

अमर उजाला
मॉडल पेपर
 केवल प्रश्न-पत्र-2017 **कक्षा-12**
MATHEMATICS **सीबीएसई बोर्ड**

Time- 3 hr. M.M-100

- General Instructions:**
 (i) All questions are compulsory.
 (ii) This question paper contains 29 questions.
 (iii) Question 1-4 in Section A are very short-answer type questions carrying 1 mark each.
 (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
 (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
 (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION-A
Questions from 1 to 4 are of 1 mark each.

- Q-1** Evaluate: $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.
Q-2 If A is a square matrix and $|A| = 2$, then write the value of $|A A^T|$, where A^T is the transpose of matrix A .
Q-3 Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$.
Q-4 Find $g \circ f, f, g: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 5x + 2$ and $g(x) = x^2 + 6$.

SECTION-B
Questions from 5 to 12 are of 2 marks each.

- Q-5** Let $A = \{(1, 2, 3)\}$. Show that the number of relations on A containing $\{(1, 2)$ and $\{(2, 3)\}$ which are reflexive and transitive but not symmetric is three.
Q-6 If a unit vector \vec{a} makes angles $\frac{\pi}{3}$ with L_1 , $\frac{\pi}{4}$ with L_2 and an acute angle θ with L_3 , then find θ and hence find the components of \vec{a} .
Q-7 If $f(x) = \begin{bmatrix} 3 \cos x & x^2 \\ 12 & e^x \end{bmatrix}$, then find $f'(x)$.
Q-8 Find the sum of the order and the degree of the following differential equations:
 $5x \left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$
Q-9 Find the vector and the Cartesian equations of line through the point $(5, 2, -4)$ and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.
Q-10 Solve the following L.P.P. graphically:
 Minimize $Z = x - 2y$
 Subject to the constraints:
 $x - 3y \leq 0, 2x + 3y \leq 6, 3x + y \geq 0, x \geq 0, y \geq 0$

- Q-11** A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is a king or a heart or a red card.
Q-12 Find the point on the curve $y^2 = 8x + 8$ for which the abscissa and ordinate change at the same rate.

SECTION-C
Questions from 13 to 23 are of 4 marks each.

- Q-13** Without expanding, prove that $\begin{vmatrix} b^2+c^2 & bc & b+c \\ c^2+a^2 & ca & c+a \\ a^2+b^2 & ab & a+b \end{vmatrix} = 0$

Prove, using properties of determinants:
 $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc.$

- Q-14** It is given that for the function f given by $f(x) = x^3 + bx^2 + ax, x \in [1, 3]$, Rolle's theorem holds with $c = 2 + \frac{a}{3}$. Find the values of a and b .
Q-15 Prove that the function f given by $f(x) = x^3 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$.
Q-16 Evaluate $\int (2x^2 + 5x) dx$ as limit of sums.
Q-17 Find the area bounded by the parabola $y = x^2$ and $y = |x|$.
Q-18 Show that the differential equation $x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0$ is homogenous, find the particular solution of this differential equation, given that $x = \frac{1}{2}$ when $y = \frac{\pi}{2}$.
 OR
 Find the general solution of the differential equation $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$
Q-19 Find the point of the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-2}{2}$ at a distance $3\sqrt{2}$ from the point $(1, 2, 3)$.
Q-20 A variable plane moves in such a way that the sum of reciprocals of its intercepts on three coordinate axes is constant. Show that the plane passes through a fixed point.
Q-21 If a 19 year old girl drives her car at 25 km/hr, she has to spend Rs. 2/km on petrol. If she drives it a faster speed of 40 km/hr, the petrol cost increases to Rs. 5/km she has Rs. 100 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express it as a L.P.P. and then solve it. **Is the girl eligible for a driving license? What is the benefit of driving at economic speed?**
Q-22 Let X denotes the number of hours your study during a randomly selected school day. The probability that X can take the values x , has the following form, where k is some unknown constant.

$$P(X) = \begin{cases} 0.1 & \text{if } x = 0 \\ kx & \text{if } x = 1 \text{ or } 2 \\ k(5-x) & \text{if } x = 3 \text{ or } 4 \\ 0 & \text{otherwise} \end{cases}$$

 (i) Determine the value of k .
 (ii) What is the probability that you study at least 2 hours? Exactly 2 hours? At most 2 hours?
Q-23 There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes tails up 25% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two-headed coin?


SECTION-D
Questions from 24 to 29 are of 6 marks each.

- Q-24** Evaluate $\int_0^{\pi} \frac{x(1-\sin x)}{1+\cos^2 x} dx$
 OR
 Evaluate $\int \frac{1}{\sin^2 x + \cos^2 x} dx$
Q-25 Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: \mathbb{N} \rightarrow S$ is invertible (where S is range of f). Find the inverse of f .
 OR
 Determine whether the function $*$ defined below is binary operation or not. If yes, determine whether it is commutative and associative. Find the identity (if exists). Find the invertible element (if any) and its inverse.
 $*$ on \mathbb{N} is defined by $a * b = LCM(a, b)$.
Q-26 $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is matrix satisfies $A A^T = 9I_3$. Find a & b .
 OR
 Verify: $A(Adj A) = (Adj A)A = |A|I$ for matrix $A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Q-27 If $y = x \log\left(\frac{x}{1-x}\right)$, then prove that $\frac{d^2y}{dx^2} = \left[x \frac{2y}{1-x} - y\right]$.
Q-28 Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$. Also show that the maximum volume of the cone is $\frac{32}{27}$ of the volume of the sphere.
Q-29 Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (i + 2j + 3k) - 4 = 0, \vec{r} \cdot (2i + j - k) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5i + 3j - 6k) + 8 = 0$

TIPS

- Read the sections thoroughly and then answer the questions.
- To complete the paper time management is essential, as the paper is quite lengthy.
- Divide the allotted time as per the questions.
- Make sure that the answers are precise.

नोट- यह प्रतिदर्श प्रश्न पत्र है जल्द ही जारी है यही प्रश्न परियोजना परीक्षा में आए।



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