A) $\mathbf{i}$ (B) $\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$

   (C) $\frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$ (D) None of these

10. If $(\mathbf{a} \times \mathbf{b})^2 + (\mathbf{a}.\mathbf{b})^2 = 144$ and $|\mathbf{a}| = 4$, then $\mathbf{b}$ is equal to

    (A) 16 (B) 8

    (C) 3 (D) 12

11. Let *p*, *q*, *r* and *s* be respectively the magnitude of the vectors $3\mathbf{i} + 2\mathbf{j}$, $2\mathbf{i} + \mathbf{k}$, $4\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Which one of the following is correct?

    (A) $r > s > q > p$ (B) $s > r > p > q$

    (C) $r > s > p > q$ (D) $s > r > q > p$

12. The position vector of three points *A*, *B* and *C* are $\mathbf{i} + \mathbf{j}$, $\mathbf{i} - \mathbf{j}$ and $l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$, respectively. The points are collinear, if

    (A) $l = m = n = 1$ (B) $l = 1, m, n \in R$

    (C) $l = 1, n = 0, m \in R$ (D) $m = 0, n = 1, l \in R$

13. If $\mathbf{a}$, $\mathbf{b}$ are two unit vectors and θ is the angle between them, then the value of $\cos\frac{\theta}{2}$ is equal to

    (A) $\frac{1}{2}[\mathbf{a} - \mathbf{b}]$ (B) $\frac{1}{2}[\mathbf{a}.\mathbf{b}]$

    (C) $\frac{|\mathbf{a} \times \mathbf{b}|}{2|\mathbf{a}||\mathbf{b}|}$ (D) $\frac{1}{2}|\mathbf{a} + \mathbf{b}|$

14. If $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is a unit vector and $x : y : z = \sqrt{3} : 2 : 3$, what is the value of z?

(A) $\frac{3}{16}$ (B) 3

(C) $\frac{3}{4}$ (D) 2

15. The projection of the vector $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ on the vector $\mathbf{b} = 4\mathbf{i} + 7\mathbf{k}$ is equal to

(A) $\frac{\sqrt{6}}{9}$ (B) $\frac{19}{9}$

(C) $\frac{9}{19}$ (D) $\frac{\sqrt{6}}{19}$

16. If the vectors $\mathbf{i} - x\mathbf{j} - y\mathbf{k}$ and $\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ are orthogonal to each other, then what is the locus of the point (x, y)?

(A) A parabola (B) An ellipse

(C) A circle (D) A straight line

17. If $\mathbf{a} = \mathbf{i} - \mathbf{k}$,
$\mathbf{b} = x\mathbf{i} + \mathbf{j} + (1 - x)\mathbf{k}$ and
$\mathbf{c} = y\mathbf{i} \times x\mathbf{j} + (1 + x - y)\mathbf{k}$.
then $\mathbf{a}.(\mathbf{b} \times \mathbf{c})$ depends upon

(A) x only (B) y only

(C) both x and y (D) Neither x nor y

18. The area of the parallelogram with $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ as consecutive sides is equal to

(A) $\sqrt{61}$ sq units (B) $2\sqrt{14}$ sq units

(C) $2\sqrt{7}$ sq units (D) $\sqrt{14}$ sq units

19. The moment about the point $A(3, -1, 3)$ of a force $\mathbf{F} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ through the point $B(5, 1, 4)$ is

(A) $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ (B) $7\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$

(C) $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ (D) $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

20. If $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 144$ and $|\mathbf{a}| = 4$, then $|\mathbf{b}|$ is equal to

(A) 3 (B) 8

(C) 12 (D) 16

21. Let $\mathbf{a}$ and $\mathbf{b}$ be two unit vectors and α be the angle between them. If $(\mathbf{a} + \mathbf{b})$ is also the unit vectors what is the value of α?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{2}$

22. What is the unit vectors parallel to xy-plane and perpendicular to the vector $4\mathbf{i} - 3\mathbf{j} \times \mathbf{k}$?

(A) $\pm(3\mathbf{i} + 4\mathbf{j})/5$ (B) $\pm(4\mathbf{i} + 3\mathbf{j})/5$

(C) $\pm(3\mathbf{i} - 4\mathbf{j})/5$ (D) $\pm(4\mathbf{i} - 3\mathbf{j})/5$

23. What is the vector in the xy-plane through origin and perpendicular to the vector $\mathbf{r} = a\mathbf{i} + b\mathbf{j}$ and of the same length?

(A) $-a\mathbf{i} - b\mathbf{j}$ (B) $a\mathbf{i} - b\mathbf{j}$

(C) $-a\mathbf{i} + b\mathbf{j}$ (D) $b\mathbf{i} - a\mathbf{j}$

24. What is the area of the parallelogram having diagonal $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$?

(A) $5\sqrt{2}$ sq unit (B) $4\sqrt{3}$ sq unit

(C) $5\sqrt{3}$ sq unit (D) $10\sqrt{3}$ sq unit